

# NUMERICAL ANALYSIS OF RC BEAMS FLEXURALLY STRENGTHENED BY CFRP LAMINATES

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**Abstract:** This paper presents the results of analytical studies concerning the flexural strengthening of reinforced concrete beams by external bonding of high-strength lightweight carbon fiber reinforced plastic (CFRP) plates to tension face of the beam. Three groups of beams were tested analytically and compared with existing experimental results. Results of the numerical analyses showed that, although addition of CFRP plates to the tension face of the beam increases the strength, it decreases the beam ductility. Finite element modeling of fifteen different beams in a parametric study indicates that steel area ratio, CFRP thickness, CFRP ultimate strength and elastic modulus considerably influence the level of strengthening and ductility.

**Keywords:** Carbon fiber-reinforced plastic; Reinforced concrete; Bonding; Flexural strength; Ductility.

## 1. INTRODUCTION

The use of externally bonded fiber-reinforced polymer (FRP) sheets and strips has recently been established as an effective tool for rehabilitating and strengthening reinforced concrete structures. CFRP strips are used to strengthen and rehabilitate deficient structures, especially in outdoor application, because of their non-corrosive characteristics. CFRP strips also offer other advantages such as high specific strength and stiffness, high durability, low creep and high fatigue resistance in comparison to conventional materials.

Several experimental investigations has been reported on the behavior of concrete beams strengthened in flexure using externally bonded FRP plates, sheets, laminates or fabrics. Saadatmanesh and Ehsani [1] examined the behavior of concrete beams strengthened in flexure using glass fiber-reinforcement polymer (GFRP) plates. Ritchie et al. [2] tested reinforced concrete (RC) beams strengthened in flexure using GFRP, carbon fiber reinforced polymer (CFRP), and G/CFRP plates. Balaguru and Foden [3]

conducted experiments on light-weight RC beams strengthened with FRP laminates. Grace [4] presented his paper on strengthening of negative moment region of RC beams using CFRP strips. Brena and Bramblett [5] tested the increase of flexural strength of RC beams strengthened with FRP laminates. These laboratory studies have demonstrated the effectiveness of externally bonded FRP plates in enhancing the flexural capacity of concrete beams. However, there is limited reported numerical modeling on the behavior of RC Beams strengthened by CFRP, especially with 3D nonlinear material modeling. This paper presents the results of FE analyses on flexural strengthening of reinforced concrete beams by external bonding of CFRP laminates to tension face of the beams.

## 2. RESEARCH SIGNIFICANCE

The continual deterioration of infrastructure world-wide highlights the need for effective means of rehabilitating structures such as bridges and buildings. The use of FRP wrapping as rehabilitation and retrofitting technique can present real advantages with

regards to durability, maintenance costs, serviceability and ultimate strength. This paper describes the strength enhancement provided to flexural capacity of reinforced concrete beams by externally bonding of CFRP laminates using finite element method. Comparison with experimental data is used to verify the capability of finite element method. Furthermore, parametric study was performed to illustrate the effect of different parameters on the beams strength and ductility.

### 3. FINITE ELEMENT STUDY

#### 3.1. Beams Fabrication

Six rectangular, under-reinforced concrete beams modeled in this study. The beams were divided into three groups of two (control and strengthened), according to the specimen's characteristics. Modeling the complex behavior of reinforced concrete which is both non-homogeneous and anisotropic is a difficult challenge in the finite element analysis of civil engineering structures. ANSYS finite element program was chosen to perform the analysis. The efficiency and accuracy, however, depends on mesh topology, material properties and element types.

Using 3D finite element method as the one presented herein, required long run time due to the large number of time steps which also depends on the elements size. The three dimensional finite element meshes in this study were developed in ANSYS and consist of solid elements to represent concrete, link elements to represent the longitudinal steel and stirrups. Fig. 1 shows a schematic of the elements used for the model development.

#### 3.2. Element Types

An eight-node solid element, *solid65*, was used to model the concrete. The element has three degrees of freedom at each node-translation in the nodal x, y and z directions and is capable of plastic deformation,

cracking in three orthogonal directions and crushing. A *link8* element was used to model the steel reinforcement. Two nodes are required for this element. Each node has three degree of freedom-translations in the nodal x, y and z directions. A layered eight nodes shell element, *shell91*, was used to model the FRP laminates. The element allows for up to 16 different material layers with different orientations and orthotropic material properties in each layer. The element has all of the six possible degrees of freedom. A four node shell element, *shell63* was used for the steel plates at the supports in the beam models. The element is defined with similar thickness (1 cm) at all of the four corners. The geometry, node location and coordinate system of the elements are shown in Fig. 2 [6].

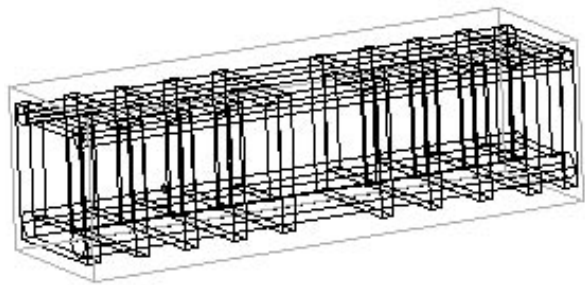
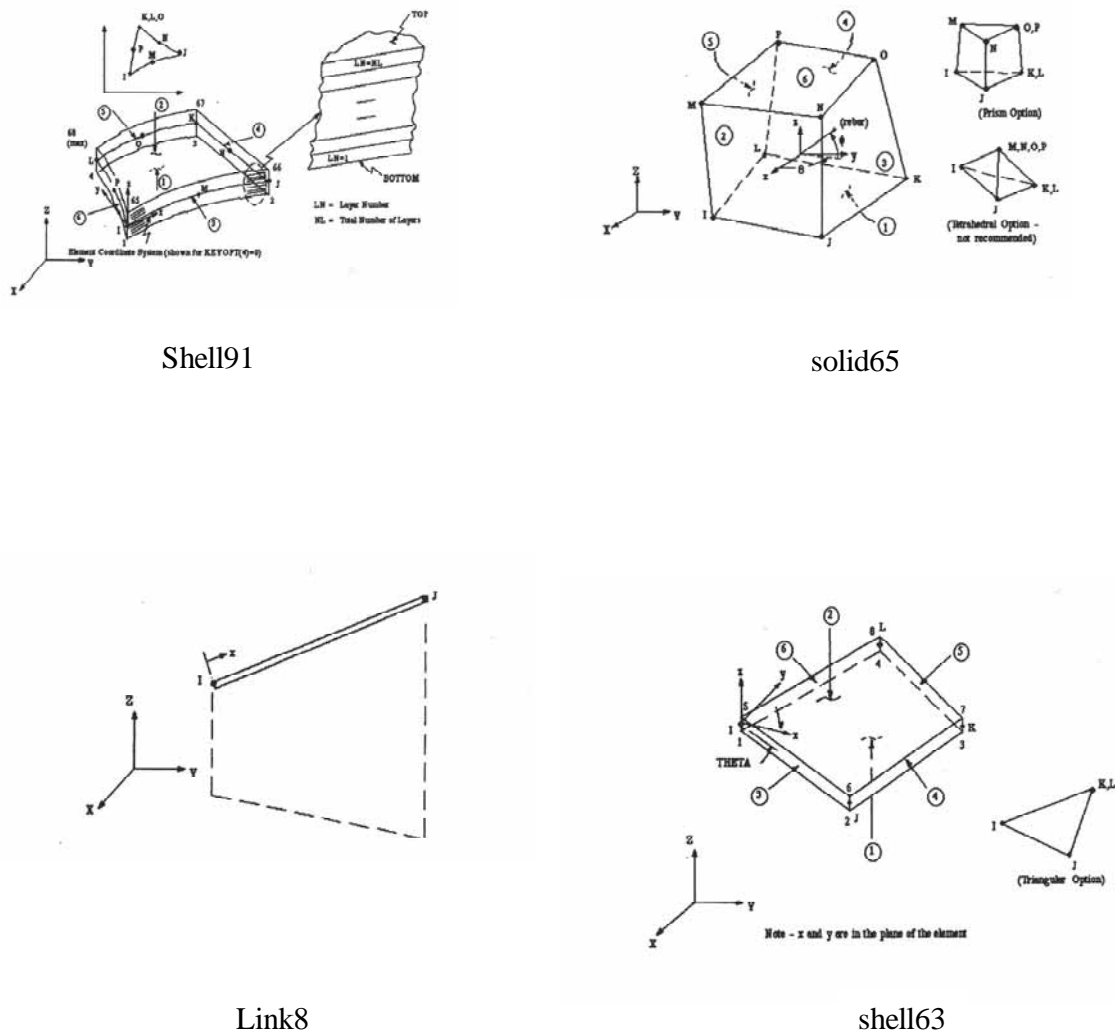


Fig. 1. schematic view of beam.

#### 3.3. Material Properties

Development of a model for the behavior of the concrete is a challenging task. Concrete is a quasi-brittle material and has different behavior in compression and tension.

In compression the stress-strain curve for the concrete is linearly elastic up to about 25 percent of the maximum compressive strength [7] as shown in Fig. 3. After the maximum strength the curve descends into a softening region, and eventually crushing failure occurs at an ultimate strain. In tension, the stress-strain curve for the concrete is approximately linearly elastic up to maximum tensile strength. After this point the concrete cracks and the strength



**Fig. 2.** The geometry, node location and coordinate system of the elements.

**Table 1.** concrete, FRP and steel material properties.

| Material  | Concrete    |             |       |                                 | Steel       |             | FRP           |                |             |     |     |         | Ultimate strength (MPa) |     |      |
|-----------|-------------|-------------|-------|---------------------------------|-------------|-------------|---------------|----------------|-------------|-----|-----|---------|-------------------------|-----|------|
|           | $E_c$ (GPa) | $F_c$ (MPa) | $n_c$ | $\sigma_c$ (KN/m <sup>3</sup> ) | $E_s$ (GPa) | $f_y$ (MPa) | No. of layers | Thickness (mm) | $E_f$ (GPa) |     |     | $\nu_f$ |                         |     |      |
|           |             |             |       |                                 |             |             |               |                | x           | y   | z   | xy      | xz                      | yz  |      |
| Group I   | 34.5        | 54.8        | 0.2   | 24                              | 200         | 410         | 1             | 0.45           | 138         | 4.8 | 4.8 | 0.2     | 0.2                     | 0.2 | 2206 |
| Group II  | 37.8        | 55.2        | 0.2   | 24                              | 200         | 415         | 1             | 1.9            | 49          | 4.8 | 4.8 | 0.2     | 0.2                     | 0.2 | 690  |
| Group III | 25          | 32          | 0.2   | 24                              | 200         | 420         | 2             | 0.52           | 62          | 4.8 | 4.8 | 0.2     | 0.2                     | 0.2 | 760  |

decreases. Concrete material was chosen from ANSYS nonlinear library. The model predicts the failure of brittle materials and accounts for both cracking and crushing failure modes. The required input data for the concrete in ANSYS are, 1) Elastic modulus, 2) Ultimate uniaxial compressive strength, 3) special weight and 4)Poisson's ratio. For each type of beams different type

of concrete were used in this study. Properties used for these models are shown in Table 1.

Steel reinforcement in finite element model was assumed to be grade 60 steel with 200 GPa elastic modulus and 0.3 Poisson's ratio. The steel yield stress ranges between 400 to 420 MPa according to the beam types. The steel behavior considered elastic-perfectly

plastic. For this purpose the Bilinear-Kinematics material was chosen from ANSYS library for modeling the steel material. Also the steel plates at the support which were added to provide a more even stress distribution over the support areas, modeled with Bilinear-Kinematics material. FRP composites are materials that consist of two constituents. The constituents are combined at a macroscopic level and are not soluble in each other. One constituent is the reinforcement, which is embedded in the second constituent, a continuous polymer called the matrix. The reinforcing material is in the form of fibers, i.e., carbon or glass, which are typically stronger and stiffer than the matrix. Therefore the FRP composites are anisotropic materials which called especially orthotropic material. In this study especially orthotropic material is also transversely isotropic, where the properties of the FRP composites are nearly the same in any direction perpendicular to the fibers. Input data required for FRP composites in finite elements models are, 1) number of layers, 2) layer thickness, 3) orientation of the fiber direction, 4) elastic modulus in three directions, 5) shear modulus or Poisson's ratio in three planes and 6) ultimate strength of FRP laminates. A summary of concrete, FRP and steel material properties for the modeling of all three groups of the beams are shown in table 1.

### 3.4. Geometry

Beams with dimensions of 3040x20x20 mm, 2652x152x254 mm and 3200x200x400 mm were modeled for group types I, II and III, respectively. Table 2 shows the dimension of each beams group, FRP and steel arrangements. The FRP meshes were connected to concrete meshes by shared

nodes. As such, a perfect bond is assumed between the concrete and the composite grid.

### 3.5. Boundary Conditions and Loading

To simulate simply supported conditions, the beam was supported on two rigid plates made of shell elements in order to avoid stress concentration problems. Moreover, a single line support was placed under the centerline of the steel plate to allow the rotation of the plate. Excessive cracking of the concrete elements above the steel plate was found to develop if the rotation of steel plate was not permitted. The finite element simulations were displacement controlled, which is usually the control method for plastic and nonlinear behavior. 8 cm for the beams. The corresponding applied load due to the prescribed displacement was then determined by monitoring the vertical reaction forces at the concrete nodes in contact with loading. Fig. 4 and Table 3 show the schematic view of loading and boundary conditions.

A displacement was prescribed on top of the beam which increased linearly from zero to

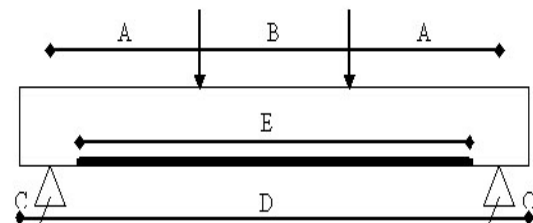


Fig. 4. Loading and boundary conditions Nonlinear solution.

In nonlinear analysis, the total load applied to a finite element model is divided into a series of load increments called load steps. At the completion of each incremental

Table 2. Beams dimensions.

| Material  | Concrete    |            |             | Steel                      |                          |              | Fiber laminates |            |             |
|-----------|-------------|------------|-------------|----------------------------|--------------------------|--------------|-----------------|------------|-------------|
|           | Length (mm) | Width (mm) | Height (mm) | Tension (mm <sup>2</sup> ) | Comp. (mm <sup>2</sup> ) | stirrups     | Length (mm)     | Width (mm) | Height (mm) |
| Group I   | 3040        | 200        | 200         | 259                        | 141                      | 102mm@152 mm | 2736            | 200        | 0.45        |
| Group II  | 2652        | 152        | 254         | 402                        | 141                      | #3@102 mm    | 2244            | 152        | 1.9         |
| Group III | 3200        | 200        | 400         | 402                        | 160                      | F12@100 mm   | 2600            | 100        | 0.52        |

solution, the stiffness matrix of the model is adjusted to reflect nonlinear changes in structural stiffness before proceeding to the next load increment.

**Table 3.** Beams loading and boundary conditions.

| Parameters all in mm | A    | B   | C   | D    | E    |
|----------------------|------|-----|-----|------|------|
| Group I              | 912  | 912 | 152 | 2736 | 2736 |
| Group II             | 816  | 816 | 102 | 2448 | 2244 |
| Group III            | 1200 | 600 | 100 | 3000 | 2600 |

#### 4. FINITE ELEMENT RESULTS

##### Group I

Figs. 5 and 6 describe the load-deflection curves of group I strengthened and control beams. Comparison of the finite element analysis results with the experimental analysis and section analysis results are also presented in the figures. The load-deflection curves in Fig. 5 indicate that prior to concrete cracking CFRP strengthening does not influence the load deflection curve. After cracking, the strengthened specimen behaves stiffer and the load at yielding of reinforcement increased by 27 percent with respect to control specimen. After yielding, the strengthened specimen carried 120 percent higher load than the control specimens.

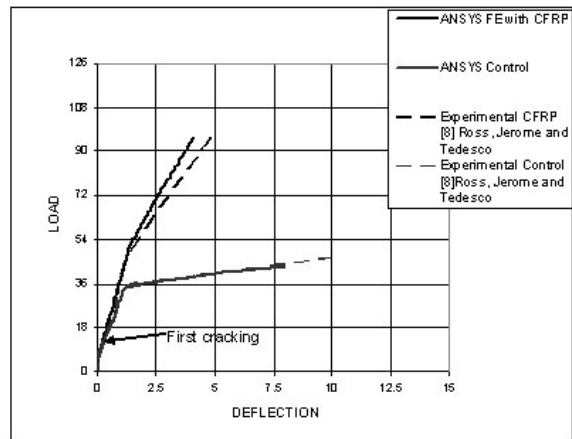
However, the ductility of the strengthened beam decreases by 48 percent due to brittle characteristics and confined geometry of CFRP laminates. As shown in Fig. 5 the calculated load-deflection curves predict the available experimental data satisfactorily. Moreover, there is good agreement between the section analysis and ADYNA 2D software analysis of Rose and Jerome [8], and ANSYS finite element results of present study (Fig. 6). The load and displacement levels corresponding to yielding of longitudinal reinforcement and ultimate strength of strengthened beam are summarized in Table 4.

##### Group II

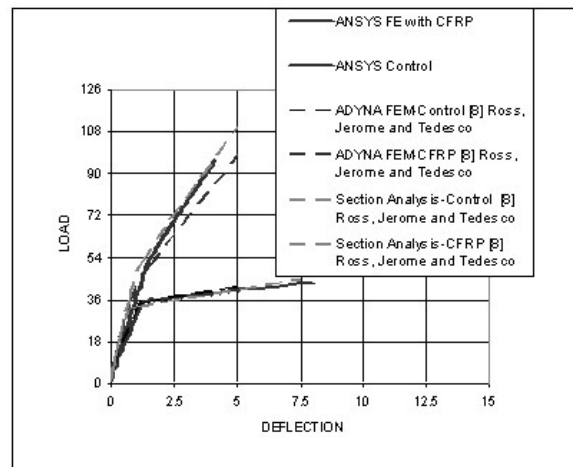
Fig. 7 shows the load-deflection diagram for beams of group II. The strengthened beam

yielded at a load of 90 KN and failed at the load of 126.7 KN due to crushing of the concrete in the compression zone. There is 13% increase in the yield load compared to that of control beam.

The ultimate strength of CFRP specimen was 41% higher than the control specimen. However, the ductility is 52% lower than control specimen (Table 4).



**Fig. 5.** Load-deflection curves based on finite element analysis and experimental results of beams group I.



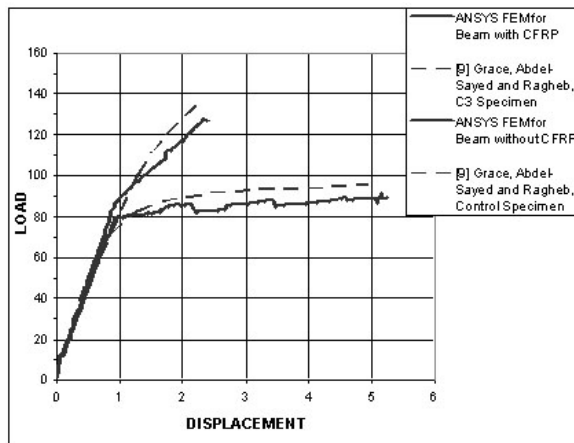
**Fig. 6.** Load-deflection curves based on finite element and Section analysis results of beams group I.

##### Group III

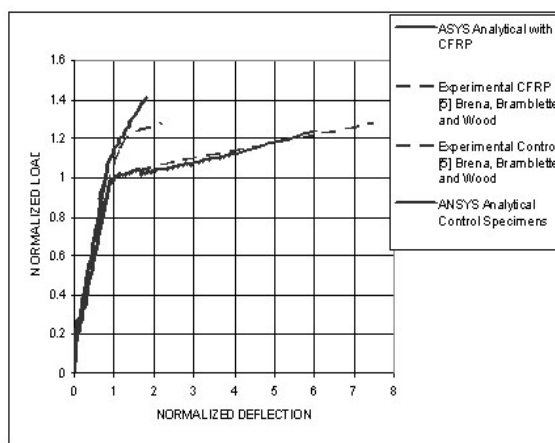
The response of the strengthened specimen was the same as the response of the control specimen before cracking of concrete. After cracking just like the other groups the strengthened specimen behaves stiffer than the control specimen. Yielding load of

**Table 4.** Load and displacement result for beams of groups I, II and III.

| Group     | Yielding   |               | Ultimate   |               | Strength increase (%) | Ductility increase (%) | Ana./Exp. At ultimate (%) |
|-----------|------------|---------------|------------|---------------|-----------------------|------------------------|---------------------------|
|           | Disp. (mm) | Strength (KN) | Disp. (mm) | Strength (KN) |                       |                        |                           |
| Group I   | 1.23       | 45.19         | 4.1        | 95.38         | 120                   | -48                    | 3                         |
| Group II  | 1.03       | 90            | 2.4        | 126.7         | 41                    | -52                    | 6                         |
| Group III | 1.18       | 1.1           | 1.8        | 1.4           | 13                    | -70                    | 9                         |

**Fig. 7.** Load-deflection curves based on finite element analysis and experimental results of beams of group.

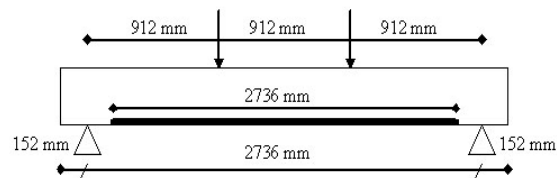
longitudinal reinforcement increased by 10% in the strengthened specimen and ultimate load capacity of specimen exceeded the capacity of control specimen by 13%. However, the displacement level was considerably less than that of the control beam. The FE analysis predicts the experimental results with acceptable accuracy (Fig. 8).

**Fig. 8.** Load-deflection curves based on finite element analysis and experimental results of beams of group.

## 5. PARAMETRIC STUDY

Flexural FE models are used to investigate the effect of main parameters on the flexural response of strengthened reinforced concrete beams. The main parameters considered are steel area ratio, thickness, ultimate strength and elastic modulus of CFRP laminates. The concrete elastic modulus, compressive strength and steel yield strength are kept constant as 33.5 GPa, 47 MPa and 410 MPa, respectively.

The beams with cross-sectional dimensions of 3040x200x200 mm and 50 mm of tensile steel cover were subjected to two point load system as shown in Fig. 9. The other beams characteristics are given in Table 5.

**Fig. 9.** loading and boundary conditions.

### 5.1. Steel Area Ratio

Three different steel area ratio of  $r=0.013$ ,  $r=0.025$  and  $r=0.032$  are used. In all cases the reinforcement ratio is kept below the allowable reinforcement ratio based on ACI design code [10]. Steel area ratio has considerable effect on the strength and ductility of the strengthened beams (Fig. 10). Increase in the reinforcement steel decreases the rate of increase in the strength with respect to the control beam. The strength of the beam with  $r=0.013$  increased by 98% with respect to control beam while the strength of the beam with  $r=0.032$

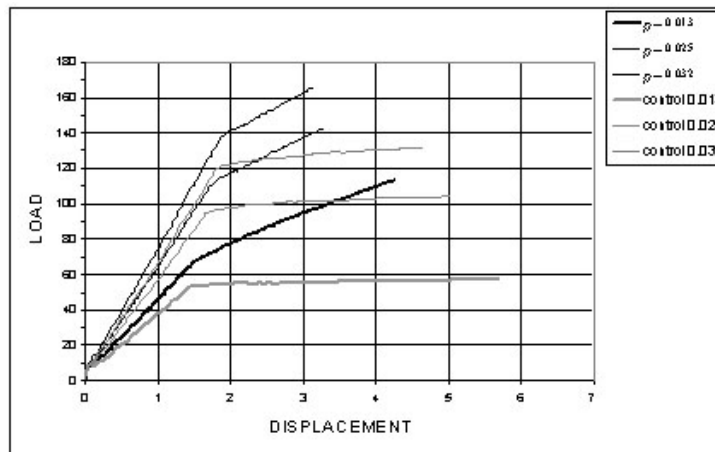


Fig. 10. Load-deflection curves for beams with different steel area ratio.

Table 5. Beams characteristics.

| Test variables               |       | Steel area                 |                                |            | CFRP properties       |                |                |                         | Thickness (mm) |
|------------------------------|-------|----------------------------|--------------------------------|------------|-----------------------|----------------|----------------|-------------------------|----------------|
|                              |       | Tensile (mm <sup>2</sup> ) | Compressive (mm <sup>2</sup> ) | Stirrups   | Elastic modulus (GPa) |                |                | Ultimate strength (MPa) |                |
|                              |       |                            |                                |            | E <sub>x</sub>        | E <sub>y</sub> | E <sub>z</sub> |                         |                |
| Steel area ratio (%)         | 0.013 | 401.9                      | 141.6                          | F12@152 mm | 138                   | 4.8            | 4.8            | 2206                    | 0.45           |
|                              | 0.025 | 759.9                      | 141.6                          | F12@152 mm | 138                   | 4.8            | 4.8            | 2206                    | 0.45           |
|                              | 0.032 | 981.3                      | 141.6                          | F12@152 mm | 138                   | 4.8            | 4.8            | 2206                    | 0.45           |
| CFRP thickness (mm)          | 0.75  | 259                        | 141.6                          | F12@152 mm | 138                   | 4.8            | 4.8            | 2206                    | 0.75           |
|                              | 1.00  | 259                        | 141.6                          | F12@152 mm | 138                   | 4.8            | 4.8            | 2206                    | 1.00           |
|                              | 1.50  | 259                        | 141.6                          | F12@152 mm | 138                   | 4.8            | 4.8            | 2206                    | 1.50           |
| CFRP ultimate strength (MPa) | 1000  | 259                        | 141.6                          | F12@152 mm | 138                   | 4.8            | 4.8            | 1000                    | 0.45           |
|                              | 1800  | 259                        | 141.6                          | F12@152 mm | 138                   | 4.8            | 4.8            | 1800                    | 0.45           |
|                              | 3000  | 259                        | 141.6                          | F12@152 mm | 138                   | 4.8            | 4.8            | 3000                    | 0.45           |
| CFRP elastic modulus (GPa)   | 60    | 259                        | 141.6                          | F12@152 mm | 60                    | 4.8            | 4.8            | 2206                    | 0.45           |
|                              | 70    | 259                        | 141.6                          | F12@152 mm | 70                    | 4.8            | 4.8            | 2206                    | 0.45           |
|                              | 80    | 259                        | 141.6                          | F12@152 mm | 80                    | 4.8            | 4.8            | 2206                    | 0.45           |

increased by only 26%. Furthermore, increase in the reinforcement steel increases the rate of decrease in ductility of strengthened beams. In other words, the ductility of beam with  $r=0.013$  decreased by 30%, however ductility of the beam with  $r=0.032$  decreased by 40% with respect to control beam.

## 5.2. CFRP Thickness, Ultimate Strength and Elastic Modulus

Figs. 11, 12 and 14 illustrate the effect of CFRP laminate thickness, ultimate strength and elastic modulus, respectively. Three different thicknesses of CFRP laminates are used as 0.75, 1.00 and 1.50 mm. Finite element modeling of beams and the P-D

diagrams show that by doubling the thickness of CFRP laminates the beam strength increases by 50%, however the ductility decreases slightly up to 10% (Fig. 11).

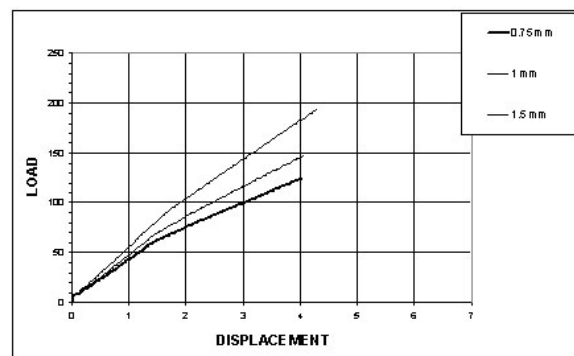


Fig. 11. Load-deflection curves for beams with different CFRP thickness.

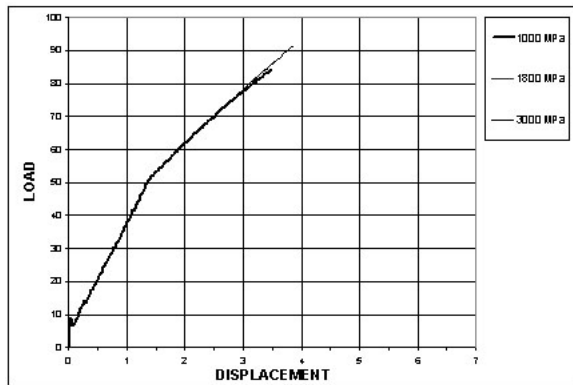


Fig. 12. Load-deflection curves for beams with different CFRP ultimate strength.

Three different ultimate strength of the CFRP laminates (i.e., 1000, 1800 and 3000 MPa) are modeled to investigate the effect of ultimate strength. Increase in ultimate strength of CFRP laminates from 1000 MPa to 3000 MPa, improves the beam strength by less than 10% (Fig. 12). Fig. 13 shows that when the concrete is crushed in compression, the fibers reach to only 60% of its ultimate strength. Therefore, it is recommended to use CFRP laminates with high strength when adequate compression reinforcement is present in the beam.

The effect of CFRP elastic modulus is investigated using three different elastic moduli. Fig. 14 shows that increasing the elastic modulus of CFRP laminates by 10 MPa, increases the beam strength and ductility by 7% and 3%, respectively.

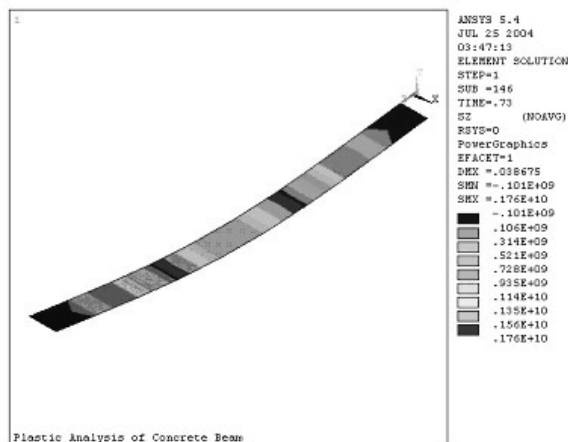


Fig. 13. Stress in CFRP laminates in the beam with 3000 MPa CFRP ultimate strength, at crushing of the concrete.

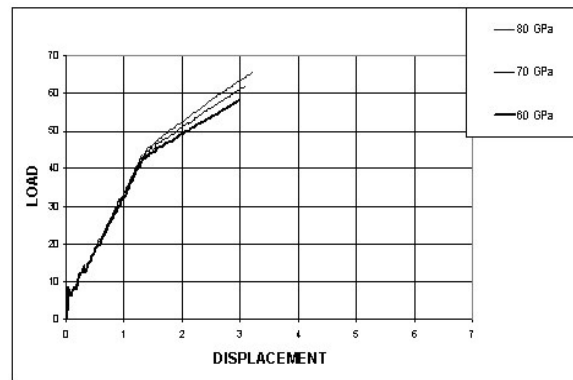


Fig. 14. Load-deflection curves for beams with different CFRP elastic modulus.

## 6. CONCLUSIONS

This study concentrated on the analytical evaluation of the behavior of RC beams strengthened in flexure with CFRP laminates. The results of finite element analyses indicate that significant strengthening of reinforced concrete beams can be obtained by bonding a relatively small amount of CFRP to the tension face of the beam. The analytical approach predicts the available experimental results satisfactorily. The degree of strength enhancement attained depends on 1) amount of tension steel reinforcement, 2) CFRP thickness, 3) CFRP ultimate strength and 4) CFRP elastic modulus. Based on results from finite element analyses the following conclusions are obtained:

1- Addition of CFRP laminates increases stiffness and ultimate strength of the beams. The maximum increase in load carrying capacity due to strengthening was 120% with respect to control beam. Generally, beams with lower steel content carried higher loads at ultimate.

2- The ductility of strengthened beams decreased up to 70% and the level of reduction depends on CFRP characteristics.

3- The general behavior of the finite element models represented by the load deflection plots at midspan show good agreements with experimental results. The ultimate load obtained by finite element analyses is lower than the experimental ultimate load by 3 to



9%. This is probably due to using assumed material properties instead of actual properties.

4-Parametric study shows that increase in the steel area ratio from  $r=0.013$  to  $r=0.032$  decreases the rate of strengthening from 98% to 26% with respect to control beam. Also increase in the thickness of the CFRP laminates increases the beam strength, however the ductility decreases slightly. Increase in ultimate strength of the fiber laminates results in increase in the strength and ductility, provided that adequate compressive steel is present. Also, by increasing the elastic modulus of the CFRP laminates the ductility and strength of beams increase slightly.

#### ACKNOWLEDGEMENT

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