Effect of Particle Crushing on Shear Strength and Dilation Characteristics of Sand-Gravel Mixtures

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Abstract: There are limitations in experimental studies on sand-gravel mixtures due to the small size of testing specimens. Due to this problem, many researchers have worked on prediction of the shear strength of mixture by testing the sandy fraction of soil alone and developed empirical relationships. Most of the previous relationships have been determined for low surcharge pressures in which particle breakage does not affect the shear strength parameters. However, the particle breakage affects the relationships in higher confinements. At the present study, the results of large scale direct shear tests on sand and sand-gravel mixtures was used to investigate the shear behavior and dilatancy characteristics in a wider range of surcharge pressures. The gravel content, relative density, surcharge pressure and gravel grain size were considered as variables in testing program. The relationships between shear strength characteristics of sand and sand-gravel mixtures were determined considering dilatation characteristics of the soil. In this regard, the minimum void ratio was found as a useful indirect index that relates uniquely to the critical state friction angle independent of soil gradation. The relations between critical state or peak friction angles of the mixture with minimum void ratio were determined as a function of surcharge pressure. The correlations could be useful for determination of the strength parameters of sand-gravel composites by testing sandy fraction of mixture.

Keywords: sand-gravel mixture, direct shear tests, dilation, friction angle, surcharge pressure, particle crushing effects, minimum void ratio.

1. Introduction

The laboratory studies on coarse grained granular soils posses a problem due to the small dimensions of test specimens. In most cases, the maximum grains size is limited to 10 mm and tests perform on finer fraction of the soil which is usually called scalping method. Also in some other cases, a parallel gradation curve is used as representative of the original soil named as parallel gradation method. However, in both methods a fraction of the soil should be ignored which can affect the mechanical properties. Another alternative is using the large scale triaxial or direct shear test which allows testing soils with larger particles and determination of the shear strength parameters. These tests have been used by several researchers to investigate the mechanical behavior of coarse grained soils.

Fragaszy et al. [1, 2] used large scale triaxial tests to investigate the effect of oversized particles on the overall density of sandy soil. They defined the concepts of near and far field density. The near field density was defined as the density of soil matrix adjacent to the oversized particles. Also the far filed density was defined as the density of the soil matrix at a distance further away from the oversized particles. It was showed that the static strength of the mixture is governed by the far filed density when the oversized particles are floated in finer matrix. They also reported that the increase in gravel content decreases the far field density of the soil and the shear strength of the mixture. This was later showed for the cyclic shear strength of sand-gravel composites by Evans and Zhou [3].

Many other researchers showed the increase in the static shear strength of sand-gravel composites in floating state by the increase in gravel content, for example Yagiz [4], Valejo [5] and Kokusho et al. [6].

One of the ways for determination of the shear strength of coarse grained soil is development of correlations between the mechanical parameters of the mixture and its finer fraction. This method has been concerned for sand-gravel mixtures by Simoni and Houlsby [7] according to the previous studies of Bolton [8, 9] on sandy soils. Bolton [8] investigated the strength and dilatancy characteristics of sands by collecting the...
extensive data of 17 sands in axisymmetric and plane strain tests in different relative densities and confinements. It was stated that the maximum friction angle of sandy soil ($\phi_{\text{max}}$) is a function of the critical state or constant volume friction angle ($\phi_{cv}$) which is itself a function of soil mineralogy, and the maximum dilation angle ($\psi_{\text{max}}$) that is dependent on the relative density and confining pressure as shown in equation (1):

$$\phi_{\text{max}} = \phi_{cv} + 0.8 \psi_{\text{max}}$$  \hspace{1cm} (1)

Bolton [8] defined a new index ($I_R$) in terms of relative density ($D_r$) in percents and effective confining level in failure ($p'$) in kPa and related it to equation (1) for plane strain condition by the following equations:

$$I_R = D_r (10 - \ln(p')) - 1$$  \hspace{1cm} (2)

$$\phi_{\text{max}} - \phi_{cv} = 0.8 \psi_{\text{max}} = 5I_R$$  \hspace{1cm} (3)

Twenty years later, Simoni and Houlsby [7] used this approach to present similar correlations for sand-gravel mixtures. They used the original method of Bolton [8] to relate the shear strength of mixture to associated values for finer fraction of the soil. In their research, the large scale direct shear apparatus was used to identify the shear strength of sandy soil mixed with gravel content up to 60 percents which was the threshold value for floating state of oversized particles. They modified the original index proposed by Bolton [8] for sand-gravel mixtures using the minimum void ratios of sand ($e_{\text{min,sand}}$) and mixture ($e_{\text{min,mixture}}$) as follows:

$$I_{R,\text{mixture}} = 5D_{r,\text{mixture}} - (1 - 4.3(e_{\text{min,sand}} - e_{\text{min,mixture}}))$$  \hspace{1cm} (4)

The modified index can be used in equation (3) to determine the shear strength of mixture. The direct shear tests reported by Simoni and Houlsby [7] were all conducted under a constant surcharge pressure of approximately 90 kPa which was less than the beginning limit of particle breakage effect. Bolton [8, 9] observed that the tendency for particles to crush under shear is not appreciable when the mean shear is lower than about 150 kPa, allowing dilation to be treated as a function only of relative density below this stress. However, the particle breakage occurs in higher surcharge pressures strongly affects the relation between shear strength and dilatancy of sand or sand-gravel mixtures. In this case, the amount of surcharge pressure should also be considered besides the relative density in the relationships.

In this paper, the results of large scale direct shear tests are used to consider the effects of surcharge pressure and relevant particle crushing on shear strength and dilatancy relationships of sand-gravel mixture. The tests are performed under surcharge values more than 150 kPa which are beyond of the beginning limit of particle breakage. The method proposed by Simoni and Houlsby [7] and Bolton [8, 9] are used to analyze the data and the mentioned effects are encountered in formulations.

2. Experimental studies

The uniform, clean, quartz beach sand with sub-rounded to sub-angular grains from the shores of Caspian Sea was used as the base material in the tests. Rounded gravel grains with maximum grain sizes of 12.5 and 25.4 mm were used as oversized particles for mixing with base soil. As a result two sand-gravel mixtures with different gravel grain sizes were considered in testing program. The gravel grains were mixed with the base sand in three weight percentages of 20, 40 and 60. Figure 1 shows the gradation curves of the sand-gravel mixtures with maximum gravel grain sizes of 12.5 and 25.4 mm. The maximum and minimum void ratios were determined for different mixtures according to ASTM-D4253 and ASTM-D4254 and the specific gravity values for sand and sand-gravel mixtures have been determined according to ASTM-D854. Table 1 shows the main properties of soils used in present study.

Figure 2 shows the variation of maximum void ratio ($e_{\text{max,mixture}}$) and minimum void ratio ($e_{\text{min,mixture}}$) with gravel content for the mixtures contain gravel grains with maximum size of 12.5 and 25.4 mm. The figure demonstrates that the threshold gravel content for the floating state is
about 60 percents. The oversized particles are floated in sandy matrix without considerable contact in gravel contents less than 60 percents. Increasing the gravel content from this value, results in a firm contact of gravel particles together and non-floated state for the mixture.

3. Direct shear tests

Consolidated-drained large scale direct shear tests were conducted on completely dry samples. The size of shear box apparatus was about 300*300*170 mm. Table 2 summarizes the variables considered in experimental study. The vertical surcharge pressures of 150, 300 and 450 kPa were used and the relative densities of samples which were prepared by dry compaction method in three layers were 35, 60 and 85 percents.

Figures 3 and 4 show typical results of direct shear tests on sand-gravel mixtures with maximum gravel grain size of 12.5 mm for two gravel contents of 20 and 60 in relative densities of 35, 60 and 85 percents under a surcharge pressure of 450 kPa. The figures include variations of normalized shear stress, dilation and dilation rate with shear displacement. As the figures show, the peak shear strength increases with increase in relative density which is mainly attributed to an increase in contribution of dilatancy to shear resistance.
The angle of dilation (\( \psi \)) has been computed using the shear displacements (\( u \)) and vertical displacements (\( v \)) by the following equation:

\[
\tan \psi = \frac{dv}{du} \tag{5}
\]

The maximum rate of dilation was found to coincide, as expected, with the peak value of the normalized shear strength as shown in figures. It was used to determine the maximum angle of dilation (\( \psi_{\text{max}} \)).

4. Analysis of results

In order to investigate the shear strength and dilatancy characteristics for sand-gravel mixtures considered in this study, and considering the effect of different variables on them, some parameters like the constant volume or critical state friction angle besides dilatancy and peak friction angle should be determined. The method used for estimation of each mentioned parameter is presented in the following sections.

4.1 Constant volume or critical state friction angle (\( \phi_c \))

The critical state friction angle shows the shear strength of soil mass that is continuously deforming at constant volume, constant normal effective stress, constant shear stress and constant rate of shear strain as defined by Poulos et al. [10]. It is an important parameter both in design and interpretation of shear test results. Haeri and Hamidi [11] showed that the critical state of gravelly sands occurs in axial strains of 20 percents and more in triaxial condition. At the present study, the direct shear tests continued up to shear strains about 15 percents. As a result, there is some difference between the friction angle determined using ultimate stresses and the
critical state one. In this regard, another method is used for determination of the critical state friction angle of sand-gravel mixtures based on the results of tests in different densities.

In this method, the peak friction angle values of sand-gravel mixtures determined in different densities (\( \phi_{\text{max}} \)) are plotted against maximum dilatancy angles (\( \psi_{\text{max}} \)) and the best fit line is drawn, giving \( \phi_{c} \) as the shearing resistance of a sample which would exhibit zero dilatancy at failure. The method requires at least two shear tests at different densities. Figure 5 shows the application of this method for sand mixed with 20, 40 and 60 percents of gravel grains with maximum grain size of 12.5 mm under a surcharge pressure of 300 kPa.

Figure 6 shows the variation of the critical state friction angle with gravel content for mixtures considered in present study. The figure shows different relations for mixtures due to the difference in gravel grains size. The critical state friction angle increases with gravel size and gravel content. Also comparison of the corresponding values in different surcharge pressures shows that the critical state friction angle decreases with the increase in surcharge pressure. This can be related to the particle crushing effects and the associated change in soil gradation.

Using the same data, the variation of the critical state friction angle (\( \phi_{c} \)) with the minimum void ratio (\( e_{\text{min,mixture}} \)) is plotted in figure 7 for different mixtures. As the figure shows, with some scatter, critical state friction angle can uniquely be correlated to minimum void ratio for mixtures with different gravel content and grain size. This is in agreement with the results of Simoni and Houlshby [7] that introduced the minimum void ratio as a useful index that relates uniquely to the critical state friction angle for mixtures with different gradation. This characteristic is mainly used in empirical relationships of the present study to relate the strength and dilatancy characteristics of the sand and sand-gravel mixtures.

4.2 Peak friction and dilatancy angle (\( \phi_{\text{max}}, \psi_{\text{max}} \))

The equation (1) proposed by Bolton [8] for sandy soils can not be fitted to the experimental results on sand-gravel mixtures. Also the...
modification proposed by Simoni and Houlsby [7] did not include the effects of particle breakage due to the increase in surcharge pressure. In order to consider these effects, a step by step calculation is performed which is illustrated in this section.

At the first step, the relation between shear strength and dilatancy with relative density of the soil are considered. These are shown in figures 8 and 9 in terms of $\varphi_{\text{max}} - \varphi_r : D_r$, and $\psi_{\text{max}} : D_r$, respectively. In each curve, the variation with surcharge pressure is presented to show the effect of resulted grain crushing on the relation. The results of tests on each mixture are generally consistent in both spaces showing an increase in $\varphi_{\text{max}} - \varphi_r$, and reduction in maximum dilatancy angle ($\psi_{\text{max}}$) with surcharge pressure. The scatter is mainly attributed to the consideration of results of tests on mixtures with different gravel content in the same plot.

At the second step, by eliminating the relative density ($D_r$) between two relationships of figures 8 and 9, the plot of $\varphi_{\text{max}} - \varphi_r : \psi_{\text{max}}$ is determined and is shown in figure 10 for different mixtures. This plot shows the form of Bolton's [8] equation for sand-gravel mixtures as the relation between shear strength and dilatancy parameters. Using linear regression passing through the origin and described by the following equation, a progressive increase in $b$ value with surcharge pressure is observed.

$$\varphi_{\text{max}} - \varphi_r = b \psi_{\text{max}}$$  \hspace{1cm} (6)

Figure 11 shows the variation of $b$ coefficient with gravel content for different surcharge pressures. It should be noted that the data for two mixtures are considered together on the same plot in these figures. Figure 11 implies a reduction in proportion coefficient $b$ with gravel content, from original value of 0.8 in Bolton's [8] equation for pure sand, which is also shown in this figure. It should be highlighted that although the contribution of the dilation on the shear strength decreases with the increase in gravel fraction, the shear strength of sand-gravel mixture usually increases by addition of the gravel content [7, 12, 13 and 14]. This can be related to the corresponding increase of the constant volume friction angle ($\varphi_{cv}$) and maximum angle of dilation ($\psi_{\text{max}}$) with increase in gravel content and their contribution in shear strength of mixture.

Fig.8. Variation of $\varphi_{\text{max}} - \varphi_r$ with relative density for different surcharge pressures

Fig.9 Variation of maximum dilatancy ($\psi_{\text{max}}$) with relative density for different surcharge pressures
Figure 12 shows the variation of maximum friction angle ($\phi_{\text{max}}$) with relative density for different mixtures tested under two surcharge pressures of 300 and 450 kPa. As the figure shows, the maximum friction angle increases with gravel content and relative density. However, it decreases with increase in surcharge pressure. This reduction can be related mainly to the particle crushing effects take place under higher surcharge pressures. It can be concluded that the overall trend in $\phi_{\text{max}}$ results from a combination of changes in surcharge pressure besides $\phi_{\text{cr}}$, $\psi_{\text{max}}$ and $(\phi_{\text{max}} - \phi_{\text{cr}})/\psi_{\text{max}}$ ratio ($b$).

Finally figure 13 shows failure envelopes of sand-gravel mixture with maximum gravel grain size of 25.4 mm in different relative densities. As figure shows the envelope moves up with increase in relative density. However, its slope decreases as surcharge pressure increases. The slope of failure envelope is an indicator of maximum friction angle. As a result it can be concluded that increase in surcharge pressure slightly decreases the maximum friction angle value. This phenomenon can directly be related to particle crushing and its major effect on shear strength characteristics of sand-gravel mixtures.

5. Empirical equations

In this part, simple empirical equations are sought to describe the strength characteristics of the sand when mixed with gravel grains. Such equations may be applicable to other similar materials. Since shear tests on coarse granular soils are difficult, expressions are sought which could be useful for practical purposes, i.e. relations based on easily measurable parameters.
As indicated in figure 7, there is a unique relationship between minimum void ratio \((e_{\min,\text{mix}})\) and critical state or constant volume friction angle \(\phi_{\text{cv, mix}}\) for mixtures with different gradations. By considering the difference between minimum void ratios of sand-gravel mixtures and pure sandy soil \((\Delta e_{\min})\), it is possible to obtain a unique relationship describing the variation for mixtures with different gradations.

This is performed in figure 14 for mixtures tested under different surcharge pressures. As the figure shows, the relation is independent of the gravel content and grain size and is unique for different gradations. However, the surcharge pressure affects the relationship between critical state friction angle and minimum void ratio of the soil.

Using a simple linear regression to data presented in the figures, the following equation can be determined which illustrates the variations of the critical state or constant volume friction angle of sand-gravel mixture \(\phi_{\text{cv, mix}}\) based on the corresponding value for sandy soil \(\phi_{\text{cv, sand}}\). Different and more complex regression types can be used to receive a better fit, but is hardly justified by the data.

\[
\phi_{\text{cv, mix}} - \phi_{\text{cv, sand}} = I_s (e_{\min,\text{sand}} - e_{\min,\text{mix}})
\]  

In this equation, \(I_s\) is a function of surcharge pressure which includes relevant effects of particle crushing on the relationship and is defined as follows for different mixtures:

\[
I_s = \sigma_v^{0.65}
\]  

The value of surcharge pressure \(\sigma_v\) should be considered in kPa at this equation. The equation proposed by Simoni and Houlsby [7] that

\[
\frac{\phi_{\text{cv, mix}} - \phi_{\text{cv, sand}}}{\phi_{\text{cv, mix}} - \phi_{\text{cv, sand}}_{\min,\text{mix}}}
\]  

Fig.13 Failure envelope of sand-gravel mixtures in different relative densities

Fig.14 Empirical equation describing the variation of constant volume friction angle of different mixtures with minimum void ratio
corresponds to surcharge pressure of 90 kPa is also included in this figure. The amount of particle crushing increases with increase in surcharge pressure. This shows direct relation of these two parameters together. Indeed the effects of particle crushing are considered by surcharge pressure value in suggested empirical equation.

To describe the complex interactions ruling the dilatancy contribution to the peak friction angle ($\varphi_{\text{max}} - \varphi_{v}$), involving peak dilatancy angle ($\psi_{\text{max}}$), relative density ($D_r$) and surcharge pressure ($\sigma_v$), the Bolton's [8] equation is considered again as starting point and is modified as required. Equation (3) effectively relates three interdependent variables $\varphi_{\text{max}} - \varphi_{v}$, $\psi_{\text{max}}$ and $D_r$. There is different ways to fit the experimental data to a correlation. The procedure adopted here is using simple linear regression applied to the data pertaining to any particular mixture in each of the different planes, $(\varphi_{\text{max}} - \varphi_{v}, \psi_{\text{max}})$, $(D_r, \psi_{\text{max}})$ and $(\varphi_{\text{max}} - \varphi_{v}, D_r)$. The coefficients obtained (slope and/or intercepts) for each mixture have been plotted against difference between minimum void ratios ($e_{\text{min,mix}} - e_{\text{min,mand}}$) and then regression analysis used to find an appropriate expression for the variations. The variation with surcharge pressure ($\sigma_v$) is also considered on each plane to determine the changes in slope and intercept of the regression fitting lines with these parameters.

In this regard, Bolton's [8] was modified by substituting for the constant 0.8 an expression describing the variation of the $b$ coefficient in equation 6 as follows:

$$\varphi_{\text{min,mand}} - \varphi_{v,\text{criterion}} = 0.8 - (0.003 \sigma_v) (e_{\text{min,mand}} - e_{\text{min,mix}}) \psi_{\text{max}}$$

(9)

In this equation, $\sigma_v$ is the surcharge pressure in kPa. The procedure follows the earlier observation that the minimum void ratio provides a useful index that related well to the critical state friction angle independent of soil gradation.

Figure 15 shows the trend of coefficient $b$ showed in equation 9 with $e_{\text{min,mand}} - e_{\text{min,mix}}$ for different relative densities and surcharge pressures. Again, data for mixtures with different gradations are used in this figure. As shown in the figure, the $b$ value increases with increase in surcharge pressure which shows the more contribution of dilation in shear strength of mixture. The fitting lines have an intercept of 0.8; hence equation (9) reduces to Bolton's [8] original equation when no gravel is present.

The relative density and surcharge has to be related simultaneously to $\varphi_{\text{max}} - \varphi_{v}$ and $\psi_{\text{max}}$. It is expected that equation (3) proposed by Bolton [8] changes to the following form to account for the surcharge pressure and relevant effects of particle breakage:

$$(\varphi_{\text{max}} - \varphi_{v})_{\text{mix}} = b \psi_{\text{max}} = 5 I_{R,S}$$

(10)

$I_{R,S}$ is the modified value of $I_r$ suggested by Simoni and Houlsby [7] in equation 4 for surcharge pressure and relevant to the slopes of linear regression in planes $(\varphi_{\text{max}} - \varphi_{v}, I_{R,S})$ and $(\psi_{\text{max}}, I_{R,S})$. This relationship is obtained by fitting the experimental data points in the mentioned.
planes resulting in the following equation:

\[ f_{k5} = 5D_{\text{max},\text{mix}} - \left[ 1 + (4.5 - 0.06 \sigma) + \left( \epsilon_{\text{mix},\text{max}} - \epsilon_{\text{mix},\text{min}} \right) \right] \]  

(11)

In this equation, \( \sigma \) is the surcharge pressure in kPa. This completes the relationships between considered variables, since the same term appears on the left hand side of equations 9 and 10. The empirical equations 7 to 11 are valid in a wide range of surcharge pressures in which particle crushing occurs. They allow prediction of the peak friction angle of sand-gravel mixture in which the gravel fraction is low enough to exist in a non-floating state; i.e. below the fraction giving the lowest \( \epsilon_{\text{min},\text{mix}} \).

To make the predictions, simple tests are required which can be performed using simple apparatus. They include shear testing of sand as the reference material to determine \( \psi_{\text{v,rand}}^{\text{min}} \) and measuring the minimum void ratio of the matrix soil \( (\epsilon_{\text{min},\text{rand}}) \) and sand-gravel mixture \( (\epsilon_{\text{min},\text{mix}}) \). As a result no shear test is performed on sand-gravel mixtures and the oversized particles are no more problematic. It should be noted that if, alternatively, the predicted \( \phi_{\text{max}} \) is calculated by means of equations 7 and 9, a better fit to the strength can be achieved, but this procedure is not practical, since the maximum dilation at failure \( (\psi_{\text{max}}) \) requires an experimental measurement. As a result, equations 7 and 10 would be more practical for determination of the maximum friction angle.

Figure 16 compares the results of empirical equations 7, 10 and 11, in terms of \( \phi_{\text{max}} \) with experimentally measured values for tests in different relative densities under different surcharge pressures. The results show a good agreement and are mainly around the 1:1 line with low error less than about 3°. According to the figure, the empirical relations can predict the shear strength of mixtures in a wide range of densities and surcharge pressures.

6. Summary and conclusion

The sandy soils including gravel size particles are quite common in nature. Furthermore, the use of natural or artificial fills containing coarse grains is increasing. As a result, the investigation of the shear and deformation characteristics of such material is important for practical applications such as embankments and road construction. However, the specimen size limitation becomes a problem in conducting standard shear tests and determination of the shear strength.

In the present study, the results of large scale direct shear tests are used to investigate the shear strength and dilatancy characteristics of sand-gravel mixtures. Two different gravel grains with maximum grain size of 12.5 and 25.4 mm were added in three different contents of 20, 40 and 60 weight percents to sandy soil, and the mixtures were tested. The direct shear tests were conducted under three different surcharge pressures of 150, 300 and 450 kPa to consider the effects of particle crushing on the results.

The addition of coarse particles to sand causes several effects to the structure of resulting material. It reduces the minimum void ratio of the mixtures up to a gravel fraction about 60%, which is the threshold limit of floating state for oversized particles in sandy matrix. The minimum void ratio can be considered as a useful index of gradation to correlate the constant volume friction angle of sand-gravel mixture to that of sandy soil. Using the results of the experimental study, empirical relationships adopted focusing on the maximum and critical state friction angles of the mixture and consideration of the surcharge pressure and relative density. Based on these correlations, the shear strength characteristics of the mixture can

![Figure 16](https://example.com/f16.jpg)
be determined based on the corresponding values for sandy soil and without performing shear strength tests on sandy soil containing oversized particles. The effects of surcharge pressure and associated particle crushing are also included in these relations and comparison with experimental results approved its accuracy.

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References


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