New Predictive Models for the $v_{max}/a_{max}$ Ratio of Strong Ground Motions using Genetic Programming

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Abstract: Although there is enough knowledge indicating on the influence of frequency content of input motion on the deformation demand of structures, state-of-the-practice seismic studies use the intensity measures such as peak ground acceleration (PGA) which are not frequency dependent. The $v_{max}/a_{max}$ ratio of strong ground motions can be used in seismic hazard studies as the representative of frequency content of the motions. This ratio can be indirectly estimated by the attenuation models of PGA and PGV which are functions of earthquake magnitude, source to site distance, faulting mechanism, and local site conditions. This paper presents new predictive equations for $v_{max}/a_{max}$ ratio based on genetic programming (GP) approach. The predictive equations are established using a reliable database released by Pacific Earthquake Engineering Research Center (PEER) for three types of faulting mechanisms including strike-slip, normal and reverse. The proposed models provide reasonable accuracy to estimate the frequency content of site ground motions in practical projects. The results of parametric study demonstrate that $v_{max}/a_{max}$ increases through increasing earthquake moment magnitude and source to site distance while it decreases with increasing the average shear-wave velocity over the top 30m of the site.

Keywords: Earthquake, $v_{max}/a_{max}$ Ratio, Genetic Programming, Predictive equation

1. Introduction

Earthquake is an unavoidable natural phenomenon of the earth. The damage potential of earthquakes depends on the characteristics of ground motion and local site condition. Three characteristics of earthquake motion including the amplitude, frequency content and duration of motion are of primary significance in earthquake engineering [1]. Ground motion parameters or intensity measures are essential for describing the important characteristics of strong ground motion and are commonly presented in quantitative forms referred to as “predictive equations” in terms of earthquake magnitude, source-to-site distance, faulting mechanism, and local site conditions. Beside the theoretical models of ground motion, most of these equations are empirically derived by the regression analysis of recorded strong motion data.

The commonly used intensity measures of ground motion involve peak ground acceleration (PGA) and peak ground velocity (PGV). Since PGV is less sensitive to the higher-frequency components of the ground motion, it is more likely to accurately characterize ground motion amplitude at intermediate frequencies[1]. Although PGA and PGV are very useful intensity measures for seismological studies, none of them can provide any information on the frequency content or duration of the motion. It is in spite of the consensus among the researchers regarding the influences of frequency content on earthquake induced deformations of civil structures. Consequently, PGA and PGV have to be supplemented by additional information for the proper characterization of a ground motion.

The ratio of PGV to PGA ($v_{max}/a_{max}$ ratio) is a ground motion parameter which provides information about frequency content of the input motion. Since PGA and PGV are usually associated with motions of different frequencies, the $v_{max}/a_{max}$ ratio should be related to the frequency content of the motion [2].

Based on the theory of one-dimensional shear wave propagation through uniform elastic medium, the equations of displacement ($u$), velocity ($\dot{u}$), and acceleration ($\ddot{u}$) for a
harmonic motion with natural circular frequency of $\omega$ can be written as:

$$u(z,t) = A \cos \left( \frac{\omega z}{V} \right) e^{i\omega t}$$

$$\dot{u}(z,t) = iA\omega \cos \left( \frac{\omega z}{V} \right) e^{i\omega t}$$ (1)

$$\ddot{u}(z,t) = -A\omega^2 \cos \left( \frac{\omega z}{V} \right) e^{i\omega t}$$

Where $z$, $v$, $u_{\text{max}}$, $v_{\text{max}}$, and $a_{\text{max}}$ refer to depth, shear wave velocity of the elastic medium, maximum displacement, maximum velocity, and maximum acceleration, respectively. According to these equations, $v_{\text{max}}/a_{\text{max}}$ is proportional to $1/\omega$.

As a measure of the frequency content of a ground motion, $v_{\text{max}}/a_{\text{max}}$ ratio is proportional to distance and earthquake magnitude. McGuire [2] summarized the magnitude and distance dependencies of the $v_{\text{max}}/a_{\text{max}}$ ratio proposed by several researchers, as presented in Table 1. According to this table, $v_{\text{max}}/a_{\text{max}}$ increases with increasing earthquake magnitude and distance for both rock and soil site classes. The influences of predictive variables such as source to site distance and site classes on the trend of $v_{\text{max}}/a_{\text{max}}$ ratio were also studied by Seed and Idriss [3] and Yang and Lee [4]. According to their work, the $v_{\text{max}}/a_{\text{max}}$ ratio increases with increasing source to site distance while it tends to be larger as the soils become softer.

The $v_{\text{max}}/a_{\text{max}}$ ratio is a very important parameter to characterize the damage potential of near-fault ground motions and indicated as being a measure of destructiveness [5]. The ground motions with higher $v_{\text{max}}/a_{\text{max}}$ values have larger damage potential [6,7]. It also has the greatest influence on the inelastic displacement ratio spectra (IDRS) for near-fault ground motions among the other parameters such as the PGV, and the maximum incremental velocity (MIV) [8]. The $v_{\text{max}}/a_{\text{max}}$ ratio of the near-fault earthquake records significantly influences the dynamic response of the bridges. The Base shear response and displacement for both the intermediate-period and short-period isolated bridges strongly depend on the $v_{\text{max}}/a_{\text{max}}$ value and the energy of the ground motion, which are related to each other [9].

Another application of $v_{\text{max}}/a_{\text{max}}$ ratio reveals in the studies indicating that the frequency content of excitation cannot be ignored in liquefaction potential assessment of soils. Orense [10] proposed an alternative approach to evaluate liquefaction potential of loose saturated cohesionless deposits. Compared to the conventional methods which are only based on PGA, the Orense’s approach depends on both PGA and $v_{\text{max}}/a_{\text{max}}$ ratio. The conventional methods which neglect the effects of frequency content are vulnerable to high frequency acceleration spikes of input motion.

The accurate estimation of ground motion parameters is an important concern in geotechnical earthquake engineering. Errors in the estimation of predictive equations can be divided into two groups: 1) the impreciseness and deficiency of input database. 2) the computational errors of regression analysis. In the recent years, new aspects of modeling, optimization, and problem solving have been evolved in light of the pervasive development in computational software and hardware. These aspects of software engineering are referred to as artificial intelligence which includes artificial neural network, fuzzy logic, genetic algorithm (GA), and genetic programming (GP).

The study presented herein employs genetic programming (GP) approach to develop predictive equations for $v_{\text{max}}/a_{\text{max}}$ ratio in terms of $v_{\text{max}}/a_{\text{max}}$ ratios proportional to distance and earthquake magnitude

<table>
<thead>
<tr>
<th>Site conditions</th>
<th>Magnitude dependence</th>
<th>Distance dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock sites</td>
<td>$e^{0.4M}$</td>
<td>$R^{0.12}$</td>
</tr>
<tr>
<td>Soil sites</td>
<td>$e^{0.15M}$</td>
<td>$R^{0.23}$</td>
</tr>
</tbody>
</table>
earthquake magnitude, source-to-site distance, and local site conditions for three types of faulting mechanisms including strike-slip, normal, and reverse. A comprehensive database of strong ground motions assembled by Pacific Earthquake Engineering Research Center (PEER) is used to derive the equations.

2. Genetic Algorithm and Genetic Programming

Genetic algorithm (GA), which is generally used as an optimization technique to search the global minima of a function, was initially developed by John Holland at the University of Michigan in 1975. Genetic algorithms work by evolving a population of individuals over a number of generations and together with “evolution strategies” and “evolutionary programming” methods form the backbone of the field of “evolutionary computation”. A genetic algorithm starts with randomly generated population of N candidate solutions (individuals). Each individual is expressed in different types such as binary strings (0,1), real strings (0,1,…,9), and representation of tree (computer programs). A fitness value is applied to each individual in the population to examine how well each chromosome solves the problem. At each generation a new set of solutions for reproduction is created by process of selecting individuals according to their fitness. The selected individuals are called parents.

Koza [11] developed a special genetic algorithm known as “genetic programming (GP)” which its population is represented by the “parse tree” (computer programs). A population member in GP is a structured computer program consisting of functions and terminals. The functions and terminals are picked out from a set of functions and a set of terminals. A function set could contain functions such as basic mathematical operators (+, -, *, /, etc.), Boolean logic functions (AND, OR, NOT, etc.), or any other user defined function. The terminal set contains the arguments for the function and can consist of numerical constants, etc.

The functions and terminals are selected randomly and put together to form a computer model in a tree-like structure with a root point with branches extending from each function and ending in a terminal. An example of a simple tree representation of a GP model is shown in Figure 1.

Each individual in the population receives a measure of its fitness. There are several methods to estimate the fitness of each individual; one of them is to minimize the error which is difference between the predicted and measured values. At each generation a new population is created by choosing individuals according to their fitness and breeding them together using the genetic

![Typical GP tree representation](https://example.com/Fig1.png)

![Typical cross-over operation in GP](https://example.com/Fig2.png)

![Typical mutation operation in GP](https://example.com/Fig3.png)
operators (cross-over and mutation).

Cross-over operator produces two new individuals for new generation by choosing two individuals of current population and randomly changing one's branch with another (Figure 2). Mutation operator produces one new individual for new generation by randomly changing a node of one of the trees in current population (Figure 3). All of these steps are repeated until a suitable solution is found or a certain number of generations have been reached [12].

The use of simple genetic algorithm in searching the critical factor of safety in slope stability analysis [13] and genetic programming in the evaluation of liquefaction induced lateral displacements of ground [14] are two applications of evolutionary approach in geotechnical engineering.

3. The Database

The data used in this study is the database compiled by Power et al [15] during PEER-NGA project. The database contains the strong ground motion data of shallow crustal earthquakes recorded at active tectonic regions of the world over a broad range of magnitude and distance. Boore and Atkinson [16] and Campbell and Bozorgnia [17] reduced the database to free-field conditions and proposed worldwide attenuation relations for PGA, PGV and response spectra of acceleration at 5% damping. Since the current study aims to propose predictive models of $v_{max}/a_{max}$ ratio for free-field conditions, parts of the data have to be excluded from the original database released by Power et al [15]. The exclusion criteria are similar to those were considered by Boore and Atkinson [16] and Campbell and Bozorgnia [17]. Finally, 1448 records from 60 mainshocks have been divided into three groups in terms of their fault types to be used as input data for three individual equations.

The predictors or independent variables include moment magnitude ($M$), $R_{jb}$ distance (closest distance to the surface projection of the fault plane) which is approximately equal to the epicentral distance for events of $M<6$ [16], $V_{s30}$ (average shear-wave velocity over the top 30m of site), fault type (i.e., normal, strike-slip, and reverse). The distribution of the data used to derive the predictive equations is shown in Figure 4 illustrating $M$ versus $R_{jb}$ per any type of faulting mechanism. The maximum and minimum values of $M$, $R_{jb}$, $V_{s30}$ and $v_{max}/a_{max}$ ratio according to their fault types are presented in Table 2.

The considerable differences among the ranges of the predictive variables may profoundly affect the fitness procedure. Thus, the predictive variables were normalized between 0 and 1.
according to Equation 2 and their minimum and maximum values cited in Table 2. These normalized data have been employed as input data for GP.

\[
X_{\text{std}} = \frac{X - X_{\text{min}}}{X_{\text{max}} - X_{\text{min}}}
\]  

(2)

Where \(X\) is a given predictive variable that will be normalized to \(X_{\text{std}}\), \(X_{\text{min}}\) and \(X_{\text{max}}\) are the minimum and maximum value of \(X\), respectively.

### 4. The Proposed Equations of \(\frac{v_{\text{max}}}{a_{\text{max}}}\) Ratio by Genetic Programming

GPLAB is a genetic programming toolbox written by Silva [18] based on MATLAB software. Several recent studies have used this toolbox for engineering estimations [e.g. 19,20]. In this study GPLAB is used to obtain the predictive equations of \(\frac{v_{\text{max}}}{a_{\text{max}}}\) ratio. The initial population of trees, consisting of functions and terminals, is created in the beginning of GPLAB run based on Ramped Half-and-Half method which is one of the initialization methods in GP [11,20]. Trees in GPLAB are subjected to a set of restrictions on depth or size (number of nodes) to avoid bloating. Bloating means an excessive code growth without any improvement in fitness. Genetic operators need parent individuals to produce their children. These parents are selected according to Lexicourg sampling method [11,18]. In this method, random numbers of individuals are chosen from the population and their best one is selected as a parent. The sum of the absolute difference between the measured values and those predicted by GP is the fitness criterion used to choose the best individuals as parent. GPLAB will run until the maximum generation or robustness defined by the user reaches.

All data associated with three types of faulting mechanisms are divided into training and validation sets with equal mean and standard deviation. 80% of data for each group have been assigned to training sets whereas the rest (20%) are for validation sets. The purpose of using validation set is to ensure that the final model obtained by GP has the ability to properly estimate \(\frac{v_{\text{max}}}{a_{\text{max}}}\) ratio for unseen or untrained cases. Training and validation sets per each type of faulting mechanisms are shown in Table 3.

The predictive equations obtained from GP for three types of faulting mechanisms including strike-slip, normal, reverse are shown in Equations 3, 4 and 5, respectively.

### Table 2. Ranges of \(M\), \(R_{jb}\), \(V_{s30}\) and \(\frac{v_{\text{max}}}{a_{\text{max}}}\) ratio according to three fault types.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>reverse (835 data)</th>
<th>normal (112 data)</th>
<th>strike-slip (490 data)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>max</td>
<td>min</td>
<td>max</td>
</tr>
<tr>
<td>(M)</td>
<td>7.62</td>
<td>5.33</td>
<td>6.9</td>
</tr>
<tr>
<td>(R_{jb}) (km)</td>
<td>193.91</td>
<td>0</td>
<td>133.34</td>
</tr>
<tr>
<td>(V_{s30}) (m/s)</td>
<td>1525.85</td>
<td>116.35</td>
<td>1000</td>
</tr>
<tr>
<td>(\frac{v_{\text{max}}}{a_{\text{max}}})</td>
<td>0.554</td>
<td>0.022</td>
<td>0.182</td>
</tr>
</tbody>
</table>

### Table 3. The number of data considered for training and validation sets per each type of faulting mechanisms.

<table>
<thead>
<tr>
<th></th>
<th>reverse</th>
<th>normal</th>
<th>strike-slip</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of cases</td>
<td>834</td>
<td>112</td>
<td>490</td>
</tr>
<tr>
<td>Training</td>
<td>667</td>
<td>90</td>
<td>392</td>
</tr>
<tr>
<td>Validation</td>
<td>167</td>
<td>22</td>
<td>98</td>
</tr>
</tbody>
</table>
Where:
\[
\sigma \left( \frac{\gamma_{\text{max}}}{a_{\text{max}}}_{\text{normalized}} \right) = 0.101
\]
\[
V = \frac{V_{\text{30}} - 116.35}{1428 - 116.35}, \quad R = \frac{R_{jb}}{199.27}, \quad M = \frac{M - 4.53}{7.9 - 4.53}
\]
\[
\frac{\gamma_{\text{max}}}{a_{\text{max}}}_{\text{normalized}} = 0.0194 + 0.569\left( \frac{\gamma_{\text{max}}}{a_{\text{max}}}_{\text{actual}} \right)
\]
\[
\frac{\gamma_{\text{max}}}{a_{\text{max}}}_{\text{normalized}} = (-0.341V^2 + 0.495V - 0.133)M^4 + (-1.023V^3 + 2.339V^2 - 1.269V + 0.372)M - (0.344V - 0.12)(R + 0.115) + \sigma \left( \frac{\gamma_{\text{max}}}{a_{\text{max}}} \right)
\]

Where:
\[
\sigma \left( \frac{\gamma_{\text{max}}}{a_{\text{max}}}_{\text{normalized}} \right) = 0.093
\]
\[
V = \frac{V_{\text{30}} - 196.25}{1000 - 196.25}, \quad R = \frac{R_{jb}}{133.34}, \quad M = \frac{M - 4.92}{6.9 - 4.92}
\]
\[
\frac{\gamma_{\text{max}}}{a_{\text{max}}}_{\text{normalized}} = 0.01 + 0.172\left( \frac{\gamma_{\text{max}}}{a_{\text{max}}}_{\text{actual}} \right)
\]
\[
\frac{\gamma_{\text{max}}}{a_{\text{max}}}_{\text{normalized}} = (-0.511V^3 + 1.375V^2 - 1.229V + 0.365)M^2 + (0.511V^3 - 1.375V^2 - 0.365V + 1.229V^2)M^4 + (1.022V^3 + 1.022V^2 + 0.167)M + \sigma \left( \frac{\gamma_{\text{max}}}{a_{\text{max}}} \right)
\]

Where:
\[
\sigma \left( \frac{\gamma_{\text{max}}}{a_{\text{max}}}_{\text{normalized}} \right) = 0.141
\]
\[
V = \frac{V_{\text{30}} - 116.35}{1525.85 - 116.35}, \quad R = \frac{R_{jb}}{193.91}, \quad M = \frac{M - 5.33}{7.62 - 5.33}
\]
\[
\frac{\gamma_{\text{max}}}{a_{\text{max}}}_{\text{normalized}} = 0.022 + 0.533\left( \frac{\gamma_{\text{max}}}{a_{\text{max}}}_{\text{actual}} \right)
\]

Where \(M\), \(R_{jb}\), \(V_{30}\) and \(\sigma\) are moment magnitude, Boore-Joyner distance, shear wave velocity over the top 30m of site soil, and standard deviation of the equations, respectively.

The proposed equations are valid only in the parameters ranges shown in Table 2. As seen, these ranges are different for any type of faulting mechanism.

In order to examine the robustness of the GP results, the values of coefficient of determination \((R^2)\), root mean squared error \((RMSE)\) and mean absolute error \((MAE)\) between actual and predicted \(\frac{\gamma_{\text{max}}}{a_{\text{max}}}\) ratio have been obtained according to Equations 6, 7, and 8 while \(RMSE\) and \(MAE\) are in second(s).

\[
\sum_{N} \left( \frac{X_{w}}{X_{p}} \right)^2 - \sum_{N} \left( \frac{X_{w} - X_{p}}{X_{p}} \right)^2
\]

\[
R^2 = \sum_{N} \left( X_{w} \right)^2 / \sum_{N} \left( X_{w} \right)^2
\]

\[
RMSE = \sqrt{\frac{1}{N} \sum_{N} \left( X_{w} - X_{p} \right)^2}
\]

\[
MAE = \frac{1}{N} \sum_{N} \left| X_{w} - X_{p} \right|
\]

<table>
<thead>
<tr>
<th>Fault types</th>
<th>Groups</th>
<th>GP results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(R^2)</td>
</tr>
<tr>
<td>strike-slip</td>
<td>Training</td>
<td>0.861</td>
</tr>
<tr>
<td></td>
<td>Validation</td>
<td>0.86</td>
</tr>
<tr>
<td>normal</td>
<td>Training</td>
<td>0.862</td>
</tr>
<tr>
<td></td>
<td>Validation</td>
<td>0.861</td>
</tr>
<tr>
<td>reverse</td>
<td>Training</td>
<td>0.866</td>
</tr>
<tr>
<td></td>
<td>Validation</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Table 4. The values of \(R^2\), \(RMSE\) and \(MAE\) for the predictive equations derived by GP
Where $N$ is the number of data, $X_m$, $X_p$ are measured and predicted values, respectively. Table 4 presents the values of $R^2$, $RMSE$ and $MAE$ for the proposed equations of ratio $v_{\text{max}} / a_{\text{max}}$.

The values of $v_{\text{max}} / a_{\text{max}}$ ratio predicted by GP were plotted versus their corresponding actual values for fault types of strike-slip, normal and reverse in figures 5, 6 and 7, respectively.

5. Comparison with Boore and Atkinson (2007)'s Attenuation Model

Since there is not any predictive equation in the literature for the direct estimation of $v_{\text{max}} / a_{\text{max}}$ ratio, the PGV and PGA attenuation relationships that were recently proposed by Boore and Atkinson [16] can be used as the estimators of this ratio. The equations proposed by Boore and Atkinson are limited to the following conditions:

\begin{align*}
M & = 5 - 8 \\
R_{j0} & < 200 \text{ km} \\
V_{s(30)} & = 180 - 1300 \text{ m/s}
\end{align*}

Therefore, the comparison between Boore and Atkinson’s equations and the proposed predictive equations is performed only for 1256 data which meet the mentioned conditions.

\begin{align*}
RMSE & = \sqrt{\frac{\sum N (X_m - X_p)^2}{N}} \\
MAE & = \frac{\sum N |X_m - X_p|}{N}
\end{align*}
Table 5 presents the values of $R^2$, RMSE, and MAE for the predictive equations derived by GP and the predictive equation estimated by Boore and Atkinson (2007) for each fault type and total data. Furthermore, Figures 8 and 9 illustrate the values of $v_{\text{max}}/a_{\text{max}}$ ratios predicted by GP and Boore and Atkinson (2007)’s model versus actual $v_{\text{max}}/a_{\text{max}}$ ratios, respectively. The comparison confirms the superiority of the proposed model with respect to the indirect use of Boore and Atkinson (2007)’s model for estimating $v_{\text{max}}/a_{\text{max}}$ ratio.

6. Parametric Study

In order to best understand the dependence of $v_{\text{max}}/a_{\text{max}}$ ratio on $M$, $R_{jb}$, and $V_{s30}$, parametric studies were performed. For this purpose, the variation of $v_{\text{max}}/a_{\text{max}}$ ratio with respect to one of the predictor variables was studied for three fault types while the other two variables are kept constant. Figures 10, 11, and 12 show three samples of the parametric study.

As observed in Figure 10 and 12, the $v_{\text{max}}/a_{\text{max}}$ ratio increases with increasing source-to-site distance and magnitude which is similar to the results of Table 1. Seismological studies have indicated that peak ground acceleration and velocity are usually caused by seismic waves of different frequencies. Peak ground acceleration is associated with high frequency waves whereas peak ground velocity is related to moderate or low frequency waves. Because of the frequency-dependent attenuation of seismic waves, peak accelerations and velocities are expected to decrease with distance from the source.

Table 5

<table>
<thead>
<tr>
<th>Fault types</th>
<th>GP results</th>
<th>Boore &amp; Atkinson results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^2$</td>
<td>RMSE (sec)</td>
</tr>
<tr>
<td>strike-slip</td>
<td>0.860</td>
<td>0.060</td>
</tr>
<tr>
<td>normal</td>
<td>0.854</td>
<td>0.033</td>
</tr>
<tr>
<td>reverse</td>
<td>0.861</td>
<td>0.067</td>
</tr>
<tr>
<td>All data</td>
<td>0.861</td>
<td>0.064</td>
</tr>
</tbody>
</table>

Fig. 8. The values of the $v_{\text{max}}/a_{\text{max}}$ ratio predicted by GP versus the corresponding actual values for all data.

Fig. 9. The values of the $v_{\text{max}}/a_{\text{max}}$ ratio predicted by Boore and Atkinson (2007) versus the corresponding actual values for all data.
ground acceleration attenuates more rapidly with distance than peak ground velocity. As a result, one would expect that ground motions experienced near an earthquake source have lower $v_{max}/a_{max}$ ratios than ground motions at a large distance from the source of seismic energy release. Moreover, the larger the magnitude of an earthquake, the longer the duration of strong

![Figure 10](image-url)

**Fig. 10.** Variations of $v_{max}/a_{max}$ ratio with respect to Boore-Joyner distance ($R_{jb}$) for strike-slip faulting mechanism and $M=4.5$, $M=5.5$, $M=6.5$, $M=7.5$ while the values of $V_{s30}$ have been kept constant at the mean value of shear wave velocities in the dataset of strike-slip faulting (i.e., 353 m/s).

![Figure 11](image-url)

**Fig. 11.** Variations of $v_{max}/a_{max}$ ratio with respect to shear wave velocity ($V_{s30}$) for normal fault while $M$ and $R_{jb}$ are equal to their mean values in the dataset of normal faulting (i.e., 5.4 and 58.58 km).
ground motion; if the distance from the epicenter remains constant. The records with low \( \frac{v_{\text{max}}}{a_{\text{max}}} \) ratios have shorter durations of strong shaking than those with intermediate \( \frac{v_{\text{max}}}{a_{\text{max}}} \) ratios. Therefore, the \( \frac{v_{\text{max}}}{a_{\text{max}}} \) ratio of earthquake records is well correlated to the strong-motion duration of the records.

It is known from the earliest studies of geotechnical earthquake engineering that the natural frequency of a level site increases with increasing the shear wave velocity of its constituting soil. Figure 11 shows that the \( \frac{v_{\text{max}}}{a_{\text{max}}} \) ratio decreases with increasing shear wave velocity. It means that the \( \frac{v_{\text{max}}}{a_{\text{max}}} \) ratio tends to be larger as the site soil becomes softer. Therefore, the greater \( \frac{v_{\text{max}}}{a_{\text{max}}} \) ratios are associated with soft soils having higher natural period and decrease with decreasing natural period (or increasing shear wave velocity) of site soil. Note that Seed and Idriss [3] and Yang and Lee [4] reported similar findings.

7. Summary and Conclusions

\( \frac{v_{\text{max}}}{a_{\text{max}}} \) ratio is a ground motion parameters which is used in seismic hazard studies. It can provide information about frequency content which affects the seismic response of structures. The accurate estimation of ground motion parameters is an important concern in geotechnical and earthquake engineering. New equations, based on genetic programming, have been presented herein to predict the \( \frac{v_{\text{max}}}{a_{\text{max}}} \) ratios of strong ground motions in free-field condition. One of the advantages of the GP approach over the black-box artificial intelligent methods such as neural network is its capability to present an explicit relationship between input and output parameters without assuming prior form of the relationship.

The data used in this study is the database compiled by Power et al. (2006) during PEER-NGA project. 1448 records from 60 mainshocks were used as input data which are divided into three groups in terms of their fault types. Three relationships between \( \frac{v_{\text{max}}}{a_{\text{max}}} \) ratio and the predictor variables including moment magnitude (\( M \)), Boore and Joyner distance (\( R_{\text{jb}} \)) and shear wave velocity (\( V_{\text{s30}} \)) for three different types of fault including strike-slip, normal and reverse have been proposed (Equations 3, 4 and 5). Figures 8, 9, and Table 5 confirm the superior performance of the proposed GP models over the attenuation models that were recently proposed.

![Fig. 12. Variations of \( \frac{v_{\text{max}}}{a_{\text{max}}} \) ratio with respect to \( M \) for reverse faulting mechanism while \( R_{\text{jb}} \) and \( V_{\text{s30}} \) are equal to their mean values in the dataset of reverse faulting (i.e., 48.8 km and 397 m/s).](image-url)
by Boore and Atkinson (2007).

The results of parametric study demonstrate that $\frac{v_{\text{max}}}{a_{\text{max}}}$ ratio increases with increasing earthquake magnitude and distance and also decreases with increasing shear wave velocity.

References


