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Optimal design of reinforced concrete frames Using big bang-big crunch algorithm

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Abstract

In this paper a discrete Big Bang-Big Crunch algorithm is applied to optimal design of reinforced concrete planar frames under the gravity and lateral loads. Optimization is based on ACI 318-08 code. Columns are assumed to resist axial loads and bending moments, while beams resist only bending moments. Second-order effects are also considered for the compression members, and columns are checked for their slenderness and their end moments are magnified when necessary. The main aim of the BB-BC process is to minimize the cost of material and construction of the reinforced concrete frames under the applied loads such that the strength requirements of the ACI 318 code are fulfilled. In the process of optimization, the cost per unit length of the sections is used for the formation of the subsequent generation. Three bending frames are optimized using BB-BC and the results are compared to those of the genetic algorithm.

Keywords: Optimization; Reinforced concrete plane frame; Big Bang-Big Crunch algorithm

1. Introduction

In optimization of structures, the main aim is to minimize the cost. For steel structures minimizing the weight is sufficient since these structures are made of a single material and the cost is proportional to structural weight. However in the case of reinforced concrete structures because of the presence of different materials, one can not simply minimize weight. In fact, more parameters, such as the cost of concrete, steel, and forming are involved, and each of these parameters influence the dimensions of the structure. If the cost per unit volume of the concrete is much higher than that of the steel, then in the process of optimization those results corresponding to minimum cross sections will be selected, and should this be not, then sections with minimum steel will be chosen.

In the last two decades, many algorithms were developed for optimal design of steel structures. Some of these algorithms were also used for reinforced concrete structures. For the members of reinforced concrete structures, Adamu et al. [1] used the continuum-type optimality criterion for minimizing the cost of reinforced concrete beams. Zielinski et al. [2] employed an internal penalty function algorithm to optimize reinforced concrete short-tied columns. Fadaee and Grierson [3] optimized the cost of 3D skeletal structures using optimality criteria. Balling and Yao [4] optimized 3D frames with a multi-level method by decomposing the problem into a system optimization problem and a series of individual member optimization problems. Rajeev and Krishnamoorthy [5] applied a simple genetic algorithm (SGA) to the cost optimization of 2D frames. Optimal design of T-shaped reinforced concrete section under bending was performed by Ferreira et al. [6]. GA optimization of RC frames under bending was carried out by Camp et al. [7] and Lee and Ahn [8]. GA were used by Govindaraj and Ramasmy [9] for the optimal design of continuous beam. Cost optimization of buildings with planar slabs was carried out by Sahab et al. [10,11] using a hybrid genetic algorithm. Kwak and Kim [12,13] used a direct search method and an integrated genetic algorithm complemented with direct search for optimal design of planar RC frames. Whilst for concrete structures genetic algorithms were mostly employed, this paper analyzes the feasibility of using the Big Bang - Big Crunch (BB-BC) method to optimally design of planar reinforced concrete frames. The BB-BC method, introduced by Erol and Eksin [14], mimics the evolution of the universe. In the field of structural optimization, Camp [15] and Kaveh and Talatahari [16] utilized BB-BC for

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optimal design of trusses. In addition, Kaveh and Talatahari [17] pursued optimal designs of Schwedler and ribbed domes via a hybrid BBBC algorithm. Other applications of meta-heuristic algorithms in RC structures can be found in Refs. [18-19].

2. Components of a reinforced concrete frame and construction of the database of frame members

2.1 Construction of database

In general a three dimensional reinforced concrete (RC) frame consists of beams, columns, floor slabs, foundations, walls, staircases. However, 2D frames to be optimized consist of beams and columns.

According to ACI 318-08 code [20], reinforced concrete members should be designed such that they have sufficient strength for sustaining the bending and torsion moments, axial forces and shear forces produced under the applied loads. In order to simplify calculations, in the present study, only bending moments are considered for the beams while bending moments and axial forces are considered for the columns. The effect of shear is ignored.

A reinforced concrete element is constructed of reinforced steel and concrete, and the specifications given in the preliminary chapters of the ACI 318-08 Code are sufficient for their design. In this paper, this code is used for the design of the frame members. Specifications for design of reinforced beams and columns are provided in Chapters 8, 9 and 10 of the above mentioned code.

Unlike steel frames where the sections of beams and columns are pre-fabricated and also limited, in reinforced concrete frames the number of sections to be considered and also different patterns of reinforcements which can be used for beams and columns is quite large. However, in practice usually the sections are considered as rectangular ones with a depth to width ratio between 1.5 to 2.5 for beams and 1 to 2 for columns. The increment of the dimensions of the sections can be considered with steps of 5cm. The sizes of reinforcing bars, similar to Ref. [8], are considered as D22 for beams and D25 for the columns. The ACI 318-08 code [20] considers some limitations on the sections. These limitations consist of the minimum and maximum of steel area in the cross sections, minimum amount of concrete cover equal to 1.5in for the members, minimum diameter of the ties and minimum distance between longitudinal reinforcement bars.

Considering the above mentioned rules, one can construct many sections for beams and columns. Lee and Ahn [8] have presented a method for generation of sections with different depth and width, and also different patterns for the reinforcement bars. Their approach to formation of beam and column cross sections is also followed in this paper.

2.2. Beams

Beams are defined as structural elements that transfer the loads from floor slabs to the supports which are at the end of the columns. Under these loads shear forces and bending moments are produced in the beams. Considering the ACI 318-08 code [20], the following constraints should be imposed on the sections of the beams.

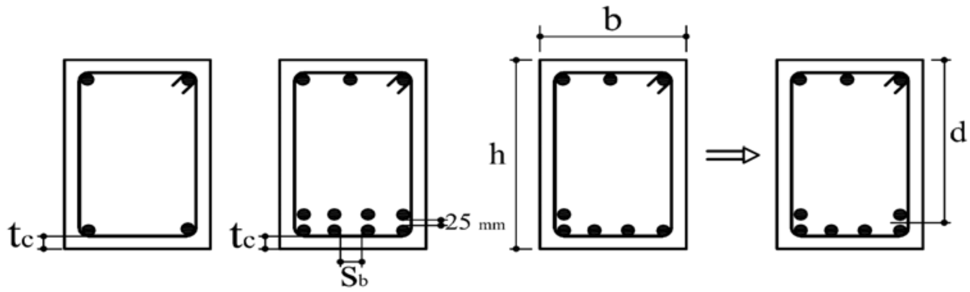
At least 4 reinforcement bars should be considered in the 4 corners of the cross section as shown in Fig. 1a. The minimum distance between the longitudinal reinforcing bars is taken as S_b=40mm. The layout of the bars is limited to at most two layers. The reinforcing bars of the top layer should be positioned on the reinforcing bars of the bottom layer and the minimum distance between two layers should be 25mm as shown in Fig. 1b. In a beam section, if additional reinforcing bars are needed, all such bars will be positioned in a second layer in a symmetric form with respect to the vertical axis of the section, and placed directly above the reinforcing bars in the lower layer. When the aforementioned symmetry does not exist, then it is made symmetric by considering an additional bar as illustrated in Fig. 1c.

In chapter 10 of ACI 318-08 code, the minimum and maximum areas of flexural reinforcement are chosen as follows:

$$A_{s,\min} = \frac{\sqrt{f_c'}}{4f_y} b.d \geq \frac{1.4}{f_y} b.d \quad [mm^2] \tag{1a}$$

$$A_{s,\max} = 0.75(0.85\beta_1)\frac{f_c'}{f_y}.\frac{600}{600+f_y} b.d \quad [mm^2] \tag{1b}$$

where b, f'_c and f_y are the width of the cross section, specified compressive strength of the concrete, and specified yield strength of the reinforcing bars, respectively. Here *d* is the effective depth of the section which is measured as the distance from extreme compression fiber to centroid of the



(a) (b) (c)

Fig. 1 . Limitations on the layout of the reinforcement bars for beam members
(a) At least four bars in the corners (b) Minimum distance between the longitudinal bars in the two layers
(c) Allowable symmetric layout of the reinforcement bars with respect to the vertical axis of the section

longitudinal tensile reinforcements of the section. The coefficient β_1 is a factor relating the depth of the equivalent rectangular compressive stress block to the neutral axis depth: it is taken from section 10.2.7.3 of the ACI 318-08 code. For the formation of the reinforcement bars of the beams, D22 is used.

Considering the above rules, 18 types of sections are constructed as follows:

300×450, 300×500, 300×550, 300×600, 350×550, 350×600, 350×650, 350×700, 400×600, 400×650, 400×700, 400×750, 400×800, 450×700,450×750, 450×800, 450×850, 450×900 mm.

A total of 1043 sections with different layouts for the reinforcing bars are generated for beams, the details of which are provided in Table 1. Details of the formation of these sections can be found in Ref. [8].

For the beams, the factored moment capacities at the middle, and near the ends are calculated using $\phi M_n = \phi A_s.f_y.(d-(a/2))$ and stored in the database. In this relation, ϕ is the strength reduction factor (ϕ=0.9), As is the area of the tensile bars and a is the depth of the equivalent rectangular compressive stress block defined as $a=A_s.f_y/(0.85f'_c b)$.

In Table 1, the database corresponding to the beam sections used for the examples of this paper are listed together with the width, depth, area and moment of inertia, number of reinforcing bars for positive and negative moments, the corresponding factored bending moment capacities, and the cost for the unit length of the beams. The calculation of the cost for the unit length will be discussed further.

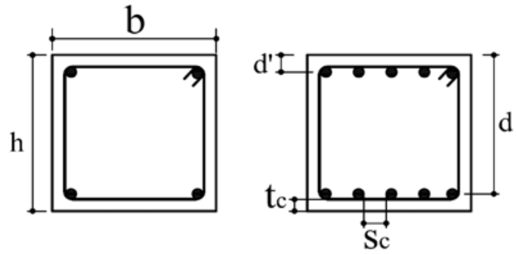
2.3. Beam columns

Beam columns are vertical elements supporting the structural floor systems. These elements are often subjected to compressive axial forces and bending moments. Using the rules from ACI- 318-08 code [20], the following constraints should be imposed on the sections of the beam columns.

The free distance between the parallel longitudinal bars is assumed to be s_c=40mm, and the minimum number of reinforcing bars is 4 which should be positioned at the four corners of the cross section as shown in Fig. 2(a). The pattern of the bars should be symmetric and in the two opposite sides of the section as illustrated in Fig. 2(b). The minimum and maximum areas of the longitudinal bars are limited to 1 to 8 percent of the total area of the cross section, respectively. In the columns only D25 bars are used. In all test problems for the columns a database consisting of 51 square cross sections with the dimensions 300mm to 850mm at steps of 50mm is used, as provided in Table 2.

The strength of a column under the applied loads (bending and axial force) is evaluated using the P-M interaction diagram. Here it is used a simplified linear P-M interaction diagram previously presented by Kwak and Kim [12], as illustrated in Fig. 3. Each interaction diagram has 4 key points consisting of the following:

The first point corresponds to the maximum compressive axial load which can be sustained by the column with zero eccentricity (point 0). The second point corresponds to the balance of strain conditions in which the concrete in compressive part reaches ε_{cu}=0.003 and the tensile steel equals yield strain ε_y. This point can be considered as the border of compressive and tensile failures (point 3). The third key point corresponds to the maximum bending moment which can be sustained by the column in the absence of the axial load (point 4). Finally, the fourth key point corresponds to the maximum



(a) (b)

Fig. 2. Limitations of the reinforcement of the column sections (a) At least 4 longitudinal bars at four corners of the section (b) Symmetric pattern of the bars and the distance and cover of the reinforcing bars

Table 1. Database of beams considered in this study

Beam number	Width (mm)	Depth (mm)	Area ($\times 10^2 mm^2$)	Moment of inertia ($\times 10^6 mm^4$)	Number of bars (D22)		Factored moment resistance (kN.m)		Cost per unit length ($)
					Center	End	Center	End	
1	300	450	1350	2278.1	2	2	98.21	98.21	74.455
2	300	450	1350	2278.1	2	3	98.21	142.31	76.096
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1042	450	900	4050	27338	12	10	1155.6	991.5	171.6
1043	450	900	4050	27338	12	12	1155.6	1155.6	174.88

Table 2. Database of columns considered in this study

Column number	Width (mm)	Depth (mm)	Number of bars (D25)	P_0 (kN)	P_1 (kN)	P_3 (kN)	P_5 (kN)	M_2 (kN.m)	M_3 (kN.m)	M_4 (kN.m)	Cost per unit length ($)
1	300	300	4	1769.8	1415.8	473.5	692.7	24.6	90.06	71.28	73.937
2	300	300	6	2025.4	1620.3	45	1039.1	28.9	112.26	102.04	78.176
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50	850	850	22	12915	10332	4748.6	3810	675.16	2134.6	1413.1	257.34
51	850	850	24	13170	10536	4741.7	4156.3	697.29	2231.2	1538.8	261.58

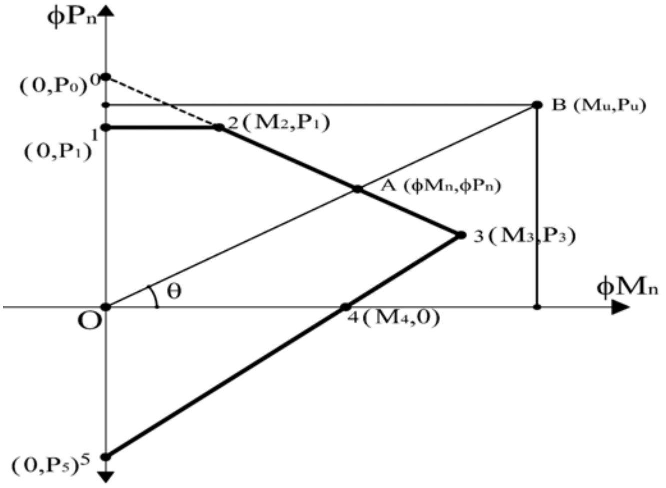


Fig. 3. Column strength rectilinear interaction diagram P-M with transitional points

tensile axial load which can be sustained by the column in the absence of the bending moment (point 5).

According to ACI 318-08 code, maximum design axial strength at zero eccentricity, i.e. P_0, is defined as

$$P_0 = \phi[0.85 f_c'(A_g - A_{st}) + f_y.A_{st}] \tag{2a}$$

Where A_g is the gross area of the column cross section and A_{st} is the total area of longitudinal reinforcement bars including tensile and compressive bars. For the sections with closed stirrup design axial strength at zero eccentricity, i.e. P_1, is defined as P_1=0.8P_0. The coordinates of point 2, i.e. (M_2,P_1), is obtained as the crossing point of the line P_n=P_1 and the line which connects the two points 0 and 3. For point (M_3,P_3) the following relationships can be used:

$$A_{st} = A'_s + A_s \tag{2b}$$

$$f_{sb} = 0.85 f_c (b.a_b - A'_s) + A'_s.f_{sb} - A_s.f_y \tag{2c}$$

$$f'_{sb} = 600 - \frac{d'}{d}(600 + f_y) \le f'_y \quad , \quad a_b = \frac{600}{600 + f_y}.\beta_1.d \tag{2d}$$

$$e'_b = \frac{1}{p_{nb}}[0.85 f_c'(a_b.b - A'_s).(d - \frac{a_b}{2}) + A'_s.f'_{sb}(d - d')] \tag{2e}$$

$$e_b = e'_b - \bar{x} \quad , \quad M_{nb} = e_b.p_{nb} \tag{2f}$$

Where b is the width of the cross section; h is the depth of the cross section; A'_s is the area of the compressive bars; A_s is the area of the tensile bars; f'_y is the yield strength of the compressive bars; f'_{sb} is the stress value in the compressive bars before yielding; d' is the distance of the center of compressive bars from the furthest compression fiber of the section; $\bar{x}$ is the distance between plastic neutral axis and centroid of tensile bars; a_b is the depth of the equivalent rectangular stress block; e_b is the eccentricity of the axial load from plastic neutral axis; e'_b is the eccentricity of the axial load from centroid of the tensile bars; P_{nb} is the nominal axial strength and M_{nb} is the nominal flexural strength of the section, in balanced strain conditions. For symmetric reinforcement $A'_s = A_s$ we have $\bar{x}$ =d-($h/2$). With simple calculations one can find the design axial strength and design flexural strength in balance strain conditions as: $p_3 = \phi p_{nb}$ and $M_3 = \phi M_{nb}$, respectively.

In order to find the bending moment M_4, with a try and error method one can first determine the magnitude of a such that the nominal axial strength of the cross section p_n, approaches to 0.

$$p_n = 0.85 f_c'(a.b - A'_s) + A'_s.f'_s - A_s.f_y \tag{3a}$$

$$f'_s = 600(1 - \frac{\beta_1 d}{a}) \le f'_y \tag{3b}$$

Numerically the magnitude of a is far less than the depth of the column, h. Once the approximate value of a is known, by using the following relationship, the magnitude of the nominal flexural strength of the column section, M_n, in the absence of the axial force can be found as:

$$M_n = 0.85 f_c'(a.b - A'_s)(d - \frac{a}{2}) + A'_s.f'_s(d - d') \tag{4}$$

Therefore, it follows $M_4 = \phi M_n$. At this point the behavior of the column is the same as a beam under the action of pure bending. The axial strength of the column under pure tension can be calculated from $p_5 = \phi A_{st}.f_y$

In the above relationship ϕ is considered according to the appendix C of the ACI 318-08 code. After finding the key points for the linear interaction diagram *P-M* of each column section, the database of column sections is completed.

In Figure 3, P_0, P_1, P_3, P_5 are the design axial strengths and M_2, M_3, M_4 are the design flexural strengths of the column cross section in different conditions of eccentricity. Also M_u and P_u are the factored moment and factored axial force of the section, respectively.

Table 2 shows the database of all the column sections used in the examples of this paper, including dimensions, number of reinforcing bars and the key points of the interaction diagram, and also the cost of the unit length of the columns. The procedure followed to compute cost will be later clarified in the paper.

3. Frame analysis

For optimal design of a frame the fitness or the cost of each frame should be computed. For this purpose the internal forces including axial force, shear force and bending moments in the frame are required. These quantities are determined via finite element analysis. In RC frames, in order to account for the effect of cracking, the moment of inertia and the area of the cross section for each member is calculated using the following relationship:

$$I_{beam} = 0.35 I_g \ , \ I_{column} = 0.7 I_g \ , \ A_{beam} = A_g \ , \ A_{column} = A_g \tag{5}$$

as suggested in Section 10.10.4.1 of [20]. Here, A_g and I_g are the gross area and the gross moment of inertia of the gross section of the beam or column, respectively. The ACI 318-08 code provides the elastic modulus of the concrete as Ec=4700$\sqrt{f_s}$ in MPa. The analysis of frame consists of controlling the slenderness of the columns, and in case a section is recognized to be slender, then the magnified bending will be considered for that column.

Slenderness

According to ACI 318-08 code [20], for compression members not braced against sidesway, the slenderness effects can be neglected, when $kl_u/r<22$. In this relation k is the effective length factor for compression members; l_u is the unsupported length of compression member; r is the radius of gyration of the cross section of a compression member.

The effective length factor of a column denoted by k depends on the ratio of the stiffness of the columns to the stiffness of beams connected at the end of the compression member. This ratio at the end of a compression member can be expressed as

$$\psi = \frac{\sum (EI/l)_c}{\sum (EI/l)_b} \tag{6}$$

Where I is the moment of inertia considering the cracked section, E is the modulus of elasticity and l is the length of the beams and columns. Indices b and c respectively refer to beams and columns connected to the ends of a column.

After calculating ψ for the ends of each compressive member, the mean value of these values, ψ_m, is obtained and the coefficient of the effective length of the compression member, k, is calculated using the following relationships:

$$\psi_m < 2: \quad k = (1-0.05\psi_m)\sqrt{1+\psi_m} \tag{7a}$$

$$\psi_m \geq 2: \quad k = 0.9\sqrt{1+\psi_m} \tag{7b}$$

For a fixed support it holds $\psi=0$. The radius of gyration is calculated as

$$r = \sqrt{\frac{I_g}{A_g}}.$$

For a slender column, the magnified bending moment can be calculated as:

$$M=M_{ns} + \delta_s.M_s \tag{8}$$

where M_{ns} is the bending moment generated by the gravity load and M_{ns} is due to lateral load and δ_s is the moment magnification factor for frames not braced against sidesway. After determining the magnified moment separately for each end of a column, the biggest one is used to design the column. The calculation of the magnification factor δ_s is performed as indicated in Chapter 10 of ACI 318-08 code [20].

4. Statement of the optimization problem and penalized objective function

4.1. Objective function

The aim of the optimal design of a structure is to minimize the cost of the structure. For optimal design of a reinforced concrete structure, the cost of the material and the cost of labor should be included in the objective function. The optimization problem of a RC frame can be stated as follows:

$$\text{Minimize: } F= F_b+F_c \tag{9}$$

$$F_b = \sum_{beams} \{C_c.b.h.L + C_s.A_{st}.L.\gamma_s + C_f.L.(b+2h)\}$$
$$F_c = \sum_{columns} \{C_c.b.h.L + C_s.A_{st}.L.\gamma_s + C_f.L.2(b+h)\} \tag{10a}$$

Subject to:

$$\text{For beams: } \begin{cases} M_u^+ \leq \phi M_n^+ \\ \left|M_{ul}^-\right| \leq \left|\phi M_n^-\right| \\ \left|M_{ur}^-\right| \leq \left|\phi M_n^-\right| \end{cases} \tag{11a}$$

$$\text{For columns: } (M_u, P_u) \leqq (\phi M_n, \phi P_n) \tag{11b}$$

Where,

F = Total cost of all members of the frame ($);

F_b, F_c = Cost of all beams and columns, respectively ($);

C_c, C_f , C_s = Unit cost of concrete, formwork (including the labor cost) and steel, respectively;

b, h, L = width, depth and length of the members (m), respectively;

A_{st} = total area of the reinforcing bars for each section (m^2);

γ_s = density of rebar (kg/m^3);

M^+_u, M^-_{ul}, M^-_{ur} = Externally applied moment at mid-span, left and right joints of beams, respectively;

M^+_n , M^-_n = Nominal flexural strength at mid-span and joints of beams, respectively;

M_u, P_u = Externally applied moment and axial force of columns, respectively;

M_n, P_n = Nominal flexural and axial strength of columns, respectively;

Since for two test cases the optimization results obtained with the BB-BC algorithm developed in this paper were compared with the results of Ref. [8], the costs of the concrete and steel and the cost of the formwork were set equal to those indicated in [8]:

$$C_c=54\ \$/m^3,\ C_s=0.55\ \$/kg,\ C_f=50.5\ \$/m^2,\ \gamma_s=7850\ kg/m^3 \tag{12}$$

As mentioned before, the cost of the unit length of the column section, F_b and F_c are calculated and stored in the last column of the corresponding database table. It should be mentioned that in the paper by Lee and Ahn [8], for positive and negative reinforcing bars of the beams, a length coefficient is considered, which means that positive and negative bars are cut in a definite length. However, in this paper in order to have more realistic costs and since in BB-BC the population production for a new generation is based on the cost per unit length, in calculating the cost per unit length of the beams, the reinforcing bars were not cut out but were assumed to span the entire beam length. In the comparison section, the cost is also calculated on this base for the results of Ref. [8] to make the comparison more meaningful. Once final sections are chosen for all the members, one can impose the suitable length coefficients for the positive and negative reinforcing bars and calculate the final cost.

4.2. Penalized objective function

In order to assess the merit of a trial design and determine its distance from the global optimum one should compute the eventual constraint violation by means of a penalty function. The penalty function consists of a series of geometric constraints corresponding to the dimensions and shape of the cross sections, and a series of constraint related to the deflection and internal forces of the members of the structure. Thus, penalty will be proportional to constraint violations and the best design will have the minimum cost and no penalty. Geometric constraints were taken into account in the definition of the database of available beam and column profiles.

The constraints considered in this paper are the same as those of Ref. [8]. For beams, three constraints are considered corresponding to the positive bending moments at the middle, and the negative bending moments at the two ends of the member:

$$g_1 = \frac{\left|M_u^+\right|}{\phi M_n^+} - 1 \geq 0 \tag{13a}$$

$$g_2 = \frac{\left|M_{ul}^-\right|}{\left|\phi M_n^-\right|} - 1 \geq 0 \tag{13b}$$

$$g_3 = \frac{\left|M_{ur}^-\right|}{\left|\phi M_n^-\right|} - 1 \geq 0 \tag{13c}$$

A column section is suitable and safe enough when the corresponding pair (M_u,P_u) under the applied loads does not fall outside the interaction diagram. In order to express this constraint in a mathematical form, the distance between the point representing the pair and the origin in the plane of interaction diagram is used (Fig. 3). Considering this figure if the position of the pair is considered at B, and A is the crossing point of the line connecting B to the origin O and the interaction diagram, then one can easily calculate the distance of the points A and B from O. The ratio of these distances can be used as the constraint of the columns. In order to specify the point A, the angle between OB and the horizontal axis should be calculated as $\theta=\tan^{-1}(P_u/M_u)$ and then considering the key points of the interaction diagram it can be found out which line of the interaction diagram will be crossed by the line OB. In this way if L_m and L_u are taken as the lengths of OA and OB respectively, then we have

$$L_u = \sqrt{(P_u)^2 + (M_u)^2} \;\;,\;\; L_m = \sqrt{(\phi P_n)^2 + (\phi M_n)^2} \tag{14}$$

Therefore the penalty function for the strength of the column an be expressed as

$$g_4 = \frac{L_u}{L_m} - 1 \geq 0 \tag{15}$$

For column sections we have another three constraints corresponding to the dimension and the number of reinforcing bars of the column sections which are in the same line (the co-linear columns). This means that the dimensions of the top column should not be larger than those of the bottom one, and also the number of reinforcing bars in the top column should not be greater than that of the bottom column. If *T* and *B* represent the top column and bottom column, respectively, then these constraints can be expressed as follow:

$$g_5 = \frac{b_T}{b_B} - 1 \geq 0 \;\;,\;\; g_6 = \frac{h_T}{h_B} - 1 \geq 0 \;\;,\;\; g_7 = \frac{n_T}{n_B} - 1 \geq 0 \tag{16}$$

Where *b, h* and *n* are the width, depth and number of the reinforcing bars of the column sections, respectively. The total penalty of each design was determined by summing over the different penalty terms for each element:

$$G = \sum_{beams}(g_1 + g_2 + g_3) + \sum_{columns}(g_4 + g_5 + g_6 + g_7) \tag{17}$$

The constrained optimization problem was transformed into an unconstrained optimization problem by collapsing the cost function and the penalty term according to literature:

$$\text{Minimize } F_p = F.(1+G)^{\varepsilon} \tag{18}$$

Where F_p is the penalized objective function, F is the cost function and ε is a parameter larger than 1 that depends on the structure type. The value ε =2 set in this study provided satisfactory results. After calculating the value of F_p for all the candidates, the optimization process is continued to obtain the optimal design using the following algorithm.

5. BB-BC algorithm

The BB–BC algorithm as developed by Erol and Eksin (2006) [14] consists of two phases: a Big Bang phase, and a Big Crunch phase. In the Big Bang phase, candidate solutions are randomly distributed over the search space. Each candidate design is a possible design for the structure. The quality of each candidate design is evaluated by computing the penalty function. The first phase of the BB-BC algorithm ends when this evaluation is done for all designs. In the Big Crunch phase, the centre of mass is defined for the population of candidate designs. In order to find the position of this centre, the mass of each candidate is considered to be proportional to the inverse of the corresponding penalized objective function. Therefore, the merit function is smaller for each candidate with small cost and low penalty, and such a candidate absorbs the mass centre towards itself. Therefore, the centre of mass is located near to more qualified candidate designs. At this point the BC stage is completed. In the new BB phase a new population around the obtained centre of mass, produced in the previous BC stage, is formed. BB and BC stages are sequentially repeated until the optimal design is obtained. In each iteration of BB and BC, the search space shrinks until reaching the convergence to the optimum design. However, the initial population of candidate designs randomly generated plays an important role. If the initial population does not cover the entire search space or most of the candidates lie inside some small region of the search space, then the optimization process may converge to a local optimum. Therefore, one should start optimization from different initial populations in order to increase the probability of finding the global optimum. The BB-BC code utilized in this study is based on the classical formulation developed by of Erol and Eksin [14]. The centre of mass is defined as:

$$X_{cm} = \frac{\sum_{i=1}^{N} \frac{1}{F_{pi}} X_i}{\sum_{i=1}^{N} \frac{1}{F_{pi}}} \tag{19}$$

where X_{cm} is the position of the center of mass; X_i is the position of individual i; F_{pi} is the penalized objective function value of the individual i; N is the population size.

The new position of the new population in the next iteration of Big Bang is obtained by a normal distribution around the centre of mass X_{cm} in the following form:

$$X_i^{new} = X_{cm} + \sigma \tag{20}$$

where X_i^{new} is the position of the new individual i ; σ is the standard deviation of a standard normal distribution. The parameter σ is defined as:

$$\sigma = \frac{r\alpha(\boldsymbol{x}_{\max} - \boldsymbol{x}_{\min})}{s} \tag{21}$$

where r is the random number from a standard normal distribution; α is a parameter not greater than 1 which limits the size of the search space around X_{cm}; x_{max} and x_{min} are the upper and lower limits on the values of the design variables; s is the number of explosions.

In order to improve the performance of BB-BC, Camp [15] presented the following formula for producing the new candidate:

$$\boldsymbol{X}_i^{new} = \beta \boldsymbol{X}_{cm} + (1-\beta)\boldsymbol{X}_{best} + \frac{r\alpha(\boldsymbol{x}_{\max} - \boldsymbol{x}_{\min})}{s} \tag{22}$$

where X_{best} is the best global solution of all the candidates obtained up to this stage of the iteration of BB-BC. The parameter β controls the effectiveness of the X_{best} in selecting the position of the new candidates.

In optimization of RC frames one can select many types of variables. These can be cross section dimensions, the diameter of the bars, number and length of reinforcement bars, etc. However, the design variables selected in this research are consistent with those indicated in Ref. [8]. In order to simplify optimization, some parameters are pre-selected and some cross sections of known dimensions and reinforcing patterns are considered and only the costs per unit length of such elements are taken as design variables. This simplification allowed optimization results to be compared with those of [8] in identical conditions. There is another reason for limiting the number of variables: as this number increases, the computational time also increases and makes the optimization unpractical.

Here the variable X is considered as cost per unit length of each element and if the members of a structure are put in different groups, for each group we should define a centre of mass. As it can be seen from Tables 1 and 2, the beam and column sections are ordered according to the cost of the unit length from smallest to the biggest. Thus for the group of beams x_{min}=74.455\$ and x_{max}=174.88\$, and for the group of columns we have x_{min}=73.937\$ and x_{max}=261.58\$. The magnitudes of X_i^{new} for different groups of beams and columns can be obtained. For each group of beams or columns, the member in the database of available structural elements with the cost per unit length closest to the computed one is selected as the new candidate design. The sorting of available elements with respect to their cost per unit length, has facilitated the updating of design.

Camp considered β =0.2 and α=1 [15]. However, β =0.3 and α=0.7 were found to be more suitable for the optimization problems considered in this study as they improved the convergence speed significantly (around 15 percent). Table 3 shows the effect of BB-BC parameters on optimal design for the three bay, six-story reinforced concrete frame.

The flowchart of the algorithm BB-BC is shown schematically in Fig. 4.

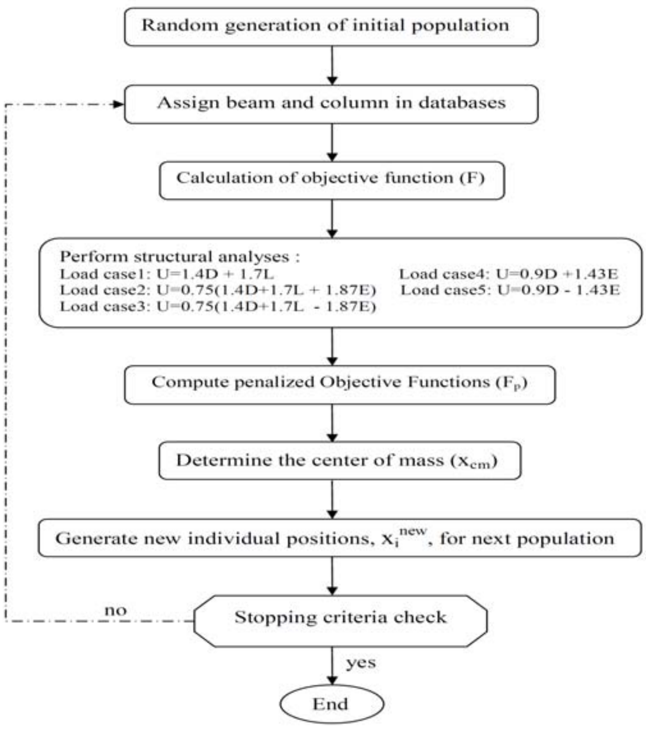


Fig. 4. Flowchart of the optimization by BB-BC algorithm

Table 3. Effect of BB-BC parameters on optimal design for three bay, six-story reinforced concrete frame

α	0.7										
β	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Optimal design (\$)	22253	22272	22253	22182	22205	22360	22360	22673	22707	22833	29122
No, of iteration	107	107	112	118	107	108	117	124	134	148	184
No. of analyses	26750	26750	28000	29500	26750	27000	29250	31000	33500	37000	46000
Standard deviation	90.18	196.87	217.74	187.63	218.37	127.44	187.20	132.48	115.09	659.81	1423.60

6. Test cases and results

In order to verify the efficiency of the BB-BC algorithm described in this paper, three examples of plane RC frames were considered. Since two of these frames were optimized previously with GA by Lee and Ahn [8], results of GA and BB-BC methods are compared.

Loading cases acting on frames consist of joint loads and uniform distributed loads. Lateral equivalent static earthquake loads (E) are applied as joint loads, and uniform gravity loads are assumed for a dead load (D) and a live load (L). In Ref. [8], five independent loading cases were considered as suggested by ACI 318-99 code [21]:

$$U=1.4D+1.7L \tag{23a}$$

$$U=0.75(1.4D+1.7L \pm 1.87E) \tag{23b}$$

$$U=0.9D\pm 1.43E \tag{23c}$$

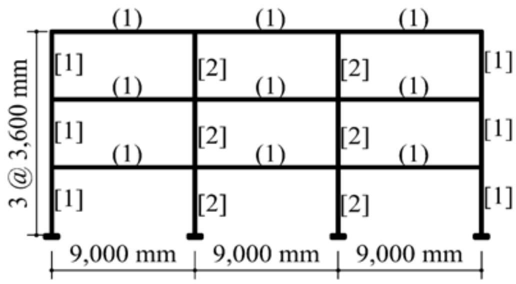
However, the ACI 318-08 code [20] for strength design indicates new loading cases which are different from those specified above. In order to compare the results, the same loading cases must be considered. However Appendix specifications of the ACI 318-08 code [20] contemplates the possibility of using the loading cases specified above provided that the strength reduction factors (ϕ) used are taken from the appendix specifications, where this requirements are considered in constructing databases.

A uniform service dead load of D=16.5 kN/m, and a uniform service live load of L=7.2 kN/m were assumed in all test problems. The assumed specified compressive strength of concrete and yield strength of reinforcement bars in these examples were set as f'_c = 23.5 MPa and f_y = 392 MPa, respectively. The BB-BC code was implemented in Matlab. Structural analyses were performed via direct stiffness method. A classical termination criterion counting at least 2,000 analyses without any change in the current best record X_{best} was used for all test cases. All optimization runs were carried out on a standard PC with a Pentium (R) Dual-Core CPU 2.60 GHz processor and 2.00 GB of RAM memory. Due to the random nature of the BB-BC all the examples were executed at least 30 times to make sure of the optimality of the results.

6.1. Three bay, three-story reinforced concrete frame

This test case was also solved with GA in [8]. Figure 5 shows that the structure includes 9 beams and 12 columns. All elements are assumed to be designed in groups: one group for all beams and two groups for columns. For this frame, only the uniform gravity loads is applied. The results of optimum design with BB-BC and GA are shown in Table 4. It should be reminded that in [8], the cost of beams was calculated based on length factors of reinforced bars for positive and negative moments, while in this research it was assumed that all reinforced bars were to be continuous along the beams. In Table 4, the cost of the beam sections of above-mentioned paper is recalculated based on this assumption.

As is shown in Table 4, BB-BC chose beams with smaller depth than GA, but the dimensions of exterior columns are bigger than GA. It should be mentioned that in [8] the cover thickness of concrete is assumed as t_c=35 mm, which is less than the value suggested by ACI 318-08 code. The present BB-BC algorithm run with t_c=35 mm, yield the cost of 10645 $, that is less than the cost of 10798 $ obtained by GA. Nonetheless, the BB-BC method run satisfying all requirements of ACI 318-08 code yield the cost equal to 10803 $ for this frame, which is only 5$ higher than the cost obtained by GA. The optimization process was completed in 24 seconds. The GA used the population with 300 individuals, while the present algorithm used 125 individuals. GA and BB-BC converged to optimum design after 59 and 75 iterations, respectively. For this frame, the ratio of the



( ) Beam group number. **[ ]** Column group number.

Fig. 5. Three-bay, three-story reinforced concrete frame

Table 4. Optimized designs for three bay, three-story reinforced concrete frame

Member type	Element group	OPTIMIZATION RESULTS							
		Lee and Ahn[8]				Present work			
		Sectional dimensions		Reinforcements		Sectional dimensions		Reinforcements	
		Width (mm)	Depth (mm)	Positive moment	Negative moment	Width (mm)	Depth (mm)	Positive moment	Negative moment
Beam	1	300	550	3-D22	5-D22	300	500	3-D22	6-D22
Column	1	300	300	6-D25		350	350	8-D25	
	2	300	300	4-D25		300	300	4-D25	
Population size		300				125			
Number of iterations		59				75			
Number of analyses		17700				9375			
Standard deviation		N/A				28.84			
Computing time (second)		N/A				24			
Frame cost		10798 $				10803 $			

sample space to the entire search space was $(125\times75)/(1043\times51\times51)=3.5\times10^{-3}$.

It should be mentioned that increasing the number of candidates increases the speed of convergence but also the CPU time required in the optimization process. Therefore a compromise between these two conflicting issues should be found.

The comparison between convergence curves for BB-BC and GA is presented in Fig. 6 while Fig. 7 shows the random spread of initial population in the design space for BB-BC method. In the latter figure, the position of each individual is marked with '*' and the position of optimum solution i.e., 10803 $ is marked with dot.

Table 5 shows the maximum values of demand capacity ratio (DCR), i.e., the maximum of (*Mu/φMn*) for beams, and the maximum of (Lu/Lm) for columns in all groups, under the critical loading case. While strength capacity of beams and exterior columns is optimally used, strength capacity of the interior columns is suboptimal, as the section optimized by BB-BC for this group of columns is the smallest one in the column database.

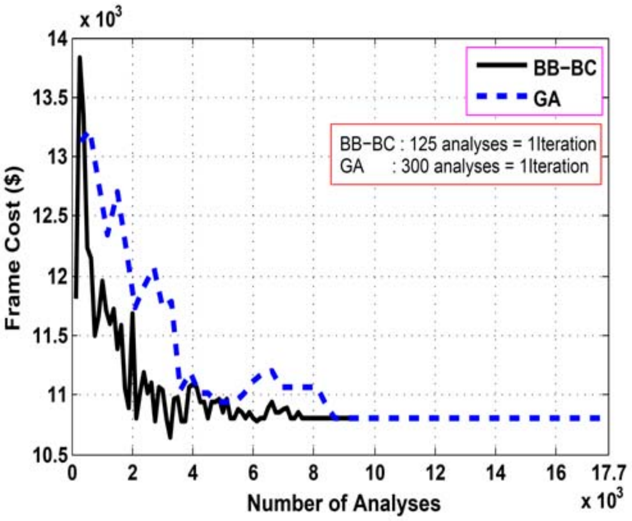


Fig. 6. Convergence history of the BB-BC and GA for the three-bay, three-story reinforced concrete frame

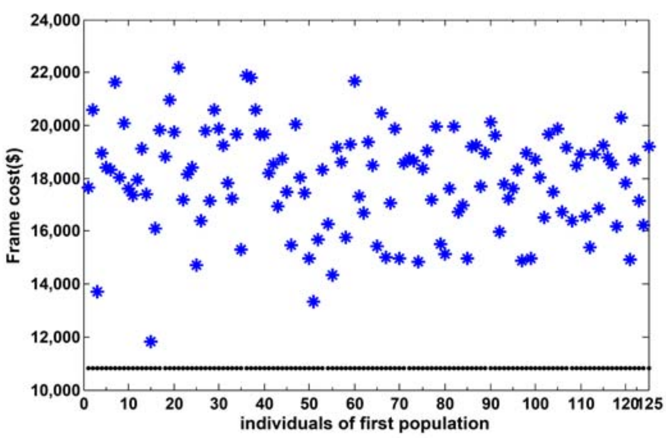


Fig. 7. Spread of individuals of the initial population of the BB-BC for the three-bay, three-story reinforced concrete frame

6.2. Three bay, six-story reinforced concrete frame

The three-bay, six-story RC frame, shown in Fig. 8, includes 18 beams and 24 columns. Beams and columns are grouped in 3 and 4 groups, respectively. Loading conditions include the uniform dead and live loads applied on beams only, and a lateral earthquake load applied as joint loads. BB-BC found the optimum solution of 22182 $ after 118 iterations by utilizing a population of 250 candidate designs. The optimization process was completed in 235.333 seconds. In this example, the order of sampling space relative to domain space was $(250\times118)/[(1043)^3\times(51)^4]=3.8\times10^{-12}$.

The optimized design and the convergence curve are shown in Table 6 and Fig. 9 respectively.

Table 7 shows the maximum values of DCR, in beams and columns for all groups, under the critical loading case. The ratios in this table show the proper usage of the strength capacity of all groups with the exception of columns group [1]. It was found that for this group of elements the design is driven by the constraints (g_5,g_6,g_7) on dimensions and number of reinforced bars in the co-linear columns but not by strength constraints. Therefore, since the section 350×350 with 8D25 was selected by BB-BC for columns group [3], columns of group [1] were forced to have at least the same section as the upper columns. As far as it concerns strength, the section 350×350 with 6D25 could withstand loads acting on the columns group [1]. However, the number of reinforced bars of this section is smaller than for the section of columns group [3] and was not selected by BB-BC for columns group [1]. The optimum section found by BB-BC for columns group [4] is the smallest section in the column database, i.e. 300×300 with 4D25. Therefore, strength capacity is not optimal also for this group of elements.

Table 5. Maximum DCR for member groups in the three bay, three-story RC frame

Member type	Group number	DCR	Critical load case
Beam	1	0.91917	Load case 1
Column	1	0.99411	Load case 1
	2	0.68733	Load case 1

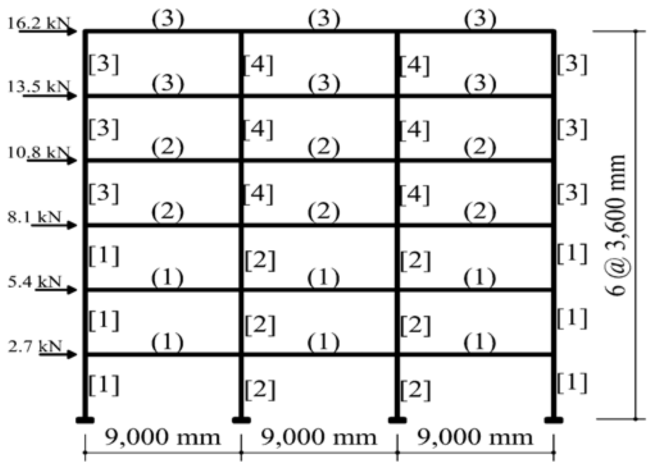


Fig. 8. Three-bay, six-story reinforced concrete frame

Table 6. Optimized designs for the three bay, six-story reinforced concrete frame

Member type	Element group	Present work			
		Sectional dimensions		Reinforcements	
		Width (mm)	Depth (mm)	Positive moment	Negative moment
Beam	1	300	500	3-D22	6-D22
	2	300	500	3-D22	6-D22
	3	300	550	3-D22	5-D22
Column	1	350	350	8-D25	
	2	350	350	6-D25	
	3	350	350	8-D25	
	4	300	300	4-D25	
Population size		250			
Number of iterations		118			
Number of analyses		29500			
Standard deviation		187.63			
Computing time (second)		235.333			
Frame cost		22182 $			

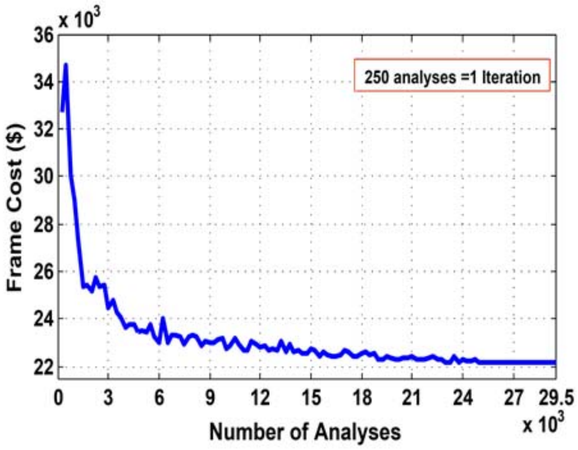


Fig. 9. Convergence history of the BB-BC and GA for the three-bay, three-story reinforced concrete frame

Table 7. Maximum DCR for member groups in the three bay, six-story RC frame

Member type	Group number	DCR	Critical load case
Beam	1	0.89969	Load case 1
	2	0.87901	Load case 1
	3	0.93705	Load case 1
Column	1	0.72672	Load case 3
	2	0.97806	Load case 1
	3	0.94687	Load case 1
	4	0.80065	Load case 2

6.3. Three bay, nine-story reinforced concrete frame

This RC frame is the same optimization problem solved by [8] with GA. Figure 10 shows that the structure includes 27 beams and 36 columns. The beams and columns are assumed to be designed in 3 and 4 groups, respectively. Loading conditions include uniform dead and live loads acting only on the beams and lateral earthquake loads applied to joints.

The BB-BC algorithm run with a population of 250 individuals found the optimum cost of 35907 $. The optimization process was completed in 128 iterations and required 7.393 minutes of CPU time. The GA of Ref. [8] found instead a higher cost, 37964 $, in more iterations (277), and required 17 minutes of CPU time. Optimized designs are compared in Table 8: the present algorithm designed a structure 5.4 % cheaper than that optimized by GA. Figure 11 shows the convergence curve for this test case. For this frame, the ratio of sampling space to domain space was $(250\times128)/[(1043)^3\times(51)^4]=4.17\times10^{-12}$.

The maximum values of DCR, in beams and columns for all groups, under the critical loading case are shown in Table 9. Strength capacity was properly assigned to all groups except for columns group [1]. It was found that for this group of

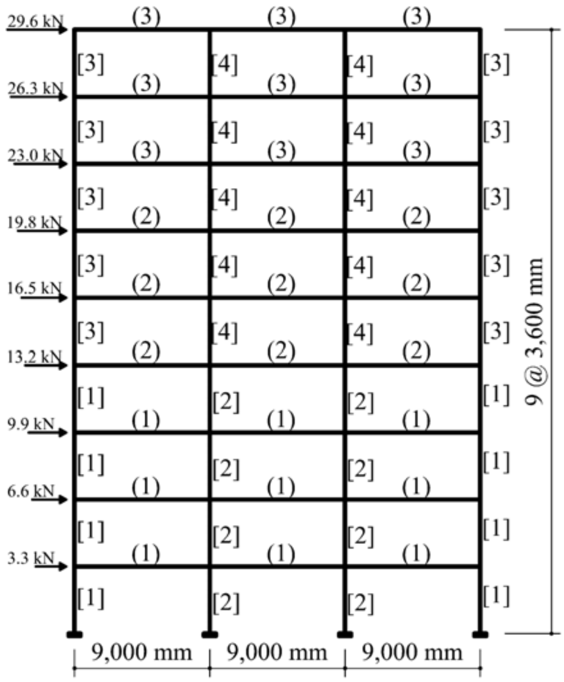


Fig. 10. Three-bay, nine-story reinforced concrete frame

Table 8. Optimized designs for three bay, nine-story reinforced concrete frame

Member type	Element group	OPTIMIZATION RESULTS							
		Lee and Ahn [8]				Present work			
		Sectional dimensions		Reinforcements		Sectional dimensions		Reinforcements	
		Width (mm)	Depth (mm)	Positive moment	Negative moment	Width (mm)	Depth (mm)	Positive moment	Negative moment
Beam	1	300	500	3-D22	6-D22	300	550	3-D22	6-D22
	2	300	500	3-D22	5-D22	300	500	3-D22	6-D22
	3	300	500	4-D22	6-D22	300	500	3-D22	6-D22
Column	1	450	450	6-D25		400	400	8-D25	
	2	650	650	10-D25		500	500	8-D25	
	3	400	400	6-D25		350	350	8-D25	
	4	500	500	4-D25		400	400	6-D25	
Population size		500				250			
Number of iterations		277				128			
Number of analyses		138500				32000			
Standard deviation		N/A				252.27			
Computing time (second)		1020				443.60			
Frame cost		37964 $				35907 $			

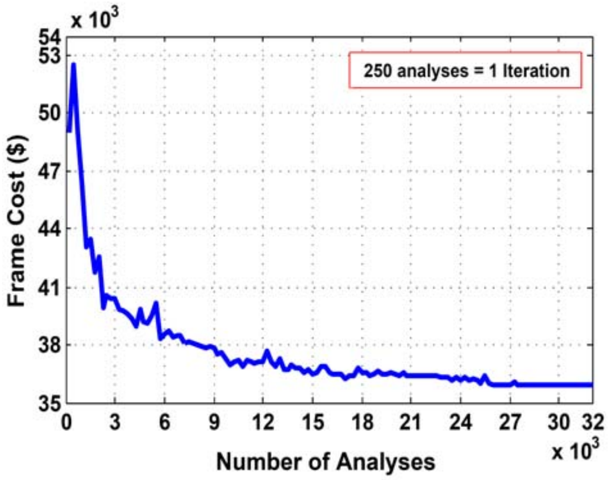


Fig. 11. Convergence history for the three-bay, nine-story reinforced concrete frame

Table 9. Maximum DCR for member groups in the three bay, nine-story RC frame

Member type	Group number	DCR	Critical load case
Beam	1	0.93421	Load case 2
	2	0.99642	Load case 2
	3	0.93354	Load case 1
Column	1	0.89399	Load case 3
	2	0.96871	Load case 2
	3	0.99294	Load case 1
	4	0.99195	Load case 2

elements the design is driven by the constraints g_7 on the number of reinforced bars in the co-linear columns but not by strength constraints. The section 400×400 with 6D25 could withstand loads acting on the columns group [1]. However, because of constraints (g_7) the number of reinforced bars for this group section could not be smaller than its counterpart for the upper columns. Therefore, BB-BC selected the 400×400 section with 8D25 for columns group [1].

7. Concluding remarks

In this paper, the Big Bang - Big Crunch algorithm was proven able to find optimal design of 2D reinforced concrete frames. BB-BC was applied for the first time to this kind of discrete optimization problems. Numerical results demonstrate the feasibility and efficiency of the proposed approach. While in steel structures design variables correspond to the member cross-sectional areas, in the case of reinforced concrete structures optimization variables are the cost for the unit length of the members.

Using some cross sections for beams and columns which occur in practice the speed of convergence was quite good. The ratios of sample space to domain space, as calculated in section 6, were low for the three-story frame, and much lower for the six-story and the nine-story frames. These ratios show that although the search space was rather small, BB-BC could find a good global optimal design. The slope of the convergence curves in the initial iterations shows that if the primary random spaces are selected properly (candidates are properly distributed in the search space and not gathered within some special region), then the BB-BC recognizes the region where the global optimum lies, after few primary searches, as then it continues to further search in this subspace.

Comparison of optimization results obtained with BB-BC and GA shows the superiority of the BB-BC which leads to better results than GA. Furthermore it can be noticed that BB-BC uses a smaller population than GA. This leads to higher convergence speed, therefore, BB-BC does not require a large number of candidate designs. If this number is very low or very high, then the performance of the algorithm decreases. In order to find a suitable number of candidate designs, the optimization program can be executed by taking different number of candidates and find the limit. It is also recommended to use different primary random search spaces to find the most suitable one.

It should also be mentioned that working with BB-BC is much easier than GA since programming is easier and also it has not many internal parameters whose values can be found by performing few preliminary optimization runs.

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References

[1] A. Adamu, B.L. Karihaloo, and G.I.N. Rozvany, Minimum cost design of reinforced concrete beams using continuum-type optimality criteria. Struct Optim, Vol. 7, pp. 91–102 (1994).

[2] Z.A. Zielinski, W. Long, and M.S. Troitsky, Designing reinforced concrete short-tied columns using the optimization technique, ACI Struct J Vol. 92, pp. 619–626(1995).

[3] M.J. Fadaee, D.E. Grierson, Design optimization of 3D reinforced concrete structures, Struct Optim Vol 12, pp. 127–134(1996).

[4] R.J. Balling, and X. Yao, Optimization of reinforced concrete frames, J Struct Eng ASCE Vol. 123, pp. 193–202(1997).

[5] S. Rajeev and C.S. Krishnamoorthy, Genetic algorithm-based methodology for design optimization of reinforced concrete frames. Computer-Aided Civil Infrastruct Eng Vol. 13, pp. 63–74(1998).

[6] C.C. Ferreira, M.H.F.M. Barros, and A.F.M. Barros, Optimal design of reinforced concrete T-sections in bending, Eng Struct Vol. 25, pp. 951–964(2003).

[7] C.V. Camp, S. Pezeshk, and H. Hansson, Flexural design of reinforced concrete frames using a genetic algorithm J Struct Eng Vol. 129, pp. 105–115(2003).

[8] C. Lee C, and J. Ahn, Flexural design of reinforced concrete frames by genetic algorithm, J Struct Eng, ASCE Vol. 129, pp. 762–774(2003).

[9] V. Govindaraj, and J.V. Ramasmy, Optimum detailed design of reinforced concrete continuous beams using genetic algorithms, Comput Struct Vol. 84, pp. 34–48(2005).

[10] M.G. Sahab, A.F. Ashour, and V.V. Toropov, Cost optimization of reinforced concrete flat slab buildings. Eng Struct Vol. 27, pp. 313–322(2004).

[11] M.G. Sahab, A.F. Ashour, and V.V. Toropov, A hybrid genetic algorithm for reinforced concrete flat slab buildings, Comput Struct Vol. 83, pp. 551–559(2005).

[12] H.G. Kwak, and J. Kim, Optimum design of reinforced concrete plane frames based on predetermined section database. Comput Aided Design Vol. 40, pp. 396–408(2008).

[13] H.G. Kwak, and J. Kim, An integrated genetic algorithm complemented with direct search for optimum design of RC frames, Comput Aided Design Vol. 41, pp. 490–500(2009).

[14] O.K. Erol, and I. Eksin, A new optimization method: Big Bang-Big crunch Adv Eng Software Vol. 37, pp. 106–111(2006).

[15] C.V. Camp, Design of space trusses using Big Bang-Big Crunch optimization. J Struct Eng ASCE Vol. 133, pp. 999–1008(2007).

[16] A. Kaveh, and S. Talatahari, Size optimization of space trusses using Big Bang–Big Crunch algorithm, Comput Struct Vol. 87, pp. 1129–1140(2009).

[17] A. Kaveh, and S. Talatahari, Optimal design of Schwedler and ribbed domes via hybrid Big Bang-Big Crunch algorithm, Constr Steel Res Vol. 66, pp. 412–419(2010).

[18] A. Kaveh, and A. Shakouri Mahmud Abadi, Harmony search based algorithm for the optimum cost design of reinforced concrete cantilever retaining walls, Int J Civil Eng IUST, Vol. 9, pp. 1-10(2011).

[19] A. Kaveh, and O. Sabzi, A comparative study of two metheuristic algorithms for optimal design of planar RC frames, Int J Civil Eng IUST, Vol. 9, pp. 193-206(2011).

[20] American Concrete Institute (ACI), Building code requirements for structural concrete and commentary, ACI 318-08, 2008.

[21] American Concrete Institute (ACI). Building code requirements for structural concrete and commentary, ACI 318-99, 1999.