1. Introduction

In this paper, the main focus is on vertical stiffness variations along a railway track. Track stiffness is defined as ratio of the load applied to the rail to vertical rail deflection. Track modulus, on the other hand, is a measure of the vertical stiffness of the track foundation, [1]. In the paper presented here, problems emanating from stiffness variations are highlighted and countermeasures are discussed. In some cases geometric irregularities of the track may result in similar inconveniences as with irregular track stiffness, but geometric variations are not included in the present study. Track stiffness variations may be more difficult to deal with because even a track with an ideal track geometry may hide irregularities that are not discovered until the track is loaded by the train.

Track stiffness irregularities may have its origin in the track superstructure (rails, railpads, sleeper, ballast) or in the substructure (foundation, subgrade soil, etc). Due to an irregular stiffness of the substructure and of the ballast, for example due to a non-uniformly compacted ballast lying on substructure with properties varying along the track, the track stiffness experienced by a train will also vary along the track. Quite often, due to the substructure, there are large changes of the track stiffness within short distances. Places along the track where track stiffness will change rapidly are for example at pile decks, embankments, bridges, transition zones etc. The transition area from an embankment to a bridge is a place where severe track settlement may occur, [2,3,4]. Also at switches and turnouts, especially at the crossings (the frogs), at insulation joints, and at hanging sleepers the track stiffness changes very rapidly.

Changes in track stiffness will cause variations in the train/track interaction forces. The force variations give rise to track degradation such as track differential settlement due to permanent deformation of the ballast and in the underlying structure. The settlement is caused by the repeated loading and the severity of the settlement depends on the quality and the behaviour of the ballast, substructure, and foundation, [5]. Also in the superstructure...
degradation will occur, for example due to fatigue of rails and sleepers and due to wear and rolling contact fatigue of the rail surface, [6].

The rate of degradation of track components and the rate of track settlement will depend on the severity of the stiffness variation. As soon as the track geometry starts to deteriorate, the variations of the train/track interaction forces increase, and this speeds up the track degradation rate. Therefore, one should be aware of the influence of track stiffness irregularities on the development of track settlement and on the deterioration of track components and materials.

Also, track stiffness irregularities will induce vibrations in the train, in the track, and in the surroundings. In many cases the stiffness variation is more or less random along the track. Long-wave stiffness variations will induce low-frequency random oscillations of the train, causing reduced ride comfort for passengers, and track vibrations may induce disturbances in nearby buildings.

At abrupt changes of track stiffness, for example at turnout crossings or at transitions from ballast to slab track, transient and high-frequency vibrations will be induced in the track. Local track deterioration may take place creating fatigue problems, cracks, wear, plastic deformation, hanging sleepers, and so on.

In Figure 1 local track stiffness variation along a 25m section of a track is shown. It is noted that the track stiffness varies with a (spacial) frequency corresponding to the sleeper distance (here about 0.65m). Due to rail bending, the track is stiffer above one sleeper than between two sleepers. It is also seen in Figure 1 that the track stiffness is much lower at three sleepers around the position 149.807 km. Most probably, these three sleepers are unsupported so that they are hanging in the rail. The reason might be that the rail has an insulated joint there, and this induces irregularities in the wheel/rail contact force. The irregular contact force creates increased loading and vibrations of the sleepers and deterioration of the ballast bed below the sleepers. Thus, the track deterioration has started at a point of discontinuous track stiffness at the insulated rail joint.

2. Literature Review

2.1. Track Stiffness

The vertical stiffness of a railway track plays an important role when considering maintenance work, but it is also an important factor when looking at dissipated energy of a train. [7] proposed optimal values of the vertical stiffness. They optimised the maintenance costs and the costs for dissipated energy of a train versus the vertical track stiffness. They used the evolution of maintenance costs versus vertical stiffness from the high-speed line between Paris and Lyon, and they estimated the annual costs of the energy dissipated at the Spanish high-speed line between Madrid and Seville (samples of passenger traffic lines, with a frequency of 23 AVE trains per direction and day). Their result shows that the optimum vertical track stiffness should be between 70 and 80 kN/mm.

Another project trying to optimise the total lifecycle costs of a ballasted track was the European project EUROBALT II, [8]. Main objectives of the EUROBALT II project were to identify the main parameters that have to be measured by maintenance track engineers in order to detect developing flaws in ballasted tracks, and to identify parameters that can be controlled for reducing track deterioration. Conclusions from the project were that relevant track parameters influencing the track behaviour are track stiffness, displacement of sleepers, and the settlement of different layers of the substructure.

[9] and [10] investigated the influence of
spatially (in the longitudinal direction) varying track stiffness on the dynamic loading of the track and differential track settlement. [11] also reported that during his research, which was based on measurement results, it was found that the differential settlement of the track was dominated by the spatial variation of the track stiffness. An equation for track settlement, taking spatially varying track stiffness into account, was formulated.

2.2. Random Track Stiffness

In general, track stiffness is randomly varying along the track. [12] investigated the response of a beam resting on an elastic support. They found that the beam response is highly dependent upon the modulus of subgrade reaction (i.e., on track stiffness). Also [13] investigated the problem of a beam resting on a Winkler foundation; the stiffness of which was a random function of the length coordinate. [14,15,16] investigated the influence of stochastic properties of the track structure. To obtain sufficient statistical information from the track structures, full-scale in-field measurements and laboratory measurements were carried out. The railpad stiffness, the ballast stiffness, the dynamic ballast-subgrade mass (a discretized equivalent mass taking part in the vibrations), and the spacing between sleepers were assumed random variables. The influence of scatter on the maximum contact force between the rail and the wheel, the maximum magnitude of the vertical wheelset acceleration, and the maximum sleeper displacement were studied. Expectations and standard deviations of these quantities were calculated.

Andersen and Nielsen [17] investigated a case with a simple track structure with randomly varying support stiffness. The vertical support stiffness was assumed to be a stochastic homogeneous field consisting of small random variations around a deterministic mean value. Response spectra were obtained and the spectra were compared with those from numerical solutions achieved with finite element simulations. Wu and Thompson [18] treated the sleeper spacing and ballast stiffness as random variables and their effects on the rail vibration and noise emission were explored through numerical simulations. It was shown that the pinned-pinned resonance phenomenon (the rail vibrates with nodes at the sleepers) may be suppressed by the random sleeper spacing, but the random foundation has no significant effect on the average noise radiated by the track. Also Moravcik [19] investigated randomly distributed ballast stiffness.

When a train moves onto a bridge abutment the effects of varying geometry and foundation stiffness are significant. To minimise the rate of track settlement growth Hunt [20] suggested that in the vicinity of bridge abutments the track should have carefully prepared variations in foundation stiffness. Li and Davis [21] state that remedies intended to strengthen the subgrade between a bridge and the approach may not be effective if they are not designed to produce consistent and acceptable track stiffness between the bridge and the approach.

Nordborg [22] found that in comparison with surface roughnesses the track support irregularities may be a significant excitation mechanism up to 100 Hz. Vibration levels increase with train speed.

2.3. Rail Joints

The vertical bending stiffness of a rail joint is generally much lower than that of the rail. A passing wheel generates larger deflections in the joint region leading to increased wheel forces and accelerated track deterioration. From this point of view, Kerr and Cox [23] analysed and tested bonded insulation joints. Koro [24] used a discretely supported Timoshenko beam and finite elements to predict the impulsive wheel-track contact force excited by the wheel passage on the rail joint. It was concluded that the rail joints are of great concern to track deterioration, the settlement of the ballast track, and the failure of track components. Different train speeds and gap size at the joint were simulated. This study was continued in Suzuki [25]. Measurements were performed and compared with analytical results and a close agreement was found. Countermeasures to reduce ballast settlement
were discussed and soft rail pads were suggested. Focusing on rolling contact fatigue and plastic deformations, track deterioration at insulated rail joints was investigated by Kabo [26].

2.4. Switches and Turnouts

As mentioned above, a switch contains several irregularities both in stiffness and in inertia; the bending stiffness of the switch rail differs from that of the stock rail, the sleepers have different lengths and distances, the crossing (the frog) is both stiffer (in bending) and has a larger mass than the surrounding rails, and so on. Andersson and Dahlberg [27,28] investigated, by use of a numerical model, the load impact at the crossing nose when a wheel moves (at the frog) from the wing rail to the nose. It was found that the severity of the load impact depends on variations of track stiffness, variations of mass distribution, and geometric irregularities at the crossing. Zarembski [29,30] performed theoretical formulation, analytical studies, and field tests. Their conclusion was that the impact load at a crossing could be eliminated by a suitable transition arrangement.

Zhu [31] investigated the effect of varying stiffness below the switch rail of a high-speed turnout. Results show that elasticity under the switch rail could effectively improve the vertical wheel-rail interaction dynamics when the train passes from the stock rail to the switch rail. Kassa [32] performed mathematical modelling and simulation of the dynamic train-turnout interaction. Comparisons with field measurements were performed.

2.5. Smoothing Track Stiffness Irregularities

Elastomeric products, such as railpads, under-sleeper pads (USP), and sub-ballast mats (SBM), and also geogrid (or geotextile) reinforcements, can be used to construct a tailor-made transition zone with the desired variation of stiffness and geometry. Track settlement in the transition zone has been studied numerically by Guiyu [33], and the influence of tensile-reinforcements on track settlement was investigated by Monley and Wu [34]. Full-scale simulation of geogrid reinforcement for railway ballast was performed by Brown [35]. In Johansson [36] the influence of under-sleeper pads on dynamic train–track interaction was investigated. Two numerical models, valid in different frequency intervals, were used to study wheel/rail contact forces, rail bending moments, rail vibrations (displacements, velocities, and accelerations), sleeper vibrations, and loads on sleepers. Frequency-dependent material properties of railpads, USPs, and ballast/sub-structure were accounted for by viscoelastic spring-damper models that were calibrated with respect to measured data. It was found that USPs influence dynamic train/track interaction mainly in the frequency range 0 – 250 Hz. In Loy [37] under-sleeper pads were used to optimise the static rail deflection in turnouts. By mounting specific sleeper pads in different sections of the turnout, the track stiffness was adjusted, and as a result the vertical rail deflection was smoothed. A numerical study of the influence of under-sleeper pads on wheel/rail contact force is reviewed below, Lundquist [38].

In Anon. [39] various track transition designs were reviewed and analysed. A number of techniques were proposed to improve track performance by providing a transition that smoothes the stiffness interface between dissimilar track types. Analyses of representative designs, as found in the existing literature, were performed.

3. Modelling Dynamic Interaction Between Train and Track

In Dahlberg [40], different aspects of track dynamics and train/track interaction were reported. Numerical modelling of the track as a whole and of different components of the track was considered. Dynamics of individual components and of the complete track structure, including dynamics of the compound train-track system, were dealt with.

In the report presented here, focus will be on dynamics due to track stiffness irregularities. First it will be shown how the wheel/rail contact force may look like at an abrupt change of track stiffness. Then a transition section is assumed in an area between two track sections with different
stiffness. Optimisation of the track stiffness in the transition section is performed. The objective of the optimisation is to minimise the maximum deviation of the wheel/rail contact force from its static mean value.

In the first example presented below (Case 1) the numerical model of the track contains one soft section and one stiff section, with a transition section of 15 sleepers between the two. The transition section is divided into five shorter sections with three sleepers each. Each section (of three sleepers) has its own stiffness that can be selected individually. Optimum values of the five stiffnesses in the transition area are sought for.

In the next example (Case 2), again, the numerical model of the track consists of one soft section and one stiff section, but now under-sleeper pads are placed under ten sleepers of the stiff section. The under-sleeper pads are then used to influence the track stiffness to get a smoother transition between the soft and stiff part of the track (the stiff part could be, for example, a bridge). Optimal stiffness of the under-sleeper pads are sought for. The objective of the optimisation is to make the transition between the two parts of the track (with different stiffness) as smooth as possible.

In another example the use of under-sleeper pads (USP) for ballast protection is investigated (Case 3), and in a final example (Case 4) the track stiffness is random and the influence of USPs on the dynamics of that track is investigated.

3.1. Train and Track Model

A finite element track model is used in this study, see Figure 2. The track model is composed of one rail (symmetry with respect to the centre line of the track is assumed), rail pads, sleepers, under-sleeper pads, and the ballast/substructure bed. The rail is a standard UIC60 rail, and the rail pads are modelled with a predefined rubber material. The stiffness of the rail pad is such that it deforms 0.33 mm when it is loaded by the force 100 kN. The sleepers are rigid bodies. The model is made up of 3-D fully integrated solid elements. Stiffnesses of the ballast/substructure or of the under-sleeper pads will be optimised.

The loading of the railway track model (half a track is modelled; symmetry is assumed also in the loading of the track) comes from a moving wheelset that simulates the load from one axle of a train. The wheel is modelled as a rigid body and it is loaded by a constant force: the dead load of the car body. The wheel mass and half of the axle mass are included, which means that inertia from the un-sprung mass, i.e. from the wheel and the axle, is taken into account. The weight of the car body is taken into account by a constant force loading the wheelset. The wheelset moves at speed v.

In the first part of this study the model has a length of 45 sleeper spans (45 sleepers) (Figure 2 shows the same model but with 30 sleepers only). This track model is so long that boundary and initial effects are eliminated so that they do not disturb the track responses investigated at the 25 sleepers at the centre of the model. The ballast/substructure bed is modelled as a continuum with elastic material properties. The ballast bed is divided into several different sections. Two sections at the ends of the model are 15 sleeper spans long each. One end section is soft and the other end section is stiff. Five shorter sections in the central part of the model are three sleeper spans long each and the ballast bed stiffnesses in these five sections are allowed to vary between certain limits. These five stiffnesses are optimised.

The model with under-sleeper pads has been made shorter. It is 30 sleeper spans long (see Figure 2). First there is one section of ten sleepers without sleeper pads and the ballast bed is soft. Then there are ten sleepers with under-sleeper pads and the ballast bed is stiff. Finally, there are
ten sleepers without pads on a stiff ballast bed. Thus, the ballast bed stiffness changes from soft to stiff between sleeper 10 and sleeper 11. The sleepers 11 to 20 are equipped with USPs to make the transition from the soft to the stiff part as smooth as possible. The stiffnesses of the ten sleeper pads are optimised, but to avoid making the optimisation too time-consuming, only five different stiffnesses are used. The under sleeper pads are, two and two, given the same stiffness.

To avoid wave reflections at the boundaries of the limited model, non-reflecting boundary conditions have been used. The non-reflecting boundary conditions absorb the shear and pressure waves so that no reflections will occur at the boundaries. Thus, these boundary conditions prevent stress waves from re-enter into the model and contaminate the results. However, the bending waves in the rail are still reflected, but it is assumed that their influence on the track responses is small.

3.2. Train/Track Interaction

In the FE-program used in this study the contact force between two contacting bodies of the structure is calculated by a penalty method, see Belytschko [41]. The contact forces could be between wheel and rail or between sleeper/USP and ballast. In the penalty algorithm, one of the contact surfaces is defined as the master surface and the other as a slave surface. If there is no contact (slave node does not penetrate the master surface), nothing is changed in the program (in the stiffness matrix). If contact is obtained between a slave node and the master surface, then the slave node will try to penetrate the master surface. Since the slave nodes are constrained to slide on the master surface after contact (they must remain on the master surface), the penalty algorithm will introduce normal interface springs between the penetrating nodes and the contact surface. The spring stiffness matrix (from the interface springs) is then assembled into the global stiffness matrix. The stiffness of the interface spring is the minimum of the master segment stiffness and the slave node stiffness. The magnitude of the interface force is thus proportional to the amount of penetration. With this contact algorithm, it is possible to simulate loss of contact and recovered contact between wheel and rail and between sleeper/USP and ballast.

3.3. Mathematical Optimisation

The finite element model was built-up using the pre-processor TrueGrid, see Truegrid manual [42], and the train/track interaction problem was solved by the commercial finite element software LS-DYNA, Hallquist [43]. The software automatically makes the time step small so that high-frequency variations in the responses are well represented. The optimisation has been made with the optimisation package LS-OPT, Stander [44].

Basically, structural optimisation can be divided into global and local methods. The notations “global” and “local” only refer to the size of the region where the optimal solution is sought for. A subgroup of the global methods can be denoted semi-global methods. These methods limit the search of an optimum to a sub-domain, a region of interest, of the design domain. The global methods use the entire design domain. In this study the semi-global Response Surface Methodology (RSM) has been used. A detailed description of the optimisation was given in Lundqvist [45].

3.4. Calculations

The numerical values used in the simulations are as follows: the wheel mass and half of the axle mass is 750 kg, the dead load of the car body is applied to the wheel as a constant force of 100 kN (thus giving a static wheel load, including weight of wheel and axle, of 107.5 kN), the track model has a length of 45 sleeper spans (45 sleepers), and the sleeper spacing is 0.6 m. The ballast and substructure consist of an elastic material with modulus of elasticity in the stiff part E = 100 MPa, and in the soft part E = 30 MPa, Poisson’s ratio is 0.1, and the density is 2500 kg/m³. The depth of the track bed is one meter. The sleeper mass (for half the sleeper) is 125 kg. As already mentioned, the rail is standard UIC60 rail, and the railpads are of rubber.
material. The wheel moves at speed \( v = 90 \text{ m/s} \) and two cases are studied, namely the case when the wheel is travelling from stiff to soft track, and when it is travelling from soft to stiff track. The objective is to minimise the dynamic part of the contact force between the wheel and the rail. A smooth wheel/rail contact force at the transition area will minimise the track deterioration. In the RSM optimisation performed only linear surface approximations have been used with an oversampling of 1.5 times the minimum number of function evaluations.

3.5. Case 1: Optimal Ballast/Substructure Stiffness

In this part of the study, the total track stiffness changes from 45 kN/mm at one end (\( E = 30 \text{ MPa} \) below the first 15 sleepers) to 90 kN/mm at the other end (\( E = 100 \text{ MPa} \) below the last 15 sleepers). Such a change of stiffness is not unusual in a track, see for example Figure 1. Stiffness of the transition zone (15 sleepers on five sections with three sleepers each and different track stiffness in each section) is optimised for the two cases that the load is travelling from stiff to soft track and from soft to stiff track, respectively.

The optimal stiffness of the transition zone, for both travelling directions, can be seen in Figure 3. When going from stiff to soft track, the stiffness change in the transition zone should be smooth in the beginning and at the end of the zone, with a more rapid stiffness change in the central part of the zone, see Figure 3(a). The optimal modulii are, from left to right, 100, 93, 82, 71, 45, 38, and 30 MPa. For the other travelling direction, from soft to stiff track, the transition zone should have a more or less linear change of stiffness, see Figure 3(b), where the optimal values became 30, 40, 50, 60, 70, 80, and 100 MPa. The wheel/rail contact force (for travelling from stiff to soft track) is shown in Figure 4. A “dip” in the contact force is noted when the wheel enters the soft region. This implies a motion downwards of the wheel, and when this downward motion comes to an end, there is a large increase of the contact force. It can be seen that the large amplitude in the contact force that is obtained when there is no transition zone has almost disappeared after the optimisation. Only small variations of the contact force are noted at every small change of stiffness in the transition zone. Going from soft to stiff track is worse than going from stiff to soft; the wheel/rail contact force variation is then larger than the variation shown in Figure 4, see Lundqvist [46].

If the transition zone is optimised for one travelling direction and the train is running in the opposite direction, then almost as good results are obtained. This means that if the transition zone is optimised for one travelling direction, then the transition when going the opposite direction is almost as smooth as if the transition zone had been optimised for that direction.

3.6. Case 2: Optimal Under-Sleeper Pad (USP) Stiffness

In the second part of the study, the length of
the track model was decreased to 30 sleeper spans, where the first ten sleepers were lying on a soft ballast bed (E = 30 MPa) and the following 20 sleepers on a stiff bed (E = 100 MPa), see Figure 2. The ten sleepers in the central part of the model were (in the model) equipped with 20 mm thick under-sleeper pads (for details, see [38]).

In order to keep the number of optimisation variables low, the same stiffness of the USP was given to two adjacent sleepers, so that the number of optimisation variables was five. The shear modulus G of the USP material was selected as optimisation parameter, and its lower limit was set to G = 10 MPa. The optimisation criterion was to minimise the variation of the wheel/rail contact force, i.e., the maximum deviation from the static load should be as small as possible.

Optimal values of the shear modulus of the USP material are shown in Figure 5. It is seen in Figure 5 that the first two USPs (on the stiff part of the track) should have a very low stiffness. The lower limit of the stiffness (i.e. the shear modulus G) was obtained during the optimisation. Then, perhaps surprisingly, there should be two sleepers with stiff under-sleeper pads, followed by four sleepers with softer pads. Finally, the two last sleepers should again have stiff pads.

The wheel/rail contact force is shown in Figure 6. Again, for comparison, one curve in Figure 6 shows the contact force if the USPs were not there. A large irregularity is seen at time t = 0.12s, where the track stiffness changes from soft to stiff. Having optimal values of the USP stiffness this irregularity is almost completely eliminated, as shown by the second curve in Figure 6.

In order to investigate the robustness of the optimal solution shown in Figure 5, two calculations with other stiffness distributions were performed. The two other stiffness distributions tried (without optimisation) had the five stiffnesses 10, 100, 125, 150, and 175 GPa, and 10, 150, 150, 150, 150 GPa, respectively. It was found that these two stiffness distributions gave almost the same result as the optimised distribution in Figure 5. The conclusion is that as long as the two first under-sleeper pads are soft, the stiffness of the following eight pads does not influence the result very much. Thus, it is concluded that it could be suitable to use USPs to smooth out the stiffness variation at, for example, the transition from a “soft” embankment to a “stiff” concrete construction at a bridge.

3.7. Case 3: Ballast Protection

In case of a stiff track, the wheel/rail contact force is transmitted down to the sleepers and to the ballast by very few sleepers. The pressure from the sleeper onto the ballast will then be high when the wheel passes. By introducing USPs, the load on the ballast can be distributed over more sleepers, thereby decreasing the pressure on the ballast. Figure 7 shows how the high contact force without sleeper pads will be reduced by the USPs. The maximum contact force of 57 kN is reduced to 48 kN if stiff (3000 kN/mm) pads are
used, 32 kN if medium pad stiffness (400 kN/mm) is used, and 22 kN for soft pads (50 kN/mm).

3.8. Case 4: Random Track Stiffness

In Figure 8, a track with a randomly varying stiffness has been modelled. Youngs modulus of the material below a sleeper has been selected randomly (within certain limits) from one sleeper to the next, see Figure 8. Level-crossing counting of the wheel/rail contact force shows in Figure 9 that no USPs and stiff USPs give almost the same result regarding the force crossing (exceeding) the different levels. The medium stiffness gives fewer high-level crossings (which is beneficial for the track) but more low-level crossings. The soft pad gives, however, more high-level crossings than the stiff pad. The reason of this is that when soft pads are used, the track structure (rails and sleepers) vibrates on the soft pads, and this induces large amplitude vibrations of the track. Thus, to keep large force variations low there is, in this case, an optimum pad stiffness that should be sought for. More details on Cases 3 and 4 are given in [47].

4. Discussion and Conclusion

A thorough understanding of the physical mechanisms causing track deterioration, and understanding of the relationship between the track design parameters and the long-term track maintenance requirement would imply that an optimised (or at least an improved) ballasted track could be constructed. The total life cycle costs of the track would then decrease, and less time would be needed for maintenance, implying more time for transport operations.

Track stiffness variations along a track will induce an irregular wheel/rail contact force. This irregularity will contribute to track structure deterioration and track settlement giving rise to, for example, unsupported sleepers. The track degradation speeds up the track deterioration rate.

It is almost impossible to build a ballasted track without any stiffness variations. In this paper, it has been demonstrated that a transition zone between two track sections of different stiffness can be achieved to obtain a smooth transition between the two sections. The optimal transition zone can be built by using elastomeric products such as under-sleeper pads and/or sub ballast mats to construct a tailor-made transition zone with desired stiffness variation and geometry, see Lundqvist [45]. Another possibility to create this stiffness variation could be by grouting.

One conclusion that can be drawn from this study is that some kind of transition zone will reduce the wheel/rail contact force variation considerably. The optimal stiffness variation in the transition zone depends on the travelling
direction, but it is not very sensitive to it. Also, under-sleeper pads with non-optimised stiffness can significantly reduce the wheel/rail contact force variation. The detrimental effects of hanging sleepers can be reduced by under-sleeper pads.

Due to the effect that USPs distribute the axle load to more sleepers, it has also been demonstrated that these pads can be used to protect the ballast.

5. Acknowledgement

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