Optimum Design of Water Conveyance Systems by Ant Colony Optimization Algorithms

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Abstract:*Water conveyance systems (WCSs) are costly infrastructures in terms of materials, construction, maintenance and energy requirements. Much attention has been given to the application of optimization methods to minimize the costs associated with such infrastructures. Historically, traditional optimization techniques have been used, such as linear and non-linear programming. In this paper, application of ant colony optimization (ACO) algorithm in the design of a water supply pipeline system is presented. Ant colony optimization algorithms, which are based on foraging behavior of ants, is successfully applied to optimize this problem. A computer model is developed that can receive pumping stations at any possible or predefined locations and optimize their specifications. As any direct search method, the mothel is highly sensitive to setup parameters, hence fine tuning of the parameters is recommended.*

Keyword:*Ant colony optimization, Design, Optimum, Conveyance System*

1.Introduction

Due to the high costs associated with construction of water conveyance systems (WCSs) much research has been dedicated to the development of techniques to minimize the capital and operational costs associated with such infrastructures.
Historically, traditional optimization techniques have been used, such as linear and non-linear programming, but within the past decade use of evolutionary algorithms, such as genetic algorithms, simulated annealing and more recently ant colony optimization (ACO) have been also of interest.

The ant colony optimization (ACO) using principles of communicative behavior occurring in real ant colonies has been applied successfully to solve various combinatorial optimization problems namely traveling salesman problem [1], the quadratic assignment problem (e.g.Maniezzo and Colorni, 1999 [2]; Gambardella, Taillard, and Dorigo, 1999a [3]), the Job shop scheduling problem [4], or the resource-constraint project scheduling problem (Merkle, et al., 2000 [5]) (for an overview of ant algorithms see Dorigo and Di Caro,1999 [6]).

So far, very few applications of ACO algorithms to water resources problems have been reported. Abbaspour et al. (2001) [7] employed ACO algorithms to estimate hydraulic parameters of unsaturated soil.[8] used ACO algorithms to find a near-global optimal solution to a water distribution system, indicating that ACO algorithms may form an attractive alternative to genetic algorithms for the optimum design of water distribution systems. [9] employed ACO to optimally operate a multi-reservoir system.

In this paper, application of ACO on the

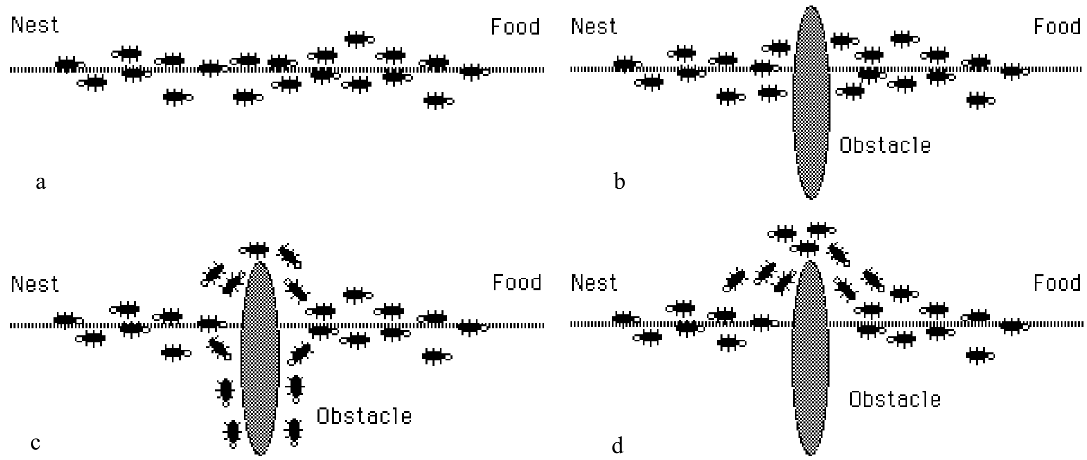

Fig. 1. Behavior of real ants.

design of a water supply pipeline system is presented. To do so, hydraulic gradient line is structured to fit an ACO model and the features related to ACO algorithms. Pressure water conveyance system and its hydraulic devices are defined with a graph and optimum solution to the system is presented.

2. Ant colony optimization, general aspects

Ant algorithms were inspired by the observation of real ant colonies. Ants are social insects that live in colonies. An important and interesting behavior of ant colonies is their foraging behavior, and, in particular, how ants can find shortest paths between food sources and their nest [1].

While walking from food sources to the nest and vice versa, ants deposit on the ground a substance called pheromone, forming a pheromone trail [10]. Ants can smell pheromone and, when choosing their way, they tend to choose paths marked by strong pheromone concentrations. The pheromone trail allows the ants to find their way back to the food source (or to the nest). Also, other ants to find the location of the food sources found by their nestmates can use it. Considering Fig. 1A, ants are moving on a straight line that connects a food source to their nest. It is well known that the primary means for ants to form and maintain the line is pheromone trail. Ants deposit a certain amount of pheromone while walking, and each ant probabilistically prefers to follow a direction rich in pheromone. This elementary behavior of real ants can be used to explain how they can find the shortest path that reconnects a broken line after the sudden appearance of an unexpected obstacle has interrupted the initial path (Fig. 1B). In fact once the obstacle has appeared, those ants that are just in front of the obstacle cannot continue to follow the pheromone trail and therefore they have to choose between turning right or left. In this situation we can expect half the ants to choose to turn right and the other half to turn left. A very similar situation can be found on the other side of the obstacle (Fig 1C). It is interesting to note that those ants which choose, by chance, the shorter path around the obstacle will more rapidly reconstitute the interrupted pheromone trail compared to those which choose the longer path. Thus the shorter path will receive a greater amount of pheromone per time unit and in turn a larger number of ants will choose the shorter path. Due to this positive feedback process, all the ants will rapidly choose the shorter path (Fig. 1D)[1]. ACO is a metaheuristic algorithm to solve combinatorial optimization problems by using principles of communicative behavior occurring in ant colonies. Ants can communicate information about the paths they found to food sources by marking these paths with pheromone. The pheromone trails can lead other ants to the food sources.

ACO was introduced by Dorigo et al.(1996). It is an evolutionary approach where several generations of artificial ants search for good solution. Every ant of a generation builds up a solution, which is found. Ants that find a good solution mark their paths through the decision space by putting some amount of pheromone on the edges of the path. Ants of the next generation are attracted by the pheromone that they will put in the solution space near good solutions.

To apply ACO to a combinatorial optimization problem, it is important to outline some basic concepts. Within ACO, the optimization problem is presented as a graph of n decision points where each decision point is connected to its adjacent decision point via a set of edges.

3.1 Ant System

Ant system (AS) [1], developed by Dorigo, Maniezzo and Colorni in 1991, was among the first generation of ACO algorithms. The decision policy used within AS is as follows: the probability that edge (*i,j*) will be selected at decision point *i* is given by:

$$P_{ij}^{k}=\begin{cases}\dfrac{(\tau_{ij})^{\alpha}(\eta_{ij})^{\beta}}{\sum\limits_{u\in N_k(r)}(\tau_{iu})^{\alpha}.(\eta_{iu})^{\beta}}\\ 0\end{cases} \qquad (1)$$

where p^{k}_{rs} is the probability that edge *(i,j)* is chosen, τ_{ij} is the concentration of pheromone associated with edge *(i,j)* , η_{ij} is the desirability of edge *(i,j)* , $N_k(i)$ is the feasible neighborhood of ant k when located at decision point *i,* and α , β are the parameters controlling relative importance of the pheromone intensity and desirability for each ants decision. If $\alpha >> \beta$ then the algorithm will make decisions based mainly on the learned information, as represented by the pheromone and if $\beta >> \alpha$ the algorithm will act as a greedy heuristic selecting mainly the shortest or cheapest edges, disregarding the impact of these decisions on the final solution quality [11].

When each ant has generated a solution (i.e. at the end of an iteration) the pheromone on each edge is updated. The pheromone updating equation for AS is given by

$$\tau_{ij}=\rho\tau_{ij}+\Delta\tau_{ij} \qquad (2)$$

where ρ is the coefficient representing pheromone persistence (note: $0 \leqq \rho \leqq 1$) and $\Delta\tau_{ij}$ is the pheromone added for edge *(i,j)*. The pheromone persistence factor is the mechanism by which the pheromone trails are decayed, enabling the colony to 'forget' poor edges and increasing the probability of selecting good edges [12]. For $\rho \rightarrow 1$ only small amounts of pheromone are decayed (evaporated) between iterations and convergence rate is slower, whereas for $\rho \rightarrow 0$ more pheromone is decayed (evaporated) resulting in faster convergence. $\Delta\tau_{ij}$ is a function of the solutions found and is given by:

$$\Delta\tau_{ij} = \sum_{k=1}^{m} \Delta\tau_{ij}^{k} \quad (3)$$

where m is the number of ants and $\Delta\tau^{k}_{ij}$ is the pheromone addition laid on edge by the ant:

$$\Delta\tau_{ij}^{k} = \begin{cases} \dfrac{Q}{f(S_k)} & if\,(i,j) \in S_k \\ 0 & otherwise \end{cases} \quad (4)$$

where Q is the pheromone additions factor (a constant) and S_k is the set of edges selected by ant k and $f(0)$ is the objective function.

3.2 Ant Colony System

Ant Colony System (ACS) is one of the first successors of AS. It introduces three major modifications into AS [11]:

1. ACS uses a different decision policy [12]. Let k be an ant located at a decision point i, $q_0 \in [0,1]$ be a parameter,and q a random value in $[0,1]$. The next decision point j is randomly chosen according to the following probability distribution
if $q \leqq q_0$

$$P_{ij}^{k} = \begin{cases} 1 & if \quad s = \arg\max_{u \in N_k(i)} (\tau_{iu})^{\alpha} \cdot (\eta_{iu})^{\beta} \\ 0 & otherwise \end{cases} \quad (5)$$

else $q > q_0$

$$P_{ij}^{k} = \begin{cases} \dfrac{(\tau_{ij})^{\alpha}(\eta_{ij})^{\beta}}{\sum_{u \in N_k(i)} (\tau_{iu})^{\alpha}.(\eta_{iu})^{\beta}} \\ 0 \end{cases} \quad (6)$$

As can be seen, the rule has a double aim: when $q \leqq q_0$, it exploits the available knowledge, choosing the best option with respect to the heuristic information and the pheromone trail. However, if $q > q_0$, it applies a controlled exploration as done in AS.

2. The pheromone update is done by first evaporating the pheromone trails on all connections used by the global-best ant (it is important to notice that in ACS, pheromone evaporation is only applied to the connections of the solution that is also used to deposit pheromone) as follows:

$$\tau_{ij} = (1-\rho)\tau_{ij} + \rho . f(S_{global-best}) \qquad \forall a_{ij} \in S_{global-best} \quad (7)$$

3. Ants apply local updating that encourages the generation of different solutions to those yet found. Each time an ant travels an edge *(i,j)* , it applies the following rule [13]:

$$\tau_{ij} = (1-\varphi)\tau_{ij} + \varphi\,\tau_0 \quad (8)$$

where $\varphi \in (0,1]$ is a second pheromone decay parameter. The application of local updating makes the pheromone trail on the connections traversed by an ant decrease. Hence, this results in an additional explorations technique of ACS by making the connections traversed by an ant less attractive to the follows and helps avoid every ant followers the same path.

4. Application of Ant Colony Optimization to Water Conveyance System Optimization

The optimization of WCSs is defined as the selection of the lowest cost combination of appropriate sizes of the pipeline and hydraulic devices (i-e, pumps) and component setting such that design constraints are satisfied.

The decision variables have primarily been selected as the friction head loss and pumping head within any branch. The design constraints on the system have normally been the requirement of minimum allowable pressures and the maximum allowable pressures at each node. In addition to the design constraint the hydraulic equations governing fluid flow must also be satisfied.

4.1 Difinition of ACO elements

ACS is applied for optimization of WCSs. The fitness function is a measure of goodness of the generated solutions according to the defined objective function. For this study, total cost (c) of pipes and pumping stations in every decision point is defined as:

$$C = \sum_{N=1}^{NS} C(N) = \sum_{N=1}^{NS} [FCP(HP(N))] + [CPD(N) \times L(N)] \tag{9}$$

where $FCP(HP(N))$ is the cost of pumping station for node N that is proportional to pumping head, $CPD(N)$ is the unit cost of pipe; $L(N)$ is the length of pipe and NS is the total number of nodes (i. e., decision point). The objective function is to minimize the total cost (i.e., cheaper options are more desirable.) The heuristic information on this problem is determined as:

$$\eta_{ij} = 1 - \frac{C_{ij}}{\sum_{l \in N_k(i)} C_{il}} \tag{10}$$

where C_{ij} is the cost of transfer from i to j, $N_k(i)$ is the feasible neighborhood of ant k when located at decision point i .

At the beginning the amount of pheromone on all paths are set equal (i.e.,τ_0=1). In other words all paths have the same preference for ants. After solutions are generated, the pheromone added to edge *(i,j)*, ($\Delta\tau_{ij}$), which belongs to global best solution, depends on its quality:

$$\Delta\tau_{ij} = \frac{Q}{C_{global-best}} \tag{11}$$

where Q is pheromone reward factor, $C_{global\text{-}best}$ is the total cost of global best solution.

4.2. Improved ACO algorithm in WCSs

Typically implementations of the ACO paradigm deposit pheromone on the links between nodes (each edge) that it can be highly memory intensive. This may cause dimensionality problem in large-scale problems. Depositing pheromone on nodes may reduce the memory overhead dramatically [14]. Therefor, the state transition rule of ACS are modified for this approach:

if $q \leqq q_0$

$$P_{ij}^k = \begin{cases} 1 & \text{if } j = \arg \max_{u \in N_k(i)} (\tau_u)^\alpha \cdot (\eta_{iu})^\beta \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

else $q > q_0$

$$P_{ij}^k = \begin{cases} \dfrac{(\tau_j)^\alpha (\eta_{ij})^\beta}{\sum_{u \in N_k(i)} (\tau_u)^\alpha . (\eta_{iu})^\beta} & \text{if } j \in N_k(i) \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

To improve the convergence of the results, Pheromone Promotion (PP) is introduced as a new approach [9]. Pheromone deposition and evaporation may cause a rapid convergence or *stagnation* problem. Now if a new solution with improved objective value is defined it may not be desirable for the agents to follow. To avoid this problem τ for all new paths with better objective value must be promoted to the maximum existing pheromone concentration.

Under some conditions, pheromone evaporation might finally abolish a solution with some locally positive decision elements. To decrease this possibility, Explorer Ants (EA) was proposed by Montgomery and Randall (2002a) [15]. EA technique divides the population of ants into two groups, with a higher proportion of normal ants than explorers. These explorer ants influence their environment by depositing pheromone in the same way as normal ants, only their preference for existing pheromone is reversed. This approach helps explore areas of the search space. EA explore poorer paths randomly to find out paths with insignificant pheromone and reasonable performance. Assigning some explorer ant at each iteration, they may take their paths as follow:

if $q \leqq q_0$

$$P_{ij}^{k} = \begin{cases} 1 & \text{if} \quad j = \arg\max_{u \in N_k(i)} [\tau_{\max} - \tau_u]^{\alpha} [\eta_{iu}]^{\beta} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

else $q > q_0$

$$P_{ij}^{k} = \begin{cases} \dfrac{[\tau_{\max} - \tau_j]^{\alpha} [\eta_{ij}]^{\beta}}{\sum_{u \in N_k(i)} [\tau_{\max} - \tau_u]^{\alpha} [\eta_{iu}(S)]^{\beta}} & \text{if} \quad j \in N_k(i) \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

where is the highest current level of pheromone in the system.

In some situations, ACO algorithms perform best when coupled with Local Search (LS) algorithms. It may cause a considerable improvement in final results, if employed after each iteration. In this approach, which is more or less similar to crossover in genetic algorithms, one or few sections of the generated solutions after each iteration are exchanged. As an example, in 3-opt local search algorithm, two elements from two different solutions are exchanged randomly which results in two new solutions. In the present work, one of the solutions for local search application is the present best solution and the other one selected randomly [16].

Finally, integration of LS, EA and PP may facilitate the convergence of the system as well as improving the final results. Explorer ants may generate some solutions with a few amounts of pheromone concentration and low desirability. Integration some sections of them with other solutions can improve the result.

5. Application of the ACS model

In order to test the performance of the proposed ACS for the optimization of WCSs, the developed computer model was applied to three benchmark problems. The first case study is a gravity system.

This case study was chosen as it is relatively simple and the global optimum solution to the problem is known. Two others are complex systems. They are chosen as relatively more complex problem as were solved by E. Jabbari & A. afshar [17] employing mixed integer programming (MIP).

Example. 1.): The gravity system consists of 12 nodes. The hydraulic gradient line is

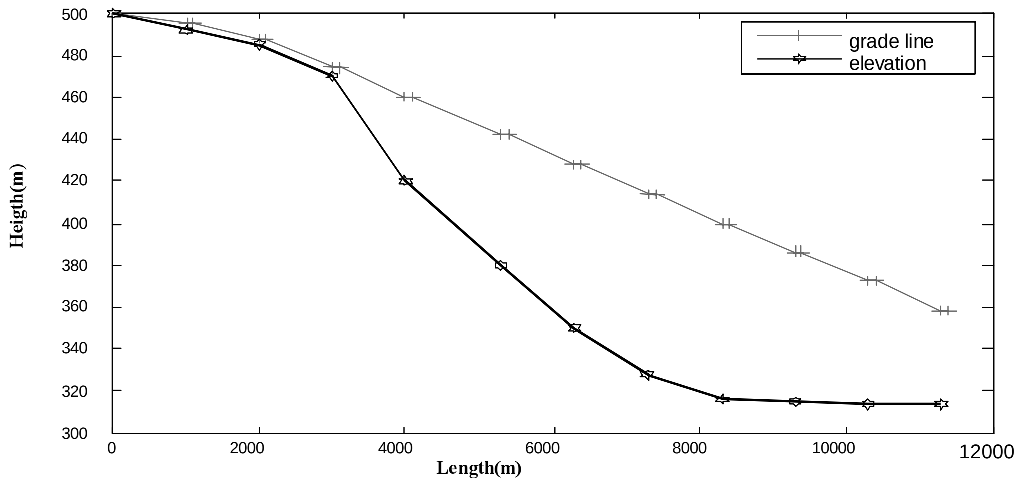

Figure 2. The optimal design of the gravity system (first example)

Table 1. Results of application ACS to the gravity system (first example)

NO. BRANCHE	Diameter(m)	Length(m)	V(m/s)
1	0.5	1000	1.562
2	0.46	1000	1.793
3	0.4	1000	2.384
4	0.4	1000	2.384
5	0.4	1300	2.373
6	0.4	1000	2.384
7	0.4	1000	2.384
8	0.4	1000	2.384
9	0.4	1000	2.384
10	0.4	1000	2.384
11	0.4	1000	2.384

Table 2. Continue results of ACS application to the gravity system (first example)

NO. NODE	Hpump(m)	Elevation(m)	NET HEAD (m)
1	0	500	0
2	0	492	3
3	0	485	3
4	0	470	4
5	0	420	40
6	0	380	62
7	0	350	78
8	0	328	86
9	0	316	84
10	0	315	71
11	0	314	58
12	0	314	44
TOTALCOST=117836			

divided into 50 classes with 1 meter interval. The min mum and max mum velocities are defined as 0.4 and 2.6 meter per second respectively. The minimum allowable pressure at each node is 3 meters. The results of applying ACS in this case study are presented in Tables 1 and 2. Figure 2 shows the hydraulic gradient line and ground elevation for the conveyance system.

As is clear from column 4 of Table 1 the maximum allowable velocity (2.4 m/s) restricts the pipe diameters in 0.4 m. So there is no possibility to further decrease the total cost and it is the global optimum solution. Two other case studies are shown in Figs. 3 and 4.

The second system consists of 12 nodes and the other has 17 nodes. The hydraulic gradient line is divided into 80 classes with 3 meters interval. Their hydraulic constraints (i-e, pressure and velocity constraints) are the same as the gravity system (i. e., example 1).

Figure 3 shows the results of applying the

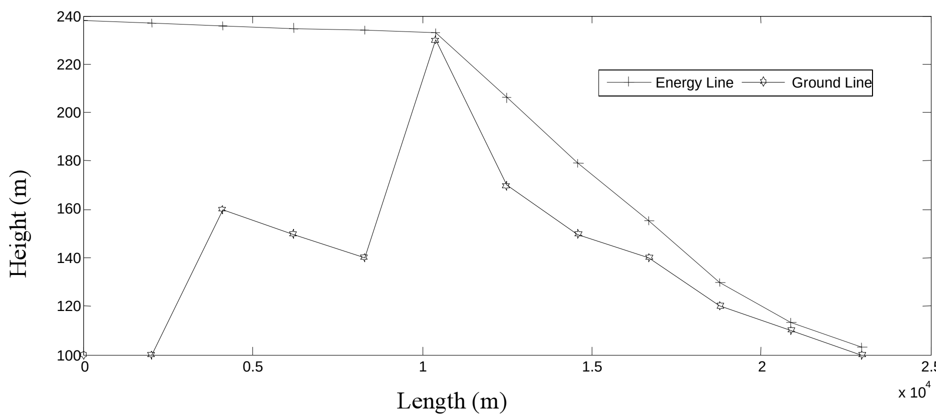

Figure 3. The optimal design of the second case study

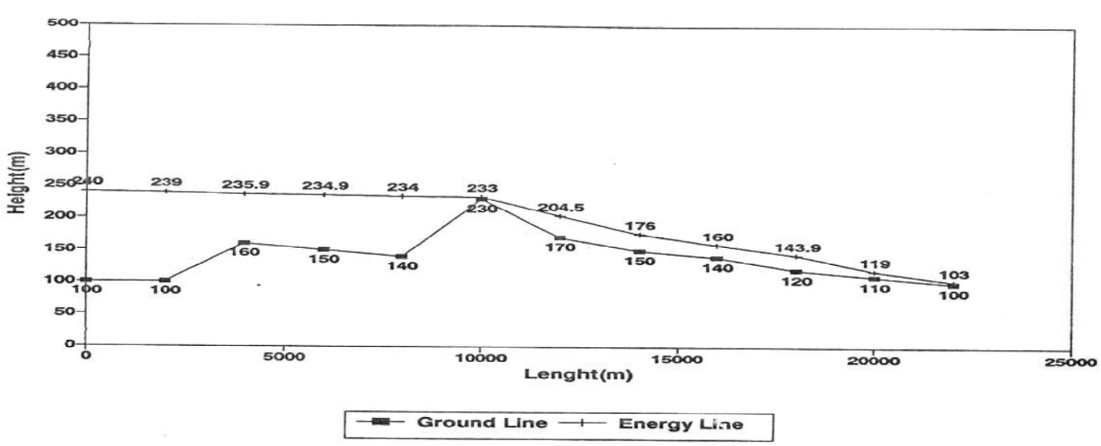

Figure 4. The optimal design of second case study through MIP model (After Jabbari &Afshar[17])

Table 3. Results of ACS & MIP application to the second case study

NO. Of reach	Length	Diameter	
		ACO	MIP
1	2000	0.794	0.8
2	840.9	0.794	0.55
	1159.1		0.8
3	2000	0.794	0.8
4	2000	0.794	0.8
5	2000	0.794	0.8
6	2000	0.403	0.4
7	2000	0.403	0.4
8	2000	0.413	0.45
9	2000	0.410	0.45
10	1421.8	0.444	0.4
	578.2		0.45
11	2000	0.495	0.45

Table 4. Results of ACS and MIP application to the second case study

NO. NODE	Hpump (m)		Elevation (m)		Net head(m)	
	ACO	MIP	ACO	MIP	ACO	MIP
1	138	140	100	100	138	140
2	0	0	100	100	137	139
3	0	0	160	160	76	75.9
4	0	0	150	150	85	84.9
5	0	0	140	140	94	94
6	0	0	230	230	3	3
7	0	0	170	170	36	34.5
8	0	0	150	150	29	26
9	0	0	140	140	15	20
10	0	0	120	120	10	23.9
11	0	0	110	110	3	9
12	0	0	100	100	3	3
				TOTALCOST	**304287**	**287795**

proposed ACS Model.Fig.4 presents the results of MIP model to this case study [17]. Results of applying ACS and MIP model in this case study are also presented in Tables 3 and 4.

Comparing the results show a little difference in their total cost. The minor differences may relate to (1) discretization scheme in ACO algorithm, and, (2) possibility of employing pipes with different diameters in any reach by MIP (see, for example branches 2 & 10 in Table 3).

The results of applying ACS in third case study have been shown in Tables 5 and 6. Figure 5 shows the hydraulic gradient line and ground elevation for the conveyance system.

Figure 6 shows the results of applying MIP model to this case study.

To make comparison easier results of applying ACS and MIP model study are presented in Tables 5 and 6.

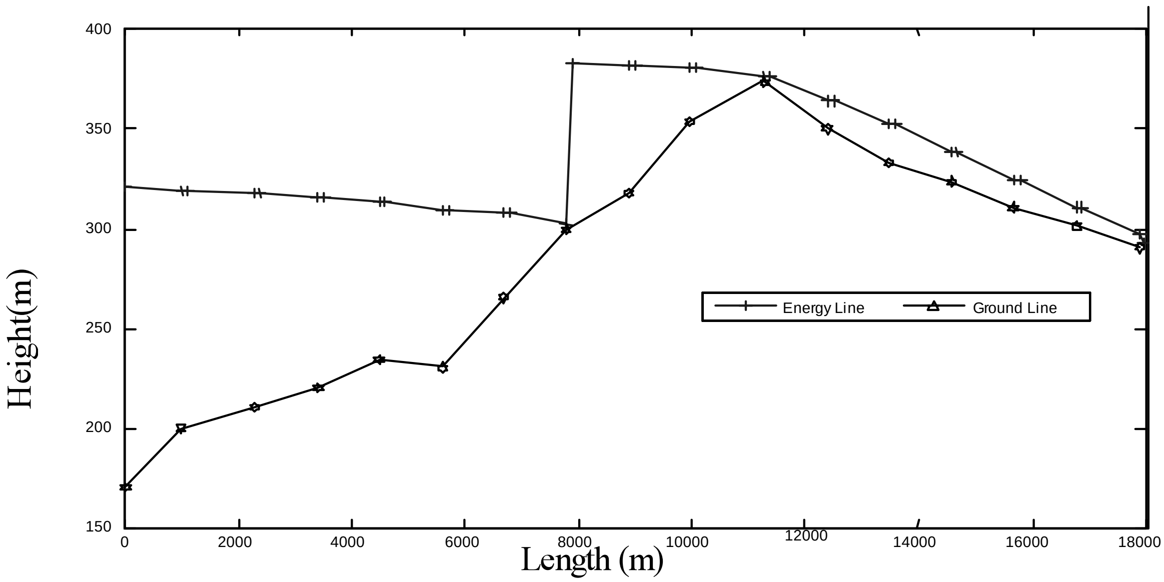

Figure 5: The optimal design of third case study(ACO model)

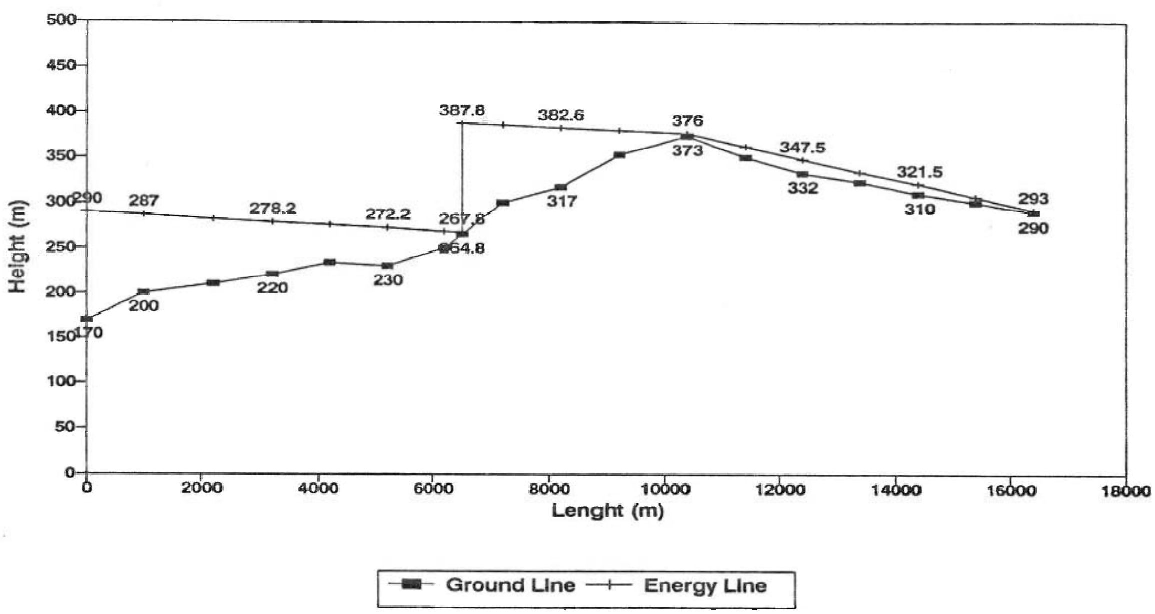

Figure 6. The optimal design of third case study obtained by MIP model

Table 5. Results of MIP and ACO model to the third case study

NO. BRANCHE	Length (m)	Diameter (m)	
		ACO	MIP
1	1000	0.5494	0.55
2	1200	0.6199	0.5
3	1000	0.5971	0.55
4	1000	0.4004	0.55
5	1000	0.4004	0.55
6	253	0.4492	0.5
	747		0.55
7	1000	0.5971	0.55
8	1000	0.5494	0.55
9	1000	0.5494	0.55
10	1200	0.4454	0.55
11	1000	0.4004	0.4
12	1000	0.4004	0.4
13	1000	0.4765	0.4
14	606.5	0.4004	0.4
	393.5		0.45
15	1000	0.4004	0.4
16	1000	0.4004	0.4

Table 6. Results of MIP and ACO model application to the third case study

NO. NODE	Hpump(m)		ELEVATION (m)		Net HEAD (m)	
	ACO	MIP	ACO	MIP	MIP	ACO
1	120	138	170	170	120	138
2	0	0	200	200	87	97
3	0	0	210	210	71.2	86
4	0	0	220	220	58.2	75
5	0	0	234	234	41.2	60
6	0	0	230	230	42.2	63
7	120	114	265	265	123	139
8	0	0	299	299	85.7	104
9	0	0	317	317	65.6	79
10	0	0	353	353	26.6	33
11	0	0	373	373	3	3
12	0	0	350	350	11.8	12
13	0	0	332	332	15.5	16
14	0	0	323	323	10.3	11
15	0	0	310	310	11.5	10
16	0	0	301	301	6.2	5
17	0	0	290	290	3	3
				TOTALCOST	337000	350000

As mentioned earlier, minor differences in total cost might be due to discrete approach employed with ACO algorithm, whereas, MIP approach benefits from continues variables. In addition, MIP allows for double pipe diameter selection in any reach. Moreover, linearization of pumping cost function is a major problem with MIP modeling in water conveyance systems.

Conclusion

During the last decade, use of metaheuristic optimization approaches to the design and operation of large systems have received considerable attention. An ant system algorithm which incorporates explorer ants, local search and pheromone promotion techniques were developed and successfully applied to three semi-benchmarked water supply pipeline systems. To test the performance of the proposed algorithm, results of the model were compared with those of mixed-integer programming approach. It was shown that ACS may be efficiently used to design such systems if the pumping stations were positioned in the line and range of pumping head in each branch was specified. Ant algorithms are generally classified as search algorithm with discrete natures, hence, the design obtained by the proposed model can not be considered as a global optimum solution. However, with fixed discretization scheme and extended number of iteration, achieving a good near optimal solution is possible. Even though in the applications, the hydraulic gradient line was discreteized and considered as search spaces, the pipe diameters may be also considered as decision variables without any complexity. Combination of a transient flow simulation model with the same optimization algorithm for design of pipeline system with dynamic pressure is an ongoing research by the authors.

References

[1]. Dorigo, M., Maniezzo, V., and Colorni, A. (1996). “The ant system: optimization by a colony of cooperating ants.” IEEE Trans. Syst. Man. Cybern., 26, 29-42.

[2]. Maniezzo, V., and Colorni, A. (1999). “The ant system applied to the quadratic assignment problem.” IEEE Trans. Knowledge and data engineering 11,769-778.

[3]. Gambardella, L.M., E.D. Taillard, and M. Dorigo. (1999a). “Ant colonies for the quadratic assignment problem.” Journal of the operational research society 50, 167-176.

[4]. Colorni, A. M. Dorigo, V. Maniezzo, and M. Trubian. (1994). “Ant system for jab shop scheduling.” Belgian journal operations research 34, 39-53.

[5]. Merkle, D., M. Middendorf, and H. Schmeck. (2000). “Ant colony optimization for resource-constraint project scheduling.” In proceedings of the genetic and evolutionary computation conference (GEECO-2000). San Mateo, CA: Morgan Kaufmann, pp. 893-900.

[6]. Dorigo, M., and Di Caro, G. (1999). “The ant colony optimization metaheuristic.” New ideas in optimization, D. Corne, M. Dorigo, and F. Glover, eds., McGraw-Hill, London, 11-32.

[7]. Abbaspour, K. C., Schulin, R., and van Genuchten, M. T. (2001). “Estimating

unsaturated soil hydraulic parameters using ant colony optimization." Adv. Water Resour., 24(8), 827-933

[8]. Maier, H. R., Simpson, A. R., Zecchin, A. C., Foong, W. K., Phang, K. Y., Seah, H. Y., and Tan, C. L. (2003). "Ant colony optimization for design of water distribution systems." J. Water Resour. Plng. and Mgmt., 129(3), 200-209.

[9]. Jalali, M. R., Afshar, A., Marino, M., Hon.M.ASCE. A. (2006). "Reservoir Operation by Ant Colony Optimization Algorithms" Iranian Journal of Science and Technology, Vol.30, No.b1.pp.107-117

[10]. M. Dorigo, G. Di Caro, and L. M. Gambardella, " Ant algorithms for discrete optimization" Artificial Life, 5(2):137–172, 1999.

[11]. O. Cordón, F. Herrera, T. Stützle, "A Review on Ant Colony Optimization Metaheuristic: Basis, Models and New Trends.", J. of Mathware and Soft Computing (9) , 1-35 ,2002

[12]. Dorigo, M., and Gambardella, L. M. (1997), "Ant colony system: A cooperative learning approach to the traveling salesman problem." IEEE Transactions on Evolutionary Computation, 1(1), 53-66.

[13]. Dorigo, M., and Gambardella, L. M. (1997), "Ant colonies for traveling sales man problem." Biosystems, 43: 73-81

[14]. Montgomery, J., & Randall, M. "Alternative Pheromone Applications for Ant Colony Optimization."

[15]. Montgomery, J., & Randall, M. (2002a). "Anti-pheromone as a Tool for Better Exploration of Search Space." Proceedings of the Third International Workshop on Ant Algorithms, ANTS 2002, Brussels, Belgium.

[16]. Dorigo, M., and Gambardella, L. M. (1997), "Ant colonies for traveling sales man problem." Biosystems, 43: 73-81

[17]. E . Jabbari and A . Afshar 2002 ."Optimum Layout and Design of a Water Supply Line ." Journal of Hydraulic Information and Management , 397-406