Fluid-structure interaction in concrete cylindrical tanks under harmonic excitations

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Abstract: *Large capacity cylindrical tanks are used to store a variety of liquids. Their Satisfactory performance during earthquake is crucial for modern facilities. In present paper, the behavior of cylindrical concrete tanks under harmonic excitation is studied using the finite element method. Liquid sloshing, liquid viscosity and wall flexibility are considered and additionally excitation frequency, liquid level and tank geometry is investigated. The results show a value for wall thickness to tank diameter ratio which may be used as a guide in the consideration of wall flexibility effects.*

Keywords: *cylindrical liquid storage tank, wall flexibility, liquid viscosity, harmonic excitation, numerical analysis*

Introduction

One of the essential structures in industrial facilities and lifeline is liquid storage tanks. The structure is being utilized in a variety of configurations such as elevated, ground supported, or partly buried. Due to their wide use and their vulnerability to earthquakes, expectedly a substantial number of incidents of damage to tanks has occurred during past seismic events [1,2,3,4].

The evaluation of dynamic behavior of cylindrical tanks has been the aim of several investigations [5,6,7,8]. Most of earlier studies have been concerned with the antisymmetrical vibrations due to lateral excitations and ignored the axisymmetrical response due to vertical component of ground acceleration. Later, attention has been conducted to the latter and led to several interesting conclusions. For example, in 1979, Marchaj [9] considered a horizontal strip of the tank wall and studied its behavior under vertical acceleration by translating the dynamically applied forces into equivalent static forces and equating the work done by such forces to the stored elastic energy of the shell. He attributed the failure of metallic tanks during past earthquakes to the lack of consideration of vertical acceleration in their design. A second study was carried out, in 1981, at Rice University by Kumar [10] in which he considered only the radial motion of partially-filled tanks and neglected the effect of axial deformations. In 1985, the dynamic characteristics of cylindrical tanks have been evaluated numerically by Haroun and Tayel [11]. The natural frequencies and the mode shapes of both empty and partially-filled tanks, as well as the corresponding hydrodynamic pressure and stress distributions were presented.

In the same year, Haroun and Tayel, also, presented an analytical method for the computation of the axisymmetrical dynamic characteristics of partially-filled cylindrical tanks [12]. The liquid was assumed to be inviscid and incompressible. The tank shell was assumed to be of uniform thickness and its material to be linearly elastic. Under these assumptions, two coupled partial differential equations govern the vibrations of the shell. Since the tank is partially-filled with liquid, two different solutions were obtained for the lower (wet) and upper (dry) portions of the shell. A system of linear homogenous algebraic equations was obtained by solving the boundary conditions at the bottom and top of the tank and the compatibility equations at the junction of the wet and dry parts of the shell. The dynamic characteristics were compared with those obtained from numerical solution.

Analytical and numerical studies of the liquid shell system [1, 7, 13, 14] and vibration tests of reduced and full scale tanks [1, 15] indicated that wall flexibility may have significant effect on the dynamic behavior of fluid containers. The hydrodynamic pressure exerted on the walls of such deformable tanks can be calculated by superposition of the impulsive and the convective pressure components to the short period component contributed by the flexible wall. Being easy to construct and efficient in resisting the hydrostatic pressure, in practice, cylindrical tanks are widely used for the vessel.

Traditionally, the fuel slosh analysis was done with a simple hand calculator [16]. However, this method is limited to gentle sloshing (i.e. no slosh contacting the tank lid, rigid tank walls, rigid baffles without hole) [17]. Thus to predict accurately the sloshing phenomenon, numerical techniques, which correctly model the free surface, were required. These numerical solution algorithms have been developed for both Lagrangian and Eulerian formulations. In general, the choice of formulation depends on the characteristics of the specific problem. For example, when the free surface undergoes large deformations, the Lagrangian formulation is not well suited and the Eulerian formulation is chosen. In 2003, Aquelet et al. [18], independent of the employed formulation, reviewed the numerical approaches that have been used to track the free surface and, in particular, discussed a new Arbitrary Lagrangian Eulerian (ALE) formulation, with an interface tracking method, for the fluid mesh to keep its integrity during the motion of the tank. Their analysis capabilities were validated against theoretical procedures using potential flow for calculating fuel slosh frequency. In 2004, Utsumi [19] developed a mechanical model for low gravity sloshing in an axisymmetric tank. Using spherical coordinates, he determined the characteristic functions for an arbitrary axisymmetric tank analytically. Parameters of his mechanical model were determined such that the frequency responses of the resultant force and moment to lateral excitation coincide with those of the actual sloshing system.

In the present paper, the behavior of cylindrical concrete tanks under harmonic excitation is studied using the finite element method. The shell and internal liquid are completely meshed by convenient elements implemented in ANSYS 5.4 software. Analyses were performed to investigate the effect of liquid viscosity, wall flexibility, excitation frequency, liquid level and tank geometry on liquid sloshing.

Numerical Studies

Models definition

In order to determine the influence of the wall flexibility, fluid viscosity, filling level and base excitation characteristic on the sloshing response in cylindrical liquid storage tanks, a numerical study has been performed. Above ground cylindrical tanks resting on rigid base, with concrete shell and filled to different levels are considered. The finite element based software, ANSYS 5.4, is used.

The finite element mesh for concrete shell consists of two dimensional 4 nodes element type SHELL43 implemented in the software while the 3 dimensional 8 nodes element FLUID80, is used for discretization of the fluid inside. Thus a nonlinear behavior may be considered for the shell's material. The Drucker-Prager failure criterion formulated as $f(\xi,\rho)=\sqrt{6}\alpha\xi+\rho-\sqrt{2}k=0$, in which a and k are material constants related to the cohesion c and the angle of internal friction F, is considered for concrete. For the concrete material with a compressive strength of 25 MPa and a tensile strength of $0.4f_c^{2/3}$, the two constants a and k are defined as follow:

$$\alpha=\frac{2\sin\Phi}{\sqrt{3}(3-\sin\Phi)},\qquad k=\frac{6c\cos\Phi}{\sqrt{3}(3-\sin\Phi)}$$

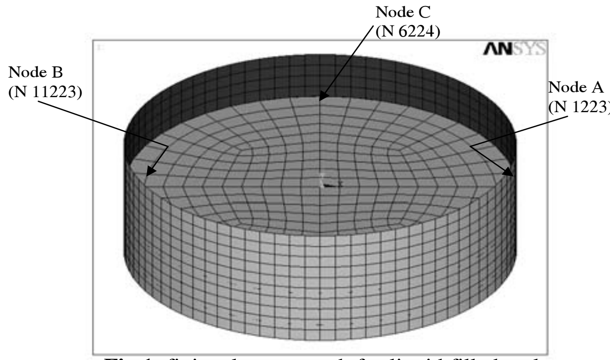


Fig.1 finite element mesh for liquid filled tank

Table 1 Geometric specification of models

Diameter (m)	Height (m)	Wall thick. (mm)	Fluid level, h/H (%)
10 20 30 40 50 60	10	300 400 505 600 800 1000 1200	10 30 50 60 70 90

Table 2 Mechanical properties of materials for models

	f_c ($N.mm^{-2}$)	E_c ($N.mm^{-2}$)	ρ (kg/m3)	ν	Bulk mod. ($N.mm^{-2}$)
concrete	25	25000	2400	0.15	
water			1000		2060

Table 3 Base excitation specifications

	1	2	3	4	5	6	7	8
A (mm)	9.3	9.3	9.3	9.3	347.9	151.5	16.9	4.3
ω (rd/s)	4.761	31.41	62.83	125.66	3.1416	4.761	14.25	28.274

$$\sin \Phi = \frac{f_c - f_t}{f_c + f_t}, \quad c = \frac{f_c(1 - \sin \Phi)}{2 \cos \Phi}$$

f_c=25*MPa*, f_t=3.42*MPa*, c=4.62*MPa*, Φ=49.4°, α =0.391, k=4.648*MPa*

A typical mesh is shown in figure 1. Joints located at the interface of the two meshes are constrained in such a way to prevent the fluid displacement across concrete shell and to allow its vertical sliding relative to the wall.

A variety of models, specified in table 1, are studied and the maximum vertical displacement of nodes, located on the fluid free surface, is obtained. Mechanical properties of concrete and the inside fluid are considered as defined in Table 2. Each model is analyzed under different horizontal harmonic excitation, specified by Table 3, at its base. As it is obvious, the first four excitations have equal amplitude and different frequencies. Amplitudes and frequencies of the other five are such arranged to induce a maximum acceleration of 0.35g (g being the acceleration of gravity). For all models, responses are observed during a five cycle forced vibration and a free vibration phase which lasts three times the excitation period.

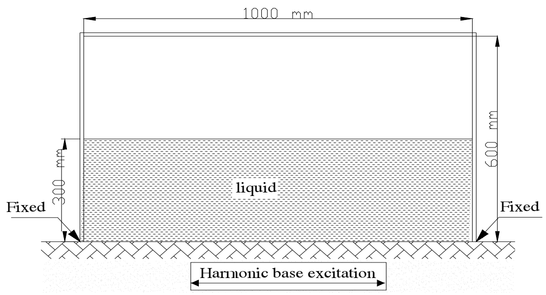


Fig.2 sketch of the sample tank

Model verification

Before presenting numerical results obtained from model analysis, the considered model should somehow be verified. Therefore, reference is made to a test result presented by Ogawa [20], which was used to validate his method of sloshing analysis. It consists of a rectangular tank 600 mm height and 1000 mm wide which is partially filled with water up to its mid height. The tank, assumed rigid, is excited with a horizontally harmonic excitation at its base with amplitude of 9.3 mm and a frequency of 4.761 rd/s. Figure 2, shows a sketch of this tank.

In the analysis, the water is modeled with Fluid80 elements and the boundaries are assumed constrained so to simulate the rigid walls. Material properties are chosen as listed in table 4. Figures 3a and 3b illustrate the water elevation at the left wall as presented in the mentioned paper and from this recent analysis using ANSYS, respectively. A comparison of them shows good agreement of the results obtained from finite element analysis and experiments. Therefore, the results demonstrate the validity of the model.

Another attempt for checking the validity of the numerical model is performed based on experimental results presented in Kruntcheva [21]. The free vibration characteristics of thin circular cylindrical shell were recorded for different filling ratios of water. The shell were made of copper and had the following geometric and material properties: height,177mm; outer diameter,88.5 mm; wall thickness,1.8 mm; Young's modulus, 104 GPa; Poisson 's ratio,0.35; and mass density, 8960 kg/m^3. Water were used

Table 4 material properties of the verification sample

Density of water	9.98 e -1 g/cm^3
Viscosity of water	1.01 e -7 N.s /cm^2
Density of air	1.23 e -3 g/cm^3
Viscosity of air	1.81 e -9 N.s /cm^2

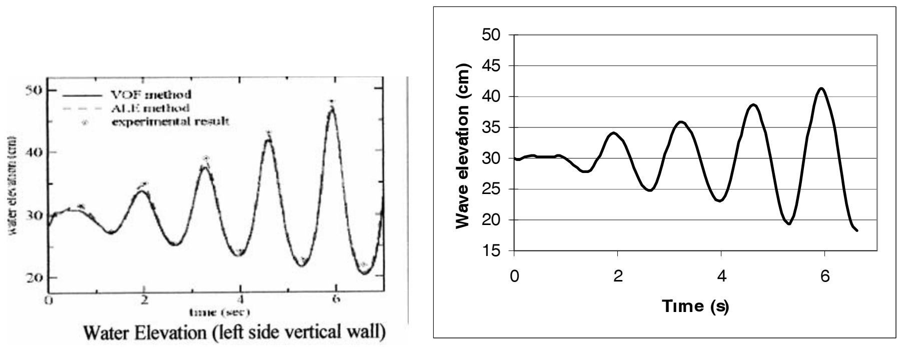
as contained fluid of mass density 1000 kg/m^3 and bulk modulus K= 2.3 GPa. For comparison purposes, the same geometry and material properties of the test tanks and the contained fluid are assumed for the finite element models. Some of the experimentally identified natural frequencies of radial shell modes are compared with the values obtained from the finite element models in figure 4. From this figure it can be seen that, in general, the finite element model produced results in good agreement with the experimental ones. The discrepancies in these results may be explained by differences in the boundary conditions for the finite element models and for the experimental setup. So, once again the validity of the numerical model is checked up.

Numerical Results

Various numerical results are obtained from the analysis of different tank models, but those presented here after, are based on the maximum free surface displacement of the inside liquid. A set of preliminary studies about the location of the maximum surface wave height, has resulted in liquid surface profiles, such as those plotted in figure 5. As it is obvious, the maximum displacement will be expected to occur at the tank wall vicinity. So, in the next sections, all results are drawn out with respect to the displacement of the water surface adjacent to the tank walls.

Effect of wall flexibility

For studying the effect of wall flexibility on liquid sloshing inside concrete cylindrical tanks, finite elements models with various diameter and wall thickness are built and analyzed under different harmonic base excitations. For these models, the wall thickness are selected in the



a b

Fig.3 free surface displacement at the left side vertical wall a)experimental results; b) numerical results

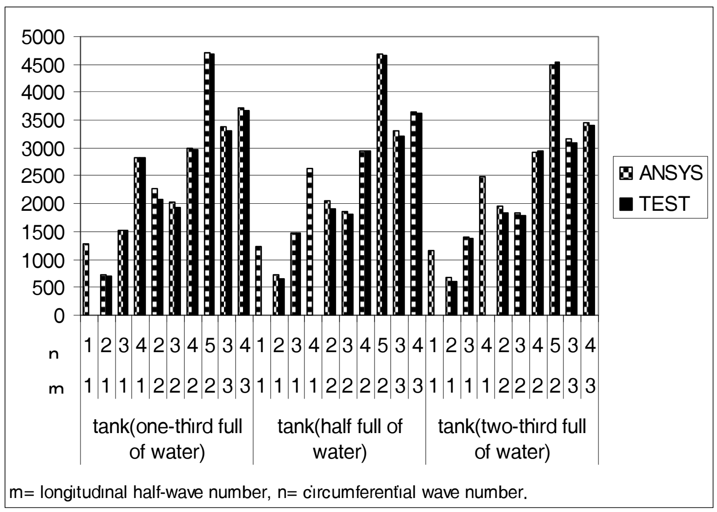


Fig.4 Radial structural frequencies [f (Hz)] obtained from numerical (ANSYS results) and Experimental (TEST results) analyses

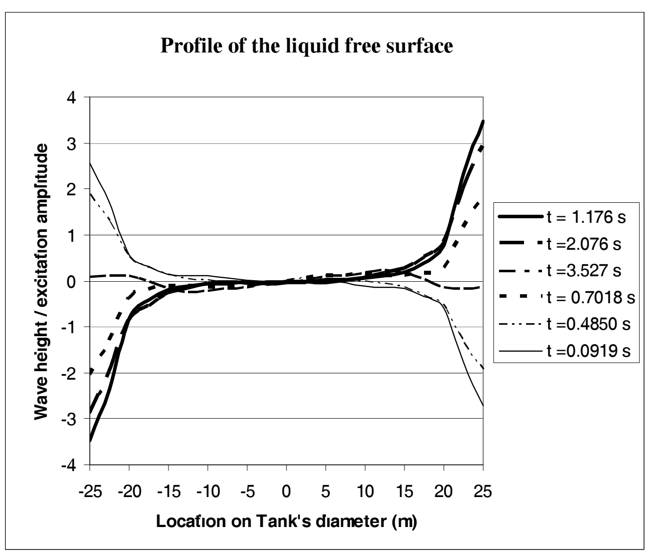
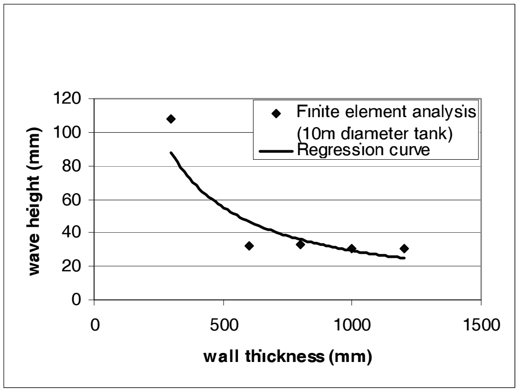
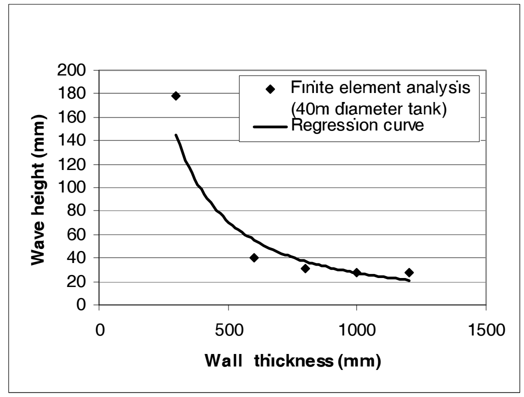


Fig.5 The liquid profile across tank diameter



(a)



(b)

Fig.6 Influence of shell flexibility of cylindrical tanks on the wave height a) 40m diameter, b) 10m diameter

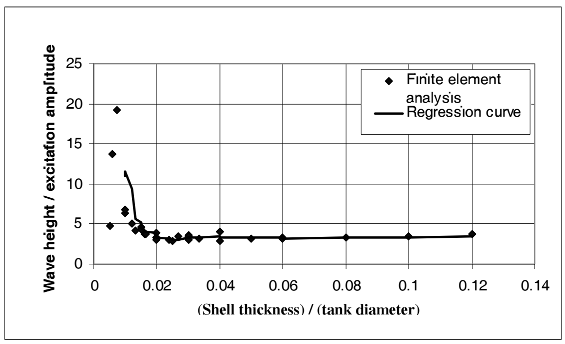


Fig.7 wall flexibility effect on surface sloshing

range of 300 (minimum required for erection) to 1200 mm and their diameter vary from 10 to 60m, as listed in table 1. As mentioned earlier, their heights are all similar and equal to 10m. The free surface level of the contained liquid is considered 6m above the tank base. As it will be explained later, this liquid level corresponds to the sloshing response with maximum amplitude. Figure 6 illustrates the variation of the wave amplitude with wall thickness for 10m and 40m diameter cylindrical tanks. Similar curves are plotted for other diameters. It could be seen that a thickness increase, which reduces the flexibility of the wall, results in a decrease of wave height and its effect is more significant on smaller thickness.

Based on the obtained results from these cases, a curve is drawn relating two dimensionless quantities, the ratio of wave amplitude to base excitation amplitude and the thickness to diameter ratio. The curve, depicted in fig. 7, shows that, as thickness to diameter ratios remain greater than 0.02, the thickness change has almost no influence on the wave amplitude induced by the base excitation. On the other side, if the mentioned ratio drops under 0.02, the surface wave amplitude increases drastically. Therefore, wall flexibility has significant effects on liquid sloshing in concrete cylindrical tanks which have a wall thickness less than 2% of their diameter.

Effect of fluid level

To investigate the role of the in-filled fluid on surface sloshing in cylindrical tanks, different models with various diameter are analyzed under similar harmonic base excitation, with 9.3 mm amplitude and a circular frequency of 4.761 rd/s. For each of them, various level of stored fluid is considered. Figure 8 illustrates the amplitude of sloshing as a function of the fluid depth for the studied cases. Obviously, maximum sloshing occurs when the tank is partly filled to 65% of its

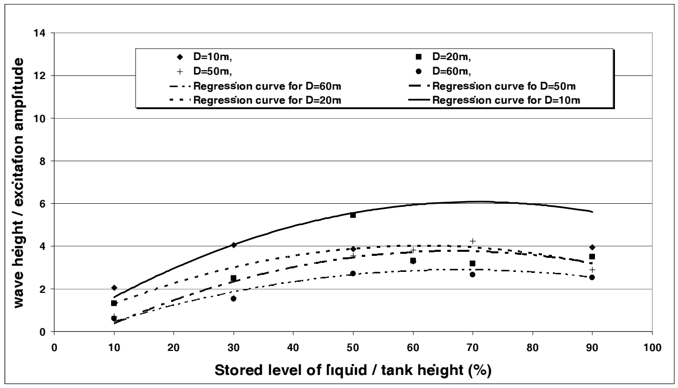


Fig.8 Storage effect on surface sloshing

height. It should be noted that for lower levels, the plotted curves have positive slope while for higher fluid level, the rate of amplitude change is near zero on the negative side. In fact, this is not unexpected since after some level, the effect of gravity forces may partly overcome the sloshing causes.

Effect of excitation frequency

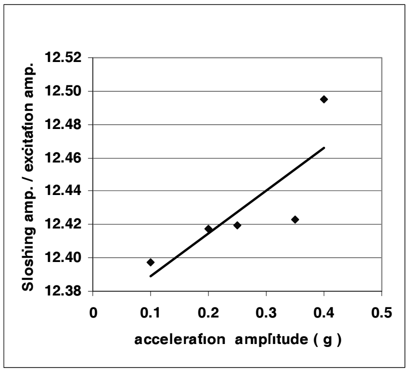
Results obtained from analysis of tanks with various diameters under different base excitations show that the acceleration amplitude affects the surface sloshing of the inside liquid. However, the change in acceleration value could result from changes in displacement amplitude or frequencies. As long as the frequency of excitation remains constant, the change in amplitude will result in a linear variation of the sloshing amplitudes (fig. 9a). But, for a defined displacement amplitude, the change in frequency affects the sloshing differently (fig. 9b).

Considering the range of peak ground acceleration presented in seismic codes, and assuming a constant value of 151.5 mm as the horizontal displacement amplitude, the models are analyzed under harmonic base excitations having 0.405, 0.573, 0.64, 0.7577 and 0.81 Hz for frequency values, corresponding to acceleration amplitudes of 0.1g, 0.2g, 0.25g, 0.35g and 0.4g respectively. Results obtained for different tank diameters and various wall thicknesses show a nearly linear relation between the ratio of sloshing amplitude to excitation amplitude and a quantity defined as whe/D^2, which depends on circular frequency of base excitation w, height of liquid inside tank h, thickness e of tank wall and tank diameter D. figure 10 illustrates this relation for some of the models studied.

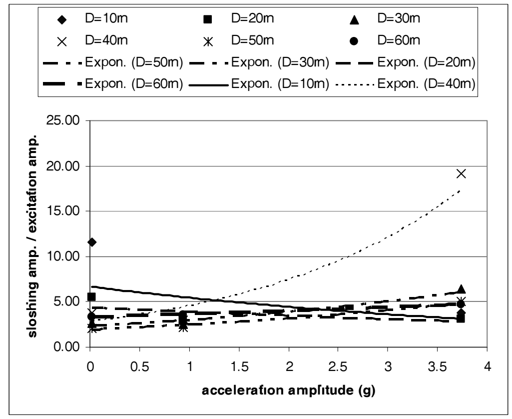
Further investigations of the overall results obtained yields a new relation, depicted in figure 11, which shows nearly constant sloshing amplitude equal to 3.6 times the excitation amplitude in cases where the quantity whe/D^2 remains below 0.03. This result may be helpful in the design and dimensioning of cylindrical concrete tanks with rigid base.

Effect of fluid viscosity

Fluid viscosity may have significant effects on surface sloshing in containments during base excitations. To investigate it, a set of analysis in which, fluid viscosity is also specified in the material property definition, is performed. Cylindrical tanks with 50m diameter, 600mm wall thickness and filled up to its 60% height of viscous fluid are considered. The obtained results show no significant influence of liquid viscosity on the sloshing amplitude (fig.12). However, a considerable damping effect should be noted in the propagation of surface waves as could be noted by figure 13. It should be mentioned that the viscosity effect is not recognizable as its value varies from 1.01x10-9 N.s/mm^2 (for water) to 7.2x10-9N.s/mm^2 (for crude oil). But for viscosity value about 7.2x10-4N.s/mm^2, the differences between surface profiles become apparent.

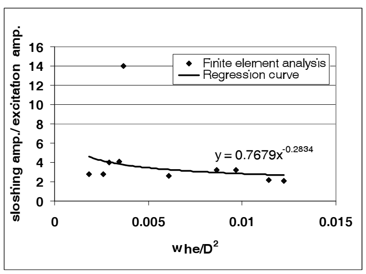


a

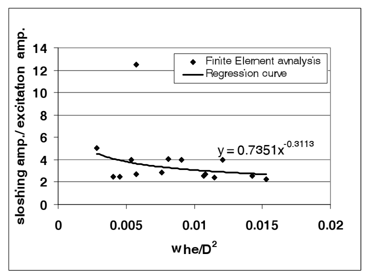


b

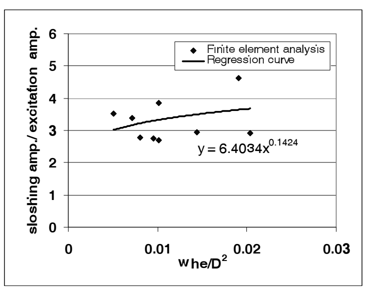
Fig.9 effect of base excitation changes on sloshing amplitude a) Amplitude; b) frequency



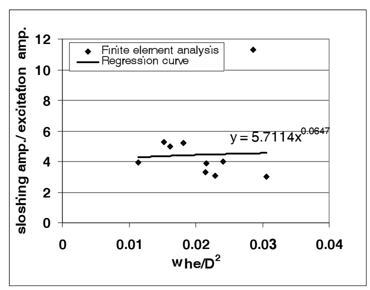
D=50m, h=6m, e=300,1000mm



D=30m, h=6m, e=300,600mm



D=40m, h=6m, e=300,600,800mm



D=20m, h=6m, e=300,400mm

Fig.10 Variation of relative sloshing amplitude with characteristic quantity whe/D^2

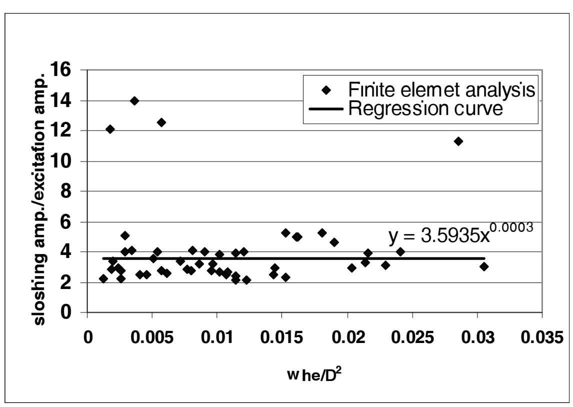


Fig.11 overall relation of relative sloshing amplitude with characteristic quantity whe/D^2

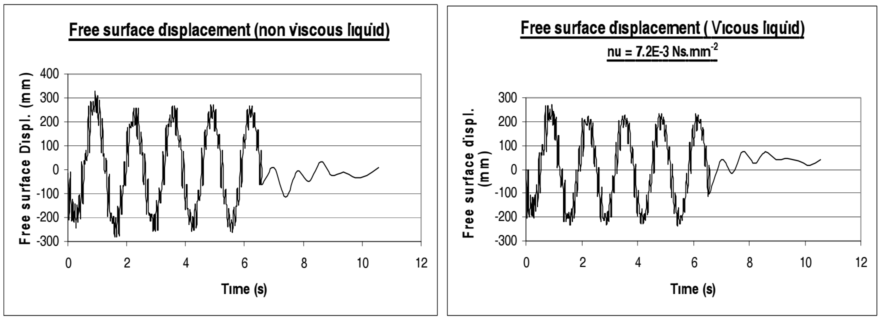


Fig.12 effect of liquid viscosity on sloshing amplitude

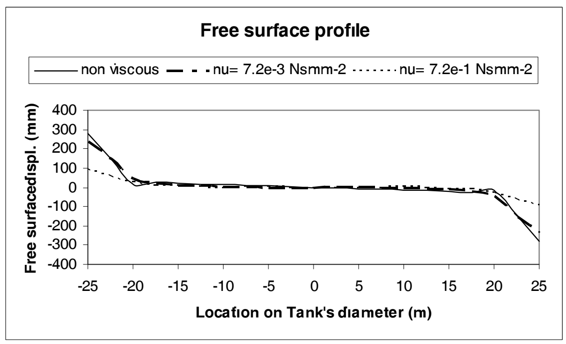


Fig.13 effect of liquid viscosity on sloshing profile

Conclusions

The study of concrete cylindrical liquid containments under harmonic base excitations yields to the following interesting results:

-.A shell thickness to diameter ratio of 0.02 is obtained as the limit above of which, wall flexibility do not affects the surface sloshing of the in-fill liquid.

- Furthermore, it has been found that the maximum amplification occurs when the storage tank is filled to its 60% height, which can be explained by the influence of gravity.

- Although the variation of displacement amplitude results in a linear variation of the free surface wave's height, varying the frequency could affect it significantly with no specific relation.

- The viscosity of the internal liquid has no valuable effect on the amplitude of the sloshing but might affects the free surface vibration shape by implying some damping, for the considered range of viscosity.

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