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An improved solution to capillary rise of water in soils

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Abstract

Evaluating the rate and maximum height of capillary rise is of prime interest in unsaturated soil mechanics. Antecedent solutions to this problem have dwelled mostly on determining the maximum capillary rise height, overlooking moisture and suction changes in the capillary region. A comprehensive improved solution for the capillary rise of water in soils is presented. Salient features of the formulation including; consideration of initial soil suction (if any) prior to capillary rise, and determination of water content variation in the capillary region are elaborately discussed. Results reveal that suction head variation within the capillary region is non-linear, where the curvature decreases as water rises to higher elevations. The solution is verified and compared with existing solutions, by means of two sets of experimental data available in the literature. The comparison suggests that the improved formulation is more accurate and versatile than previous solutions for capillary rise.

Keywords: Unsaturated soils, capillary rising, unsaturated transient seepage, suction

1. Introduction

Capillary rise, that is the movement of pore water against the gravity force, is a prominent phenomenon in unsaturated soil mechanics. Well known samples of capillary rise in geotechnical structures are above the phreatic surface in embankment dams and beneath road pavements. Also many environmental consequences such as the transportation of saline materials to the ground surface are attributed to capillary rise [1].

The capillary rise of water significantly affects physical and mechanical soil properties such as matric suction and soil virtual cohesion [2]. Hence, the rate, maximum height, and fluid storage capacity of capillary rise are of paramount interest in unsaturated soil mechanics. Heretofore, the aforesaid characteristics of capillary rising have been extensively investigated by applying experimental measurements, theoretical formulations, and numerical modeling. However, there are some unaddressed problems such as; the role of initial moisture on water rising in soils and the variation of soil moisture below the wetting front (i.e. in the capillary region) during capillary rise.

This treatment aims to develop an enhanced solution for capillary rise in soils. To this end the paper is organized in three main parts: (1) reviewing available theoretical solutions for the rate of capillary rise in soils while debating their hypothesis, advantages and limitations, (2) elaborately introducing an improved solution for the capillary rise of water in soils and (3) assessing the accuracy of the improved solution with reference to antecedent experimental data.

2.Theoretical background

2.1. Principles of capillary rise

When two fluids such as water and air are nested contiguously, the imbalanced intermolecular forces result in a pressure difference (U_a-U_w) across the interface. Considering the classical capillary tube schema in Figure 1, to achieve mechanical equilibrium, surface tension (T_s) is mobilized at the

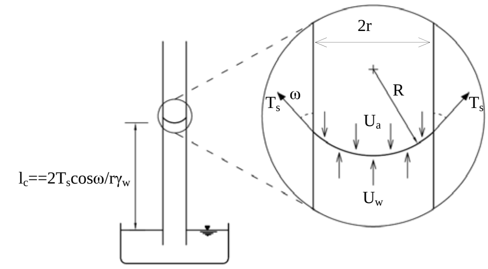

Fig. 1. The capillary rise in narrow tube and the equilibrium condition at meniscus

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air-water boundary forcing the interface to form a meniscus (with curvature radius of R). Based on the Young -Laplace Equation, the following relation is valid at the interface:

$$U_a - U_w = \frac{2T_s}{R} \quad (1)$$

Accordingly, water is driven up the capillary tube via the pressure difference at the meniscus that provides hydraulic head, equivalent to maximum height of water rise (l_c). Hence, the following relation is derived for water's maximum rising height in the capillary tube [3]:

$$l_c = \frac{2T_s \cos\omega}{r\gamma_w} \quad (2)$$

Where; r= radius of capillary tube, γ_w= unit weight of water, and ω= wetting angle between water and the wall material of tube.

2.2. Capillary tube model for soils

In porous media, such as soils, water can rise through the network of pore spaces and pore constrictions. In order to model capillary rise in soils, some researchers have considered the pore constrictions as a system of unconnected, parallel capillary tubes with a radius equivalent to the soil's average pore size. Based on this analogy and recalling Equation 2, coarse soils, i.e. gravel and sand, have low capillary rise heights, while silts and clays, with very small pore sizes, can theoretically give very high capillary rises.

Considering the capillary tube model (Equations 1 and 2) and implementing Poiseulle's Law, the wetting front reaches l at time t, defined by [4, 5, 6 & 7]:

$$t = \frac{8\mu}{\gamma_w r^2}\left(l_c \times ln\frac{l_c}{l_c - l} - l\right) \quad (3)$$

Where, l= elevation of wetting front (i.e., border demarcating the initially dry soil and the region wetted by capillary) equivalent to the capillary rising height and defined positive upward from the water table, μ= water's dynamic viscosity, and l_c= ultimate wetting front elevation (maximum height of capillary rise). Equation 3 is straightforward, yet has some limitations as the assumptions are only valid for homogenous soils with uniformly distributed pore sizes and constant moisture content. Regarding the former, in reality soils have interconnected tortuous pore spaces [8]; and concerning the latter, practically some time (approximately 2 hours) after capillary commencement the soil's moisture content can no longer be considered constant [6 & 9]. Therefore the capillary tube model has limited accuracy in simulating real soil behavior.

2.3. Unsaturated seepage in soils

Terzaghi [10] postulated that Darcy's Law is approximately valid for unsaturated seepage during capillary rise. Thus discharge velocity (q) of the wetting front is presented by:

$$q = n\frac{dl}{dt} = k_s i \quad (4)$$

Where, k_s and n are the saturated hydraulic conductivity and porosity of the soil respectively, and i is the hydraulic gradient of capillary rise (i.e., hydraulic gradient of the wetting front located at elevation l) defined as:

$$i = \frac{l_c - l}{l} \quad (5)$$

The unsaturated seepage framework (Equations 4 and 5) alleviates limitations of the capillary tube model. Nevertheless, incorporating k_s considerably overestimates the rate of capillary rise; since, the soil region above the water front is unsaturated and possesses a hydraulic conductivity significantly lower than k_s [11].

Lu and Likos [11] amended Terzaghi's [10] solution by substituting the saturated hydraulic conductivity (k_s) with Gardner's [12] unsaturated hydraulic conductivity (k); which is an exponential function of soil suction head (h), as follows:

$$k(h) = k_s\, exp(ah) \quad (6)$$

Where, a= parameter defining the rate of hydraulic conductivity decrease. As illustrated in Figure 2, a is interpreted as the inverse of air-entry head (i.e., capillary fringe height) value (l_a) defined from the Soil-Water Characteristic Curve (SWCC); i.e., $a=1/l_a$.

By substituting Equations 5 and 6 in Equation 4, Lu and Likos [11] suggested the following equation for the rate of capillary rise:

$$\frac{dl}{dt} = \frac{k_s\, exp(a l)}{n} \times \frac{l_c - l}{l} \quad (7)$$

Recently, Jitrapinate et al. [14] proposed a semi empirical relation for the capillary rise rate, where Philip's Equation is implemented for upward infiltration occurring in capillary rise. Thus, for any elevation (z) at any time (t) the following is valid:

$$z = b_0(\theta)t^{1/2} - b_1(\theta)t + b_2(\theta)t^{3/2} \quad (8)$$

Where; $b_0(\theta)$, $b_1(\theta)$ and $b_2(\theta)$ are functions of soil moisture (θ), experimentally determined from at least three capillary tests. This relation; does not consider the maximum height of capillary rise (l_c) or the driving capillary gradient (i), and is unable to explicitly determine the wetting front location (l) during capillary rise.

Anticipatively, Equation 7 is more versatile than other solutions; however, it has the following limitations:

1. The suction head (h) implemented in unsaturated hydraulic conductivity is considered equal to the wetting front elevation (that is; $h=l$ in Equation 7). This perception, illustrated in

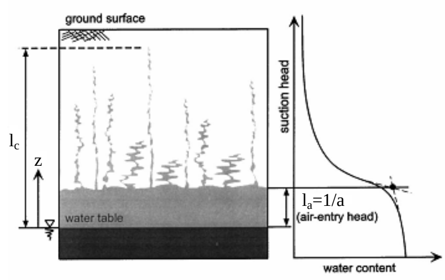

Fig. 2. The capillary fringe (air-entry head) and ultimate capillary height in capillary rising- redrawn after Lu and Likos [13]

Figure 3-b, is only true when the wetting front has stopped progressing and reached ultimate elevation of capillary rise (l_c), where conditions of hydrostatic equilibrium are satisfied. The physical reasoning is that, in equilibrium conditions, water is stationary in the capillary region (i.e. there is no water flow), therefore, spatial variation of total head in the soil profile is zero (i.e. total head is identical at any arbitrary elevation in the capillary region). Since total head is equal to sum of suction head and elevation head at any point in the unsaturated soil, at equilibrium suction head varies linearly with elevation. In reality, however, the wetting front is rising continuously, thus equilibrium is not reached, and water seeps upward in the capillary region, imposing a non-linear suction head variation with elevation, as illustrated in Figure 3-c. Capillary tests by Jitrapinate et al. [14], demonstrated moisture variation in capillary region below (i.e., lower) the wetting front; hence, validating the aforesaid hypothesis. Based on the assumptions of Equation 7, the wetting front rises continuously, albeit, there will be no flow in the capillary region and this contradicts continuity law for flow.

2. Equation 7 overlooks the role of soil initial suction head (prior to capillary commencement) in the rate of capillary rise; that is, U_a-U_w for the unsaturated region above the water table is not considered. Interestingly, capillary tests on uniform glass spheres by Lu et al. [15] revealed that the presence of initial moisture between grains increases the rate of capillary rise.

3. In Equation 7, porosity (n) is implemented to define seepage velocity (dl/dt). For unsaturated soils, not all pore spaces in the capillary region are filled with water, thus the volumetric water content (θ) is preferable in computing seepage velocity [16 & 17].

4. Equation 7 is incapable of tracing water content variation in the capillary region during capillary rise, since the suction head at the wetting front and capillary region is equal to elevation head. The knowledge of water content variation in the capillary region is important for some theoretical and practical applications.

3. Improved approach for the capillary rise of water in soils

In order to overcome the limitations, and enhance the accuracy of previous solutions for capillary rise of water in soils, an improved solution is presented.

In this solution, akin to the Lu and Likos [11] solution (Equation 7) [11], Darcy's Law, Terzaghi's definition of capillary hydraulic gradient (Equation 5) and Gardner's unsaturated hydraulic conductivity are employed. Besides, the volumetric water content is implemented in lieu of porosity. Therefore seepage velocity of the wetting front is obtained by:

$$\frac{\Delta l}{\Delta t} = i\,\frac{k(h)}{\theta(h)} \tag{9}$$

Where, $\theta(h)$ = soil's volumetric water content at the wetting front, related to suction head (h) by the following Soil-Water Characteristic Curve (SWCC) model presented by Van Genuchten [18]:

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{1}{\left[1+(bh)^c\right]^d} \tag{10}$$

Where; Θ is the dimensionless water content, θ, θ_s and θ_r are the soil's; water content, saturated water content and residual water content respectively, b is related to the suction head corresponding to a volumetric water content of $(\theta_s - \theta_r)/2$, and c and d are fitting parameters related to the soil's pore size distribution [19].

For a precise definition of suction head (h) at the wetting front, applied in identifying $k(h)$, the transient water flow in the capillary region at any moment is attained by solving Richards' Equation [20]. The solution to Richards' equation provides the suction head variation in the capillary region and its value at the wetting front.

Predicated on the preceding perceptions, capillary rise of water is modeled in an incremental scheme. Any increment

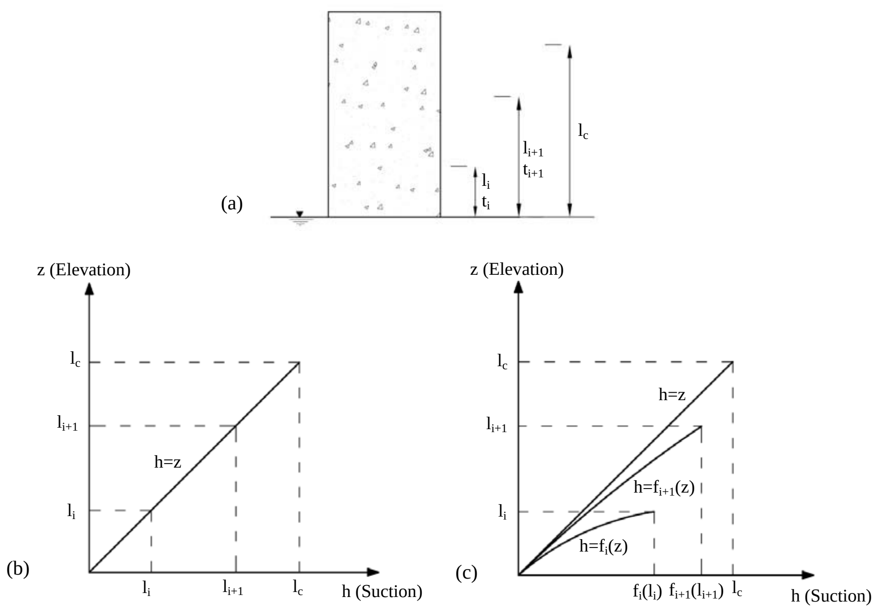

Fig. 3. Suction head variation versus elevation; (a) location of wetting front at different times, (b) assumptions of Lu and Likos [11], and (c) formulation of this study

stage includes the following steps:

1. Considering that at time t_i the wetting front elevation is l_i (cf. Figure 3-a), suction head variation in the capillary region (between $z=0$ and $z=l_i$) is a function of elevation defined as (cf. Figure 3-c):

$$h = f_i(z) \quad 0 \le z \le l_i \tag{11}$$

2. At the next time step, i.e. $t_{i+1}=t_i+\Delta t$, the wetting front rises to its new location at l_{i+1}, where:

$$l_{i+1} = \left(\frac{k_s \exp\left[a\, f_i(l_i)\right]}{n} \right) \left(\frac{l_c - l_i}{l_i} \right) \Delta t + l_i \tag{12}$$

3. As the wetting front is rising, water seeps upward in the capillary region. For the new wetting front location, water flow in the capillary region during time increment Δt is modeled via Richards' transient flow equation, thus;

$$\frac{\partial}{\partial z}\left[k(h) \times (\frac{\partial h}{\partial z} + 1) \right] = C(h) \frac{\partial h}{\partial t} \quad 0 \le z \le l_{i+1} \text{ and } 0 \le t \le \Delta t \tag{13}$$

Where; $C(h)$= soil's water storage capacity; that is, the inverse of the SWCC slope, i.e. $d\theta/dh$.

4. The initial and boundary conditions for Equation 13 are defined as:

$$t = 0 \Rightarrow h = f_i(z) \tag{14}$$

$$z = 0 \Rightarrow h = 0 \tag{15-a}$$

$$z = l_{i+1} \Rightarrow k(h) i_{i+1} = k(h)(\frac{\partial h}{\partial z} + 1) \tag{15-b}$$

The initial condition of Equation 13 is identical to suction head variation at time t_i.

The first boundary condition (Equation 15-a) implies that the soil is saturated at the water table. The second boundary condition (Equation 15-b) satisfies the continuity law, that is, discharge velocity of the wetting front (the left hand of Equation 15-b) is equal to discharge velocity of water flow in the capillary region at the wetting front elevation ($z=l_{i+1}$). Noteworthy, the hydraulic gradient of wetting front (i_{i+1}) is predefined at the new time (t_{i+1}) from:

$$i_{i+1} = \frac{l_c - l_{i+1}}{l_{i+1}} \tag{16}$$

In Equation 16, l_c is usually determined via experimental measurements. The following empirical relation can also be implemented [21]:

$$l_c = \frac{C}{eD_{10}} \tag{17}$$

Where; e= soil's void ratio, D_{10}= effective soil particle size (in mm), and C is a function of soil particle shape (between 10 to 50 mm^2).

5. The solution to Equation 13 is the suction head variation in the capillary region at new time, t_{i+1}, which is a function of capillary region elevation as follows (cf. Figure 3-c):

$$h = f_{i+1}(z) \qquad 0 \le z \le l_{i+1} \tag{18}$$

By repeating the scheme from 1 to 5, the wetting front increment at the next time step (t_{i+2}) is calculated. To implement the new formulation, the initial suction head at time $t=0$ should be defined, therefore, contrary to previous formulae; the soil's initial unsaturated condition is incorporated in the new approach.

4. Experimental verification

The formulation presented in this paper is verified and compared with two sets of data available in the literature.

4.1. Lane and Washburn

Lane and Washburn [22] measured the capillary rise of water in compacted sand columns. Table 1 presents specifications and parameters of four tested natural sands. Direct measurements of l_a (note that $a=1/l_a$) were available for soil classes 5 and 6 (cf. Table 1), however, for soil classes 2 and 4, an appropriate value of a is determined by the "best fit" of our solution to experimental data. Also, the initial ($t=0$) dimensionless water content (Θ) was defined by fitting the solution, while considering that soils were initially air dried [20], and initial water content should be close to residual (θ_r). Therefore, the initial Θ for all soil samples is near zero; except for soil class 6 where from the "best fit", Θ is slightly more than θ_r. The parameters for the Van Genuchten SWCC function are analogously defined from similar soils [19 & 23].

Figure 4 illustrates the experimental results of Lane and Washburn along with predictions by the Terzaghi, and Lu and Likos solutions, besides our improved formulation. According to Table 2, and based on regression (R^2) analysis, predictions by the improved formulation show best confirmation with test data. For soil class 6, results agree well with the experiment for low and high time durations, however there is some discrepancy for median times. This may be attributed to non-uniform specimen compaction leading to faster rising of water.

Table. 1. Specifications of tested soils

Investigator	Soil		k_s (m/s)	n	l_a (m)	l_c (m)	γ_{dry} (kN/m^3)	SWCC Parameters				Initial Θ at t=0
	Name	Type						b	c	d	Reference	
Lane & Washburn [22]	Class 2	Coarse sand	1.6×10^{-4}	0.31	0.62*	0.284	-	4.5	16	0.1	Poorly graded sand [23]	0.13*
	Class 4	Fine sand	4.6×10^{-6}	0.31	0.16*	1.06	-	3.5	5.87	0.4	Superstition sand [19]	0.01*
	Class 5	Fine sand	1.1×10^{-6}	0.21	0.41	0.82	-	3.5	5.87	0.4	Superstition sand [19]	0.10*
	Class 6	Sandy silt	6.2×10^{-7}	0.41	1.75	2.396	-	1.4	10.68	0.15	Touchet silt loam [19]	0.69*
Malik et al. [6]	Rawalwas	sand	44×10^{-6}	0.45	0.432	0.775	15	2.31	3.8	0.4	Lakland sand [19]	0.03*
	Bhiwani	sand	40×10^{-6}	0.41	0.44	0.625	15	1.66	10.68	0.35	Columbia sandy loam [19]	0.19*
	Rewari	sand	43×10^{-6}	0.45	0.352	0.609	15	1.66	10.68	0.35	Columbia sandy loam [19]	0.19*
	Yamuna	sand	88×10^{-6}	0.61	0.222	0.341	16	1.66	10.68	0.35	Columbia sandy loam [19]	0.19*

* Estimated by the "best fit" of new formulation to experimental data

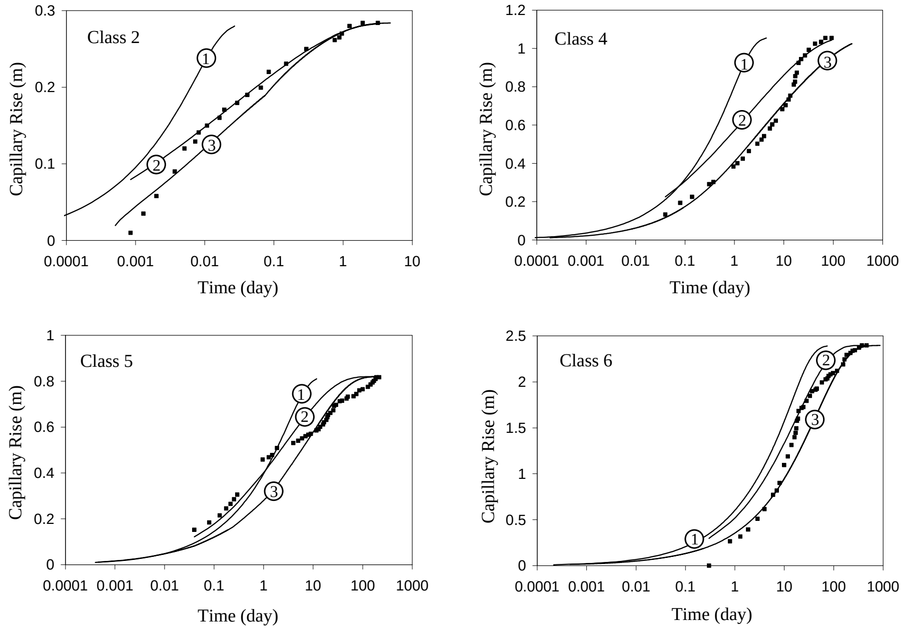

Fig. 4. The capillary rise vs. time for the experimental results (▪) of Lane and Washburn [22] using different formulations; (1) Terzaghi [8], (2) Lu and Likos [11], and (3) this study

Table. 2. R^2 of estimations by different solutions

Solution	Lane & Washburn [22]				Malik et al. [6]			
	Class 2	Class 4	Class 5	Class 6	Rawalwas	Bhiwani	Rewari	Yamuna
This Study	0.95	0.93	0.85	0.90	0.97	0.97	0.98	0.99
Lu & Likos	0.92	0.75	0.84	0.93	Out of range compared to experimental data			

One valuable feature of the improved solution; is the ability to trace suction head variation in the capillary region during wetting front rise. As an example, for soil class 2, variations of suction head with respect to elevation, for different wetting front locations (*l*), are plotted in Figure 5. As the wetting front elevates, the slope of the function increases and the curvature decreases; that is to say, $dh/dz \xrightarrow{decrease} 1$. The clue is that, according to Equation 5, the hydraulic gradient of capillary rise is higher for lower wetting front elevations. Hence, the wetting front rising rate and flow velocity in the capillary region are higher. This results in a high variation of hydraulic gradient in the capillary region. Therefore there is a high suction difference between different points which causes high *dh/dz*. Gradually, the rate at which the wetting front rises, and the flow velocity in the capillary region, decline; thus *dh/dz* decreases. Finally when the wetting front stops at the ultimate capillary elevation (i.e., $l=l_c$), there will be no flow and no hydraulic gradient in the capillary region, hence, $dh/dz=1$.

4.2. Malik et al.

The laboratory tests of Malik et al. [6] were performed on sandy soils (cf. Table 1). The air-entry head values had been measured for all specimens, whilst values of Θ and the SWCC parameters were determined as previously described for Lane and Washburn's results. Figure 7 compares the experimental results of Malik et al. [6] with our formulation and other aforementioned solutions. Apparently, the improved for mulation is much more accurate (cf. Table 2).

Noticing that the Malik et al. [6] tests have shorter durations and lower capillary rise elevations compared to Lane and Washburn's [22] tests, is a further indication for the versatility and accuracy of our formulation.

5. Conclusions

An improved formulation for the rate of capillary rise of water in soils is presented, having the paramount features of; considering unsaturated hydraulic conductivity for the capillary region, transient modeling of flow in the capillary region during water rising while defining suction head/water content variation at any elevation, and incorporating initial

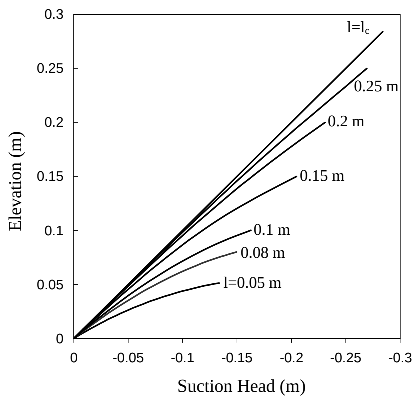

Fig. 5. The profiles of suction head (h) versus elevation (z) at various locations of wetting front (l); for soil class 2 of Lane and Washburn's [22] experiments

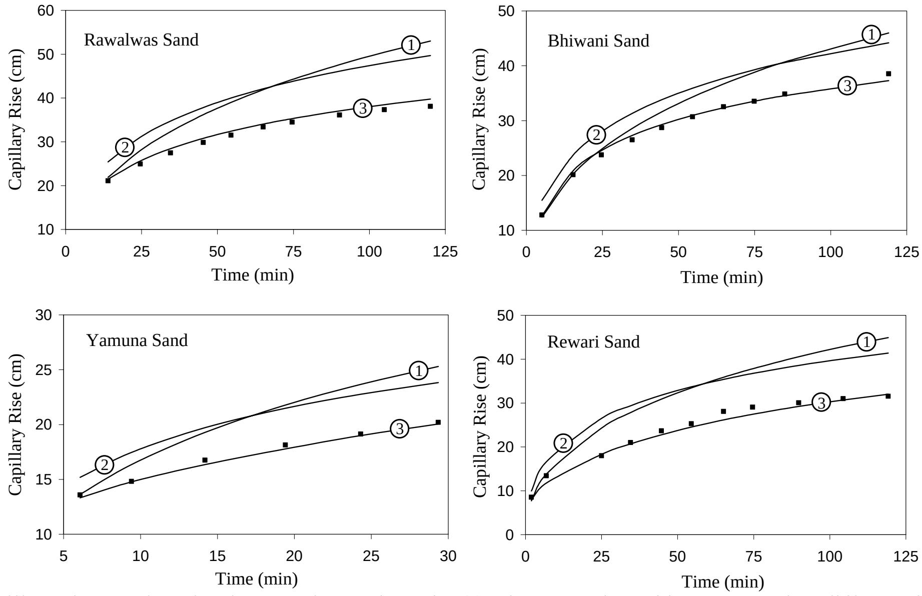

Fig. 6. The capillary rise vs. time for the experimental results (▪) of Lane and Washburn [22] using different formulations; (1) Terzaghi [8], (2) Lu and Likos [11], and (3) this study

suction (if any) present in the soil (due to partial saturation) prior to capillary rising.

By employing the incremental scheme presented in this paper, the rate of capillary rise can be calculated precisely. Comparison of results from the improved formulation, along with the Terzaghi [10] and Lu and Likos [11] predictions for experiments of Lane and Washburn [22] and Malik et al. [4] prove that the improved formulation is more accurate.

The incremental scheme presented can easily be incorporated in a mathematical software for the solution of capillary rising in soils.

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Notation

U_a-U_w = Matric suction
T_s = Surface tension
R = Radius of meniscus curvature
γ_w = Unit weight of water
ρ_w = Mass density of water
γ_{dry} = Dry unit weight of soil
r = Radius of equivalent capillary tube
μ = Dynamic viscosity of water
l = Elevation of wetting front (distance upward from water table)
g = Gravimetric acceleration
l_c = Ultimate wetting front elevation
l_a = Suction head of air-entry value
q = Discharge velocity
n = Soil porosity
i = Hydraulic gradient of wetting front in capillary rise
k_s = Saturated hydraulic conductivity
$k(h)$ = Unsaturated hydraulic conductivity
a = Parameter of Gardner unsaturated hydraulic conductivity function
h = Suction head
$\theta(h)$ = Volumetric water content of soil
θ_r = Residual volumetric water content of soil
θ_s = Saturated volumetric water content of soil
Θ = Dimensionless volumetric water content of soil
b, c, d = Parameters for Van Genuchten's SWCC model
z = Distance of points within capillary region from water table
l_i = Location of wetting front at time t_i
i_i = Hydraulic gradient of wetting front at time t_i
$f_i(z)$ = Suction head profile in capillary region at time t_i
$C(h)$ = Moisture capacity of soil
Δt = Time increment
$M\epsilon\chi\eta\alpha\nu\iota\chi\sigma$ = wetting angle between water and the wall material of tube