An Iterative Penalty Method for the Optimal Design of Pipe Networks

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Abstract: *This paper presents the application of an iterative penalty method for the design of water distribution pipe networks. The optimal design of pipe networks is first recasted into an unconstrained minimization problem via the use of the penalty method, which is then solved by a global mathematical optimization tool. The difficulty of using a trial and error procedure to select the proper value of the penalty parameter is overcome by an iterative use of the penalty parameter. The proposed method reduces the original problem with a priori unknown penalty parameter to a series of similar optimization problems with known and increasing value of the penalty parameters. An iterative use of the penalty parameter is then implemented and its effect on the final solution is investigated. Two different methods of fitting, namely least squares and cubic splines, are used to continuously approximate the discrete pipe cost function and are tested by numerical examples. The method is applied to some benchmark examples and the results are compared with other global optimization approaches. The proposed method is shown to be comparable to existing global optimization methods.*

Keywords: *Pipe Networks; Water distribution; Design; Optimization.*

1. Introduction

A typical water distribution network is a collection of pipes, reservoirs, pumps, and different kinds of valves connected to each other in order to meet specified demand at nodes. Basically, the optimal design of pipe networks is a multi-objective task involving hydraulics, reliability, and water quality. The multi-objective design of water distribution networks necessitates the development of proper algorithms before it can be effectively applied in practice. However, investigations indicate that much can be gained by handling the matter as a single objective problem of component design in which only the size optimization of different components such as pipes, tanks, etc with a least cost objective is considered.

Optimal design of pipe networks, when formulated mathematically, is clearly one of constrained minimization where the hydraulic requirements constitute some of the constraints of the problem. Various investigators have addressed this problem in a number of different ways during the past decades. Enumeration techniques, though reliable, suffer from limited practical application due to an extraordinary wide search space and consequently an enormous computational time required when applied to real-world size networks, where optimization is mostly needed (Yates et al. 1984). The class of constrained minimization, in particular decomposition methods, has become popular in recent years. These algorithms can be divided into two main groups, namely linear and nonlinear programming methods. The first linear decomposition method, called linear programming gradient (LPG) suggested by Alperovits and Shamir (1977) and later extended by Kessler and Shamir (1989), assumes the length of the pipes in each arc to be the decision variable for a given flow distribution and solves the linear programming problem so constructed. Modifications and comments to the original LPG method were made by Quindry et al. (1979), Saphir (1983), and Fujiwara et al. (1987). Quindry et al. (1981) presented an analogous approach to the LPG method in which the problem is solved for a given set of hydraulic heads. The first of the nonlinear decomposition

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models was devised by Mahjoub (1983) who started with an assumed initial link flow rate and then solved for link head losses. The obtained optimal link head losses were then fixed and the cost minimized for link flows. The whole procedure was repeated until no improvement could be attained. An improvement to the above method was suggested by Fujiwara and Khang (1990), who used the Lagrange multipliers of the optimal solution obtained in the first phase to modify the flow distribution, achieving a reduction in system cost before the second phase is started. Recently, genetic algorithms (GA) have been applied to the design of pipe networks (Simpson and Goldberg 1993; Simpson et al. 1994; Savic and Waters 1997, Wu and Simpson 2002; Wu et al. 2002). Some problems associated with GA are the uncertainty about the termination of the search and, as in all random search methods, the absence of a guarantee for the global optimum.

Global (unconstrained) optimization methods are among the first proposed for pipe network design (Abebe and Solomatine, 1999). . These methods convert the original constrained problem to an unconstrained problem via the use of a penalty function or a Lagrange multiplier method. The solution of the resulting problem is therefore much easier but the final solution is inferior to methods so far described. However, a global optimization formulation of the pipe network design problem has the advantage of simplicity and practicality for engineering use. Furthermore, all the random search and evolution methods such as GA employ some sort of global optimization formulation of the pipe network optimization. Abebe and Solomatine (1999) used the penalty method to define the pipe design problem as an unconstrained optimization problem, which was then solved by a global optimization package, GLOBE (Solomatine 1998), incorporating four different algorithms.

In this paper, the optimal design of pipe network is addressed from a hydraulic point of view. Only steady state condition is considered here; however, the extension to the case of dynamic condition is straightforward. The paper addresses the determination of optimal diameters of pipes in a network with a pre-determined layout in order to provide the required pressure and quantity of water at every demand node. A penalty method is used to formulate the optimal design of the pipe network as an unconstrained optimization problem, which is then solved by a general purpose optimization package called DOT (Vanderplaats, Miura and Associates 1994). The formulation of the problem is described next. The effect of the penalty parameter and the cost function approximation on the final solution is then investigated and finally some verifying numerical examples are presented.

2. Problem Formulation

The optimal design of a pipe network with a pre-specified layout in its standard form is described as follows:

$$Min \quad \sum_{l=1}^{m} L_l \, C_l \tag{1}$$

Subject to:

2.1. Hydraulic constraints:

$$\sum_{l\in k} q_l = Q_k \qquad k = 1,2,\ldots,n \tag{2}$$

$$\sum_{l\in p} J_l = 0 \qquad p = 1,2,\ldots,P \tag{3}$$

$$q_l = Kch_l d_l^{\alpha} (J_l / L_l)^{\beta} \tag{4}$$

2.2. Head and flow constraints:

$$H_k \geq H_{\min} \qquad k = 1,2,\ldots,\mathrm{n} \tag{5}$$

$$q_l \geq q_{\min} \qquad l = 1,2,\ldots,m \tag{6}$$

2.3. Pipe size constraints:

$$d_{\min} \leq d_l \leq d_{\max} \qquad l = 1,2,\ldots,m \tag{7}$$

where L_l = length of the lth pipe ; C_l = per unit cost of the lth pipe ; d_l = diameter of the lth pipe ; q_l = flow in the lth pipe; J_l = head loss in the lth pipe ; H_k = nodal head at the kth node; Q_k = consumption at node k ; H_{min} = minimum allowable hydraulic head; q_{min} = minimum allowable pipe flow; d_{min} and d_{max} = minimum and maximum allowable pipe diameter, respectively; and n, p, m = total number of nodes, loops, and links in the network, respectively; K = Hazen-Williams constant whose value depends on the system of units used; ch = Hazen-Williams roughness coefficient whose value depends on the pipe characteristics; $\alpha = 2.63$ and $\beta = 0.54$. The first set of constraints (2)-(4) describes the flow continuity at nodes, head loss balance in loops, and the Hazen-Williams equation. The second set (5)-(6) refers to the minimum nodal head and pipe flow requirements while the last constraint (7) requires the optimal pipe diameters to be between the maximum and minimum available pipe diameters, respectively. Equation (1) describes the total cost of the pipes in the network.

In this work, hydraulic constraints are satisfied via the use of an element by element simulation program, which explicitly solves the set of hydraulic constraints for nodal head and pipe flows (Afshar 2001). The second set of constraints are, therefore, included in the objective function via the use of an exterior penalty method resulting in the following penalized problem:

$$Min \sum_{l=1}^{m} C_l L_l + \sum_{l=1}^{m} \alpha_l (q_l - q_{\min})^2 + \sum_{k=1}^{n} \alpha_k (H_k - H_{\min})^2 \quad (8)$$

Where α_1, α_k = penalty parameters with large values when corresponding constraints are violated, and zero when the constraints are satisfied. For each network with the known pipe size diameters obtained, the distribution of the nodal heads and pipe flows are calculated by the simulation program and used to calculate the penalty terms in Eq. (8). The pipe size constraints are handled by the optimization package, DOT, as box constraints when the pipe diameters are taken as decision variables. Next, we consider the numerical treatment of the penalty term and the type of the analytical function used to approximate the cost per unit length of the pipes and present the results obtained.

3. Penalty prameter treatment

It is a common practice in the penalty formulation of the pipe network design to assume a large number for the penalty parameter and minimize the penalized objective function (Abebe and Solomatine 1999). The first difficulty with this approach is that the numerical value of the penalty coefficient is not known a priori and hence a trial and error procedure should be used to get the proper value, ensuring the enforcement of the constraints included in the objective function. Secondly, this large value of the penalty coefficient could change from one application to another, requiring the proper setting of the penalty parameter for each application. From a mathematical point of view, the penalty method is an iterative approach, which asymptotically converges to the solution of the original constrained problem as the value of the penalty coefficient increases. Iterative nature of the method eliminates the need for choosing the numerical value of the penalty coefficient via a trial and error process. Furthermore, iterative implementation of the penalty method can improve the effectiveness of the method as will be shown later when numerical examples are considered. In this paper, the following strategies are used to set the value of the penalty parameter:

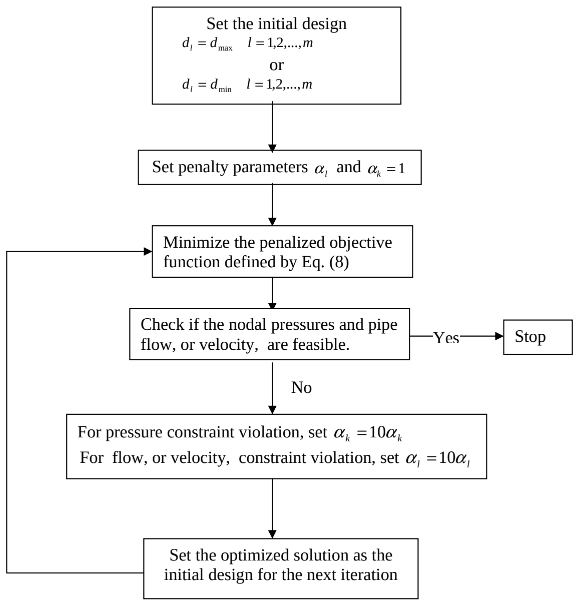

Fig. 1 Iterative penalty method flow diagram

Table 1 Cost data for the two-loop network

Diameter(cm)	2.54	5.08	7.62	10.16	15.24	20.32	25.4	30.48	35.56	40.64	45.72	50.8	55.88
Cost (units/m)	2	5	8	11	16	23	32	50	60	90	130	170	300

(1) Choose a very large value for the penalty parameter (herein, 10 C_{max}, where C_{max} is the cost of the most expensive design); and (2) use an iterative approach in which the value of the penalty parameter is increased by an order of magnitude starting from unity, until the constraints are exactly satisfied. The iterative procedure employed here is depicted in Fig. 1.

4. Pipe cost function

An analytical function of the form $a\,\mathrm{d}^b$ is usually used to approximate the cost per unit length of commercial pipes when a continuous method of optimization is employed for the optimum design of the networks. Unit cost parameters, *a* and *b*, are usually obtained via a least- squares fit of the aforementioned function to the discrete cost data. It is obvious that this fit would not be exact in terms of both function value and its gradient. Most of the common mathematical search methods use function value or its derivative to find the optimum value of the objective function. Thus, it is expected that improving the accuracy of the approximate cost function would improve the optimization results. In this work, a piecewise cubic spline is used to approximate the cost per unit length of the pipes and the results are compared with the usual least-squares form. Cubic splines are, by definition, third-order functions enjoying first and second derivative continuity. A piecewise cubic spline fit to a set of N discrete data can therefore be easily constructed by defining a different cubic spline for each N-1 reach such that their first and second derivative at N discrete points are continuous. This leads to 2(N-1) equations stemming from the fact that each of the N-1 piecewise functions at their two end points should equate the corresponding discrete data plus 2(N-2) equations stemming from the continuity of the first and second derivative at N-2 internal points, summing up to 4(N-1) - 2 equations in terms of 4(N-1) unknowns defining N-1 piecewise third-order functions. Two boundary conditions regarding the curvature of the spline at the two extreme points of the set are used to close the system of simultaneous linear equations. The size of the system of equations can be reduced by a proper formulation of the problem in terms of the value of the second derivatives at the N discrete points. The solution of this system yields the value of the second derivatives required to define each of the cubic splines at N-1 reach. This function can then be used to approximate the value of the discrete function at any arbitrary points, including the discrete data points. It is obvious that cubic spline approximation would always yield exact values at the discrete data points and therefore is a more accurate representation of the approximated function compared to a least-squares fit. Figure 2 shows the least squares and piecewise cubic spline approximations of the discrete cost function used in the literature for the two-loop network (Table 1).

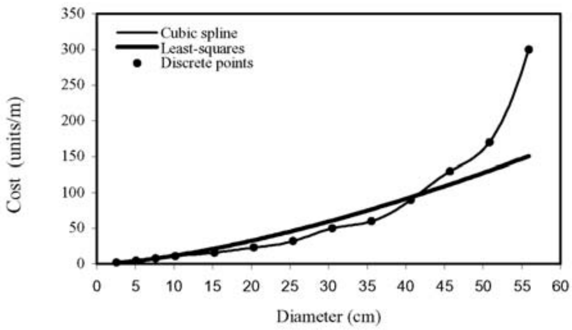

Fig. 2 Cubic spline and least-squares fit to the data of Table 1

5. Test problems

The first problem addresses a two-loop network with 8 pipes, 7 nodes, and one reservoir as shown in Fig. 3 (Abebe and Solomatine 1999). All the pipes are 1,000 m long and the Hazen-Williams coefficient is assumed to be 130 for all the pipes. The minimum nodal head requirement for all demand nodes is 30 m. There are 14 commercially available pipe diameters as listed

Table 2. Nodal demand and elevation data for the two-loop network

Node	Demand (m^3/h)	Ground level (m)
1	----	210.0
2	100.0	150.0
3	100.0	160.0
4	120.0	155.0
5	270.0	150.0
6	330.0	165.0
7	200.0	160.0

in Table 1 while Table 2 shows the nodal elevation and demands for the network.

First, the effect of the penalty parameter treatment is investigated. For this, the problem is solved using strategies (1) and (2) described earlier, with two different initial solutions. Table 3 compares the results obtained and the number of function evaluations required by these methods. Here d_{min} and d_{max} refer to the initial designs in which diameters of all the pipes are taken as the minimum and maximum available diameters, respectively. It can be seen that the iterative use of the penalty parameter significantly improves not only the solution for all cases but also reduces the number of function evaluation to get the solution. This is particularly significant for the cases in which the most expensive solution is taken as the initial guess for the optimization process, where the network cost is reduced from 469556 to 431630. This strategy, therefore, will be used in all the examples presented hereafter unless it is stated otherwise. It should be noted that the results presented in Table 3 were obtained with a least-squares fit to approximate the cost function.

The problem is solved again, using a cubic spline approximation of the pipe cost function. To make the comparison possible, these solutions along with the solutions shown in Table 3 are converted to a discrete set of commercially available diameters using the standard split pipe techniques (Fujiwara and Khang 1990) and shown in Table 4. In this method, each link with a given diameter d is replaced by two pipe with diameters d^u and d^l where d^u and d^l are the upper and lower commercially available diameters, respectively. The lengths of these pipes are then calculated such that the head loss in the link remains unchanged. It can be easily seen that cubic spline approximation of the cost function yields a better solution for all different initial solutions. It should, however, be noted that much of the improvements of the obtained solutions are due to the use of proposed iterative penalty method. The results shown in Tables 3 and 4 also indicate that using the most expensive design as an initial guess for the optimization procedure, represented by d_{max} in the Table 3, yields the better solution and hence would be used in all other tests presented hereafter. The near optimality of the solution obtained with the proposed method presented in Table 4 is evident by the fact that the nodal heads at four nodes ,ie; nodes 3, 5, 6 and 7, of the network is approximately equal to the minimum head requirement set by the problem definition. Table 5 compares the cheapest solution obtained here with the results obtained by some other optimization methods for this problem. It is clearly seen that the proposed method yield the second best solution amongst all the methods used in the literature. These solutions are

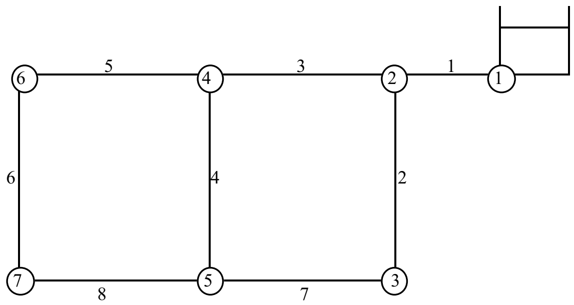

Fig. 3 Two-loop network

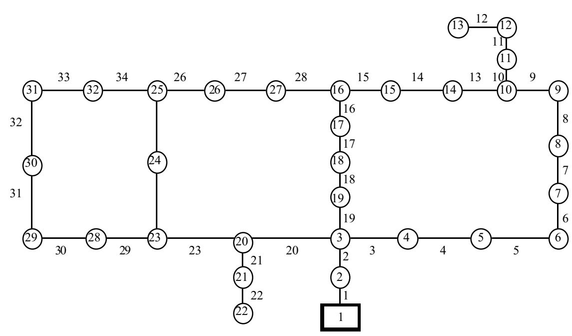

Fig. 4 Hanoi Network

Table 3 Optimal pipe diameters and corresponding nodal heads for the two-loop network

Pipe data					Nodal data				
Strategy	(a)		(b)			(a)		(b)	
Initial Guess*	d_{max}	d_{min}	d_{max}	d_{min}		d_{max}	d_{min}	d_{max}	d_{min}
Pipe	Diameter (cm)				Node	Head (m)			
1	49.66	50.11	47.77	49.19	1	------	------	------	------
2	38.30	41.30	26.13	36.24	2	55.48	55.68	54.55	55.27
3	32.46	33.98	39.62	33.78	3	40.91	42.63	31.43	39.29
4	2.54	8.07	2.54	2.54	4	43.83	45.15	44.57	44.80
5	32.30	30.40	37.31	30.86	5	46.80	42.26	30.01	43.79
6	2.54	2.54	25.29	2.54	6	30.00	30.01	30.01	30.01
7	36.37	32.00	23.85	34.26	7	30.01	30.00	30.01	30.01
8	23.70	24.99	2.54	26.72					
Cost (units)	469,556	477,266	431,630	462,170					
Evaluations**	1,100	1,532	968	903					

* Initial diameters used for the pipe diameters to start the optimization process. Note that NLP methods require an initial solution to start the process. d_{max} refers to an initial guess in which all the pipe diameters are taken as the maximum diameter possible while d_{min} refers to the minimum diameters.

** The number of network evaluations required by the NLP method to converge to the final solution which can be considered as a measure of computational cost of the method.

obtained on a P4-800 MHZ with 5 second of CPU time.

The second test problem considers the Hanoi network with 34 pipes, 31 demand nodes, and one reservoir as shown in Fig. 4 (Fujiwara and Khang 1990; Abebe and Solomatine 1999). The minimum nodal head requirement at all demand nodes is 30 m. Table 5 shows diameters of commercially available pipes. The cost of the pipes per unit length are calculated based on the analytical cost function 1.1d1.5 (Fujiwara and Khang 1990). Solutions to this problem using strategies (a) and (b) are shown in Table 7 in its split form along with some of the results obtained by other investigators for this problem. It is observed that much more can be gained by the use of the strategy (b) in larger problems. These solutions are obtained using the most expensive design as the initial guess. It should be remarked that the solution of Fujiwara and Khang (1990) to this problem was reported to be infeasible by Dandy et al. (1996). It is again seen that the proposed method has been able to produce the best solution for this problem. These solutions

Table 4 Optimal split pipe diameters and corresponding nodal heads for the two-loop network.

Pipe data									Nodal data				
Fit	Cubic spline				Least squares					Cubic spline		Least squares	
Initial Guess	d_{max}		d_{min}		d_{max}		d_{min}			d_{max}	d_{min}	d_{max}	d_{min}
Pipe	Solution [length (m) and diameter (cm)]								Node	Head (m)			
1	474.90	45.72	799.87	45.72	516.59	45.72	250.19	45.72	1	------	------	------	------
	525.02	50.80	200.13	50.80	483.41	50.80	749.81	50.80					
2	936.27	25.40	732.71	35.56	777.51	25.40	815.41	35.56	2	54.66	53.78	54.55	55.27
	63.73	30.48	267.29	40.64	222.49	30.48	184.59	40.64					
3	235.01	35.56	183.32	30.48	143.50	35.56	253.97	30.48	3	30.14	38.06	31.43	39.29
	764.99	40.64	816.68	35.56	856.50	40.64	746.03	35.56					
4	999.96	2.54	1000.00	2.54	999.77	2.54	1000.00	2.54	4	44.31	43.64	44.57	4480
	0.04	5.08	0.00	5.08	0.23	5.08	0.00	2.54					
5	470.22	35.56	461.45	30.48	563.35	35.56	889.85	30.48	5	30.00	43.62	30.01	43.79
	529.78	40.64	538.55	35.56	436.65	40.64	110.15	35.56					
6	9.98	20.32	999.95	2.54	9.84	20.32	999.94	2.54	6	30.01	30.01	30.01	30.01
	990.02	25.40	0.05	5.08	990.16	25.40	0.06	5.08					
7	105.25	20.32	931.40	35.56	183.70	20.32	177.93	30.48	7	30.01	30.01	30.01	30.01
	894.75	25.4	68.60	40.64	816.30	25.4	822.07	35.56					
8	999.97	2.54	566.88	25.4	1000	2.54	627.08	25.4					
	0.03	5.08	433.12	30.48	0.00	5.08	372.92	30.48					
Cost (units)	409,954		425,430		410,395		435,025						
Evaluations	1,711		1,558		968		903						

are obtained on a P4-800 MHZ with 20 second of CPU time.

The third test problem concerns the rehabilitation of the New York City water supply network with 21 pipes, 20 demand nodes, and one reservoir as shown in Fig. 5 (Dandy et al. 1996). Commercially available pipe diameters and their corresponding costs are shown in Table 8 while pipe and nodal data of the network are shown in Table 9. The inclusion of zero diameter with zero cost in the table of available pipe diameter is intended to widen the search space to include the 'no pipe' option for all of the links present in the network. Solutions to this problem using strategies (a) and (b) are shown in Table 10 in its split form along with the cheapest continuous and discrete pipe solutions reported by Dandy et al. (1996). Again, the improvement

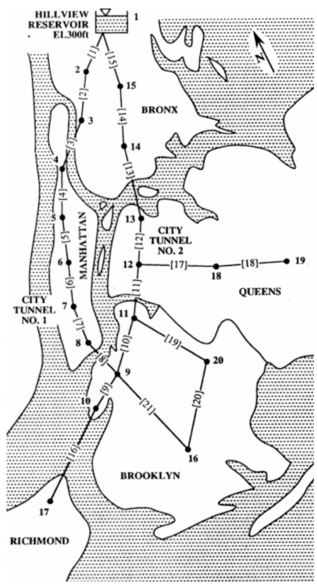

Fig. 5 New York tunnel network

Table 5 Optimal pipe diameters obtained with various methods for the two-loop network.

Pipe	Pipe length (m) and/or diameter (cm)									
	Kessler and Shamir (1989)		Eiger et al. (1994)		Savic and Walters (1997)		Abebe and Solomatine (1999)		Present work	
					GA1	GA2	GA	ACCOL*		
1	1000.00	45.72	1000.00	45.72	45.72	50.80	45.72	55.88	474.98	45.72
									525.02	50.80
2	66.00	30.48	238.02	30.48	25.40	25.40	35.56	45.72	936.27	25.40
	934.00	25.40	761.98	25.40					63.73	30.48
3	1000.00	40.64	1000.00	40.64	40.64	40.64	35.56	50.80	235.01	35.56
									764.99	40.64
4	713.00	7.62	1000.00	2.54	10.16	2.54	2.54	7.62	999.96	2.54
	287.00	5.08							0.04	5.08
5	836.00	40.64	628.86	40.64	40.64	35.56	35.56	40.64	470.22	35.56
	164.00	35.56	371.14	35.56					529.78	40.64
6	109.00	30.48	989.05	25.40	25.40	25.40	2.54	10.16	9.98	20.32
	891.00	25.40	10.95	20.32					990.02	25.40
7	819.00	25.40	921.86	25.40	25.40	25.40	35.56	45.72	105.25	20.32
	181.00	20.32	78.14	20.32					894.75	25.40
8	920.00	7.62	1000.00	25.40	25.40	25.40	30.48	40.64	999.97	25.40
	80.00	5.08							0.03	5.08
Cost (units)	417,500		402,352		419,000	420,000	424,000	447,000	409,954	

* Adaptive Cluster Covering with Local Search

Table 6 Available pipe diameter for the Hanoi network

Diameter (cm)	30.48	40.64	50.80	60.96	76.20	100.16

made in the solution by iterative setting of the penalty parameter is self-evident. This strategy yields one of the cheapest solution ever achieved for New York tunnels emphasizing on the ability of the proposed method for the optimal solution of large scale pipe network optimization problems. These solutions are obtained on a P4-800 MHZ with 10 second of CPU time.

A note should be made here regarding the size of the problems considered. Although the search space size of these problems are rather large, but they can only be considered as small to medium scale size problems compared to real world pipe network examples. Though the applicability of the proposed method does not basically depend on the size of the problem, its performance might be affected by the size of the problem to be solved.

6. Concluding remarks

A penalty method was presented for converting the optimal design of pipe networks to an unconstrained problem, which is then solved

Table 7 Optimal pipe diameters obtained by various methods for the Hanoi network.

Pipe length (m) and/or diameter (cm)								
Pipe	Present work				Fujiwara and Khang (1990)		Abebe and Solomatine (1999)	
	Strategy (a)		Strategy (b)				GA	ACCOL
1	0.3	76.2	0.01	76.2	100.0	101.6	101.6	101.6
	99.7	101.6	100.0	101.6				
2	0.62	76.2	0.9	76.2	1350.0	101.6	101.6	101.6
	1349.4	101.6	1349.1	101.6				
3	13.3	76.2	1.3	76.2	50.0	76.2	101.6	101.6
	886.7	101.6	898.7	101.6	850.0	101.6		
4	381.3	76.2	1.7	76.2	60.0	76.2	101.6	101.6
	768.7	101.6	1148.3	101.6	1090.0	101.6		
5	289.7	76.2	2.3	76.2	150.0	76.2	76.2	101.6
	1160.3	101.6	1447.8	101.6	1300.0	101.6		
6	65.1	76.2	.64	76.2	90.0	76.2	101.6	76.2
	385.0	101.6	449.4	101.6	360.0	101.6		
7	312.6	76.2	71.9	76.2	350.0	76.2	101.6	101.6
	537.4	101.6	778.1	101.6	500.0	101.6		
8	392.6	76.2	140.5	76.2	460.0	76.2	76.2	101.6
	457.3	101.6	709.5	101.6	390.0	101.6		
9	471.4	76.2	217.8	76.2	570.0	76.2	76.2	61.0
	328.6	101.6	582.2	101.6	230.0	101.6		
10	184.3	60.96	51.5	60.96	690.0	61.0	76.2	101.6
	765.7	76.2	898.5	76.2	260.0	76.2		
11	902.5	76.2	400.2	60.96	190.0	50.8	76.2	76.2
	297.5	101.6	799.8	76.2	1010.0	61.0		

Table 7 Continued

Pipe	Present work				Fujiwara and Khang (1990)		Abebe and Solomatine (1999)	
	Strategy (a)		Strategy (b)				GA	ACCOL
12	118.4	61.0	189.9	50.8	329.0	50.8	76.2	101.6
	3381.6	76.2	3310.1	61.0	210.0	61.0		
13	89.1	61.0	388.7	40.6	110.0	40.6	40.6	40.6
	713.7	76.2	411.3	50.8	690.0	50.8		
14	317.6	61.0	200.0	30.5	100.0	30.5	61.0	40.6
	128.4	76.2	300.0	40.6	400.0	40.6		
15	427.5	61.0	546.9	30.5	550.0	30.5	76.2	76.2
	122.6	76.2	3.1	40.6				
16	2439.8	76.2	2712.5	30.5	30.0	40.6	76.2	30.5
	290.2	101.6	17.5	40.6	2700.0	50.8		
17	1233.2	76.2	716.0	40.6	260.0	50.8	76.2	50.8
	516.8	101.6	1034.0	50.8	1490.0	61.0		
18	747.4	76.2	233.8	50.8	330.0	61.0	101.6	61.0
	52.6	101.6	566.3	61.0	470.0	76.2		
19	124.5	61.0	112.0	50.8	150.0	61.0	101.6	76.2
	275.5	76.2	288.0	61.0	250.0	76.2		
20	143.5	76.2	3.4	76.2	620.0	76.2	101.6	101.6
	2056.5	101.6	2196.6	101.6	1580.0	101.6		
21	999.6	50.8	587.8	40.64	1260.0	40.6	50.8	76.2
	500.4	61.0	912.2	50.8	240.0	50.8		
22	348.1	50.8	311.0	30.48	500.0	30.5	50.8	76.2
	151.9	61.0	189.0	40.6				

Pipe length (m) and/or diameter (cm)

Table 7 Continued

Pipe	Present work				Fujiwara and Khang (1990)		Abebe and Solomatine (1999)	
	Strategy (a)		Strategy (b)				GA	ACCOL
	Pipe length (m) and/or diameter (cm)							
23	456.0	76.2	245.7	76.2	110.0	61.0	76.2	101.6
	2194.1	101.6	2404.3	101.6	2540.0	76.2		
24	720.1	50.8	685.2	76.2	110.0	40.6	40.6	101.6
	509.9	61.0	544.8	101.6	1120.0	50.8		
25	1147.2	50.8	1251.5	76.2	1070.0	40.6	50.8	101.6
	152.8	61.0	48.5	101.6	230.0	50.8		
26	88.9	40.6	750.9	50.8	850	30.5	30.5	61.0
	761.1	50.8	99.2	61.0				
27	128.8	50.8	222.4	30.5	240.0	50.8	61.0	76.2
	171.2	61.0	77.6	40.6	60.0	61.0		
28	210.4	76.2	750.0	30.5	210.0	50.8	50.8	30.5
	539.6	101.6	0.0	40.6	540.0	61.0		
29	149.0	40.6	3.07	30.5	250.0	40.6	61.0	40.6
	1351.0	50.8	1496.9	40.6	1250.0	50.8		
30	464.7	40.6	1018.8	30.5	1160.0	40.6	76.2	101.6
	1535.3	50.8	981.2	40.6	840.0	50.8		
31	419.2	30.5	1600.0	30.5	300.0	30.5	76.2	40.6
	1180.8	40.6	0.00	30.5	1300.0	40.6		
32	55.01	30.5	41.5	30.5	150.0	30.5	76.2	50.8
	95.0	40.6	108.5	40.6				
33	534.2	30.5	442.6	40.6	860.0	30.5	76.2	76.2
	325.9	40.6	417.4	50.8				
34	461.9	40.6	191.9	50.8	60.0	40.6	30.5	61.0
	488.1	50.8	758.1	61.0	890.0	50.8		
Cost ($)	7,086,466		6,142,063		5,562,000		7,000,000	7,836,000

Table 8 Pipe cost data for the New York tunnel network

Diameter (cm)	0	91.4	121.9	152.4	182.9	213.4	243.8	274.3
Cost ($/m)	0	306.8	439.6	577.4	725.1	876.0	1036.7	1197.5
Diameter (cm)	304.8	335.3	365.8	396.2	426.7	457.2	487.7	518.2
Cost ($/m)	1367.1	1538.7	1712.6	1893.0	2073.5	2260.5	2447.5	2637.8

Table 9 Pipe and nodal data for the New York tunnel network.

Pipe data					Nodal data		
Pipe	Start Node	End node	Length (m)	Existing diameter (cm)	Node	Demand (*l*/s)	Min. total head (m)
1	1	2	3535.6	457.2	1	reservoir	91.4
2	2	3	6035.0	457.2	2	2616	77.7
3	3	4	2225.0	457.2	3	2616	77.7
4	4	5	2529.8	457.2	4	2497	77.7
5	5	6	2621.2	457.2	5	2497	77.7
6	6	7	5821.6	457.2	6	2497	77.7
7	7	8	2926.0	335.3	7	2497	77.7
8	8	9	3810.0	335.3	8	2497	77.7
9	9	10	2926.0	457.2	9	4813	77.7
10	11	9	3413.7	518.2	10	28	77.7
11	12	11	4419.6	518.2	11	4813	77.7
12	13	12	3718.5	518.2	12	3315	77.7
13	14	13	7345.6	518.2	13	3315	77.7
14	15	14	6431.2	518.2	14	2616	77.7
15	1	15	4724.4	518.2	15	2616	77.7
16	10	17	8046.7	182.9	16	4813	79.2
17	12	18	9509.7	182.9	17	1628	83.1
18	18	19	7315.2	152.4	18	3315	77.7
19	11	20	4389.1	152.4	19	3315	77.7
20	20	16	11704.3	152.4	20	4813	77.7
21	9	16	8046.7	182.9			

Table 10 Optimal pipe diameters obtained by various methods for the New York tunnel network.

Pipe	Present work Strategy (a)		Present work Strategy (b)		Gessler (1982)	Morgan and Goulter (1985)	Bhave (1985)	Dandy et al. (1996)
	Pipe length (m) and/or diameter (cm)							
1	0		0		0	0	0	0
2	0		0		0	0	0	0
3	0		0		0	0	0	0
4	0		0		0	0	0	0
5	0		0		0	0	0	0
6	0		0		0	0	0	0
7	0		0		254	365.8	0	0
8	0		0		254	0	0	0
9	0		0		0	0	0	0
10	0		0		0	0	0	0
11	0		0		0	0	0	0
12	0		0		0	0	0	0
13	0		0		0	0	0	0
14	4087.3	213.4	0		0	0	0	0
	2344.2	243.8						
15	0		2832.9	43.8	0	0	136.4	304.8
			1891.5	274.3				
16	2826.4	365.8	695.8	213.4		243.8	87.4	213.4
	5220.3	396.2	7350.9	243.9				
17	1618.7	274.3	110.8	213.4	254	243.8	99.2	243.9
	7891.1	304.8	9399.0	243.9				
18	3179.7	152.4	116.1	182.9	203.2	213.4	78.2	213.4
	4135.5	182.9	7199.1	213.4				
19	0		529.3	182.9	203.2	152.4	54.4	182.9
			3859.9	213.4				
20	912.5	243.8	0		0	0	0	0
	10791.9	274.3						
21	2770.8	365.8	2096.9	152.4	203.2	213.4	81.5	182.9
	5275.9	396.2	5949.8	182.9				
Cost ($M)	66.90		38.95		41.80	39.20	40.18	38.80

by a general purpose optimisation code. The performance of the method was improved by an iterative use of the penalty parameter, which significantly reduces the design cost compared to the conventional use of the penalty method. The method was further improved by using a cubic spline fit to continuously approximate the discrete pipe cost function. Numerical experiments showed that the use of cubic spline fit reduces the design cost compared to the design obtained via the use of conventional least-squares fit. Simplicity of both the basic method and the modification presented herein as well as comparability of the results with other methods makes it a suitable choice for engineering use. Research into finding ways to use the same method for discrete optimization of pipe networks is underway.

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