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Investigation of the surf zone hydrodynamics in the vicinity of reflective structures by taking the nonlinearity of waves and wave-current interactions into account

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Abstract

In this paper, a new approach is presented for estimating the vertical and horizontal distribution of undertow in the surf zone for reflective beaches. The present model is a modification of the original model presented by Okayasu et al., (1990) for natural, non-reflective beaches to include the effect of partially reflected waves. The nonlinearity of waves, wave-current interaction and nonlinear mass drift of the incident wave are also included in the present model. The results of experimental investigation and model development show that existence of reflective conditions on beaches results in a reduction in the magnitude of undertow and modifies its distribution across the beach profile. Comparison of the results by those obtained from the experiments clearly indicates that by taking the nonlinearity and wave-current interaction, the predictions of undertow in the surf zone are much improved. In particular, due to the effect of turbulence induced by wave breaking for nonlinear waves, the predicted results show more consistence with the measurements.

Keywords: *Nonlinear wave, Reflective structure, Surf zone, Undertow, Wave-current interaction.*

1. Introduction

One of the serious problems in coastal engineering is beach erosion because this phenomenon often compromises structural integrity or alters the hydrodynamic characteristics of the flow adjacent to the structure. Although beach erosion occurs frequently at coastal structures, engineering procedures for predicting scour potential are relatively underdeveloped. Scour can be caused by combinations of hydrodynamic phenomena involving wind waves, currents, and low – frequency water level changes interacting with structures.

This paper develops Okayasu's model (1990) by taking the nonlinear theory equation (Fenton's theory (1990)) and wave-current interaction for estimating time mean flow into account. The time mean flow, or undertow, is considered one of the dominant mechanisms in the erosion of beaches [1, 2].

The undertow is generated by momentum flux due to the wave breaking in the surf zone [3]. In order to predict the sediment transport in the surf zone, it is necessary to estimate the cross-shore distribution of the undertow.

Although a number of advanced models which are able to predict the undertow for natural beaches are developed, surprisingly there has been only a limited number of works on estimating the undertow in the case of reflective beaches where partially standing waves could be found in the literature [4]. System identification techniques are applied in many fields in order to model and predict the behaviors of unknown and/or very complex systems based on given input–output data. The group method of data handling (GMDH) algorithm is a self organizing approach by which gradually complicated models are generated based on the evaluation of their performances on a set of multi-input–single-output data pairs. Abedimahzoon et al. (2010) show that GMDH-type neural networks could be used to estimate the undertow velocity based on specified variables [5].

The vertical and horizontal distributions of undertow in front of a partially reflective seawall in a series of random wave experiments were measured by researchers [6, 7]. Their investigation revealed that the magnitude of the undertow is reduced in the presence of partially standing waves. This conclusion is in agreement with the work of Rakha and Kamphuis (1997) indicating a reduction in undertow which caused by reflected waves [8].

The work presented in this paper is a new modification of the originally-based model developed by Okayasu et al., (1990) and modified by Neshaei et al (2009). This study is a contribution to compensate for the lack of information concerning the effect of reflective beaches on the

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distribution of undertow in the surf zone due to regular wave attack. The results of the present work can be used for the cross-shore sediment transport and beach evolution models where reflective conditions exist.

2. Theoretical Development

A model was presented to estimate the distribution of undertow in the surf zone for arbitrary beach topography [1]. Here, a modification of that theory is presented which takes the effects of reflected waves into account. Radiation stress, wave set up and mass flux due to the wave motion are modified to take the effects of reflected waves into consideration.

The basic equation to calculate the vertical distribution of undertow in the surf zone is given by [1]:

$$U = \frac{a_\tau'}{a_v}\left(z' - \frac{d_t}{2}\right) + \frac{a_v b_\tau' - a_\tau' v}{a_v^2}\left(1 + \log\frac{a_v z' + v}{a_v d_t + v} - \frac{v}{a_v d_t}\log\frac{a_v d_t + v}{v}\right) + U_m \quad (1)$$

Where:

U = undertow at elevation z' from the bed; U_m = mean undertow below trough level (calculated from equation 7); v = kinematic viscosity of water; d_t = water depth at wave trough and a_v, a_τ' and b_τ' are calculated based on the following equations [1]:

$$a_v = 0.06 h \rho^{1/3} D_B^{1/3} d_t^{-1} \quad (2)$$

$$a_\tau' = \frac{1}{15}\rho^{1/3} D_B^{2/3} d_t^{-1} + \frac{va^2\sigma k}{2h^2 \sinh^2 kh}\left(3kh\sinh 2kh + \frac{3\sinh 2kh}{2kh} + \frac{9}{2}\right) \quad (3)$$

$$b_\tau' = -\frac{1}{75}\rho^{1/3} D_B^{2/3} - \frac{va^2\sigma k}{4h\sinh^2 kh}\left(2kh\sinh 2kh + \frac{6\sinh 2kh}{2kh} + 9\right) \quad (4)$$

In which:

ρ = density of water; a = wave amplitude; σ = wave angular frequency; k = wave number

h = mean water depth and D_B = rate of energy dissipation by wave breaking (calculated from equation 5).

The energy dissipation rate is calculated based on the linear wave theory on a constant slope and can be expressed as [9]:

$$D_B = \frac{5}{16}\rho g^{3/2} \tan\beta \gamma_H{}^2 h^{2/3} \quad (5)$$

Where:

g = acceleration of gravity; $\tan\beta$ = bottom slope and γ_H = ratio of wave height to water depth.

It has to be noted that in the present model the concept of variable eddy viscosity through depth is used to derive the vertical distribution of undertow. The eddy viscosity can be expressed as:

$$\nu_e = 0.06\rho^{-1/3} D_B{}^{1/3}\frac{h}{d_t}z' \quad (6)$$

The vertically averaged value of the undertow is calculated based on the following equation:

$$U_m = -\frac{1}{d_t}M_t \quad (7)$$

In which the total mass flux by the breaking wave, M_t, is calculated as:

$$M_t = M_w + M_v \quad (8)$$

It can be seen that inside the surf zone, the total mass flux by the breaking wave is caused by the mass flux due to the wave motion, M_w, and organized large vortexes, M_v. Outside the surf zone, however, the mass flux is caused only by the wave motion. The following equations are used to calculate the mass fluxes:

$$M_w = \frac{1.6c}{\rho g h}E_p \quad (9)$$

$$M_v = 0.09\rho H c \quad (10)$$

Where:

c = wave celerity; H = wave height (in case of random waves, sum of the broken wave heights [10]) and E_p = potential energy of wave motion $(\frac{\rho g H^2}{16})$.

In case of wave reflection, the mass flux by the reflected wave, M_w, is subtracted from the total mass flux to modify the mass balance in the surf zone. It is assumed that the reflected waves reduce the total mass flux in the surf zone with the consequence of reduction in the mean undertow according to equation 7. Therefore, for reflective beaches, the total mass flux by breaking and reflected waves can be written as:

$$M_t = (M_w + M_v)_i - (M_w)_r \quad (11)$$

In which subscripts i and r represents the incident and reflected waves, respectively. The reflected wave height can be calculated as RH_i where R is the reflection coefficient of the beach. Although in case of random waves the reflection coefficient is a function of frequency and varies for different wave components, at this stage of calculations an average reflection coefficient is used to modify the wave height.

Finally, calculation of the wave set up in the surf zone is based on the mean balance of the onshore momentum equation [9]:

$$\frac{ds_{xx}}{dx} + \rho g h\frac{d\overline{\eta}}{dx} = 0 \quad (12)$$

Where:

x = horizontal coordinate in cross-shore direction; S_{xx} =

radiation stress and $\overline{\eta}$ = wave set up.

The component of radiation stress tensor normal to the shore can be calculated based on linear wave theory as:

$$S_{xx} = (\frac{1}{2} + \frac{2kh}{\sinh 2kh})E \tag{13}$$

Where:

E = total energy of the wave $(\frac{\rho g H^2}{8})$ and $h = d + \overline{\eta}$, d being still water depth.

It is to be noted that for the case of reflected waves the radiation stress component should be modified by adding an extra term representing the radiation stress due to the reflected wave. This will result in a modified wave set up in the surf zone. Therefore, the total radiation stress can be written as:

$$S_{xx} = (S_{xx})_i + (S_{xx})_r \tag{14}$$

In which $(S_{xx})_r$ represents the radiation stress for reflected wave and can be calculated from the reflected wave characteristics. In summary, it can be concluded that reflected waves modify the radiation stress; wave set up and mass flux in the surf zone, which will accordingly change the undertow.

The present model also considers the nonlinear effect of the incident wave and wave-current interaction in calculation of undertow inside the surf zone. An accurate steady wave theory may be developed by numerically solving the full nonlinear equations with results that are applicable for short waves (deep water) and for long waves (shallow water). This is the Fourier approximation method. Fenton's Fourier approximation wave theory satisfies field equations and boundary conditions to a specified level of accuracy [11].

The governing field equation describing wave motion is the two-dimensional (x,z in the Cartesian frame) Laplace's equation, which in essence is an expression of the conservation of mass:

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} = 0 \tag{15}$$

Fenton's solution method uses the Fourier cosine series in kx to the governing equations. It is clearly an approximation, but very accurate, since results of this theory appear not to be restricted to any water depths. The dependent variable is the stream function Ψ represented by a Fourier cosine series in kx, expressed up to the Nth order as [11]:

$$\psi(x,z) = \overline{u}(z+d) + (\frac{g}{k^3})^{0.5} \sum_{j=1}^{N} B_j \frac{\sinh jk(z+d)}{\cosh jkd} \cos jkx \tag{16}$$

Where B_j are dimensionless Fourier coefficients. The truncation limit of the series N determines the order of the theory. The nonlinear free-surface boundary conditions are satisfied at each of M+1 equi-spaced points on the surface.

The horizontal and vertical components of the fluid particle velocity are [12]:

$$u(x,z) = \frac{\partial \psi}{\partial z} = -\overline{u} + (\frac{g}{k})^{0.5} \sum_{j=1}^{N} jB_j \frac{\cosh jk(z+d)}{\cosh jkd} \cos jkx \tag{17}$$

$$w(x,z) = -\frac{\partial \psi}{\partial x} = (\frac{g}{k})^{0.5} \sum_{j=1}^{N} jB_j \frac{\sinh jk(z+d)}{\cosh jkd} \sin jkx \tag{18}$$

The stokes drift caused by the nonlinearity of the incident wave is also included in the model which reads as [13]:

$$\overline{U} = \frac{1}{2} c k^2 a^2 \frac{\cosh 2k(d+z)}{\sinh^2 kd} \tag{19}$$

Where:

c= wave celerity; a = wave amplitude; k = wave number and d= mean water depth.

Finally, the wave-current interaction is also implemented in the proposed model according to the following equation [13]:

$$L = \frac{c_0 T}{2} \tanh \frac{2\pi d}{L} \left(1 \pm \sqrt{\frac{4u}{c_0} \coth \frac{2\pi d}{L}} \right) \tag{20}$$

Where:

T= wave period; L = wave length; c_0= wave celerity in deep water and u= mean current velocity.

It is to be mentioned that an iteration process was applied in the model to modify the wave height based on the above criteria for wave-current interaction.

3. Experimental Investigation of Undertow in the Surf Zone

3.1. Experimental Apparatus and Procedure

In order to consider the effect of reflective beaches on the distribution of the mean flow, series of experiments were performed in the Coastal Engineering Laboratory of Kagoshima University, Japan, the magnitude and distribution of undertow were obtained from different cases of reflective beaches [14]. Figure 1 shows the experimental set up. Monochromatic waves were generated and water particle velocities were measured in the surf zone using an Electromagnetic current meter.

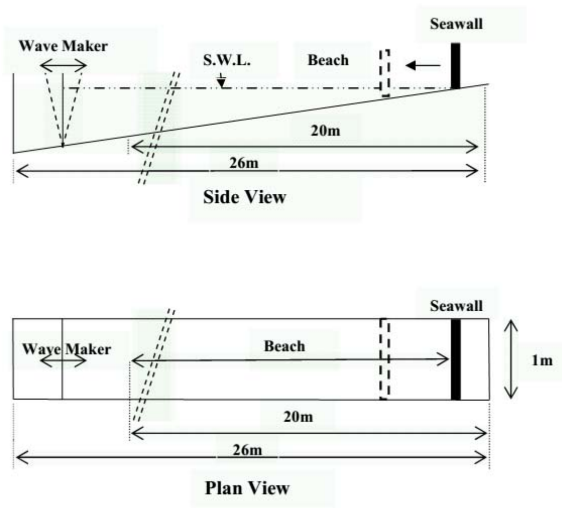


Fig. 1 Illustration of the experimental set up for regular wave experiments.

The overall dimensions of the tank were 1.00 m wide, 1.2 m deep and 26 m long. A plane beach profile with a constant slope of 1:20 was built at the end of the tank. Waves were generated at one end of the tank by a wave generator controlled by an electro-hydraulic system and the water particle velocities were measured in the surf zone using a two-component Electro-magnetic current meter. Different horizontal locations were chosen in the surf zone, where the measurements were made at several points between the bottom and still water level.

The measured points were in the middle of the flume (0.50 m from inside face of the side-wall). Additionally, a resistance type wave gauge measured the water surface elevation (synoptic with the velocity measurements) at each location. Also, a deep water wave gauge was mounted further offshore to measure the deep water incident wave spectrum. Data was acquired and analyzed using a personal computer with a sampling rate of 20 Hz for each channel. The recording length was 3.5 minutes allowing approximately 200 waves into account. A solid-reflective wall was placed at different locations across the surf zone and the velocity measurements were repeated in front of the structure.

3.2. Monochromatic Wave Experiments

Table 1 summarized the different wave conditions (wave heights and wave periods), which were used in the experiments. The reflection coefficient of the beach was measured using the moving probe method to detect the envelope of partially standing waves formed in front of the seawall.

Table 1 Characteristics of monochromatic waves used in the experiments.

Deep Water Wave Height (m)	Wave Period (s)	Deep Water Wave Length (m)	Deep Water Wave Steepness
0.100	2.0	6.24	0.016
0.125	1.5	3.51	0.036
0.150	1.0	1.56	0.096

Figure 2 shows the variation of the wave height across the beach for the one of wave conditions used in the experiments. As can be seen, the wave attenuation model based on the linear wave theory can predict the measured data with a reasonable level of accuracy. However, at the breaking point, a discontinuity can be observed in the prediction, which is due to the different criteria used in the wave transformation phase of the model to predict the attenuated wave heights before and after the breaking point, respectively.

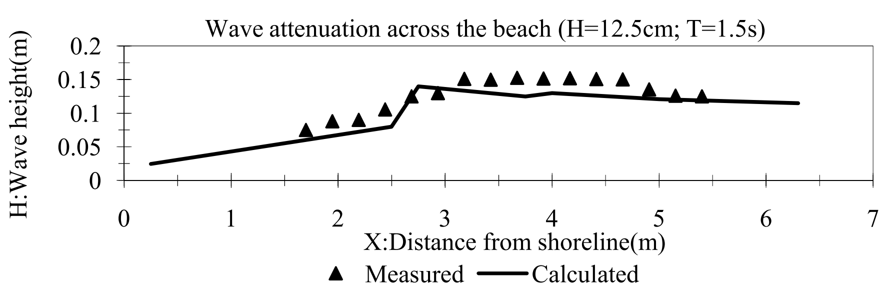


Fig. 2 Comparison between calculated and measured wave heights across the profile for one of the wave conditions

In the present model, the wave height in the offshore region of the breaking point, i.e. outside the surf zone, was computed based on the linear shoaling coefficient [13]; whereas inside the surf zone a wave decay model, which was based on a linear relationship between the broken wave height (H_b) and water depth (h), was used ($H_b = \gamma h$ in which γ is the breaker index normally taken as 0.78). Therefore, just at the breaking point, a discontinuity in the model prediction can be observed.

The velocity was measured for three cases of seawall location; i.e., without seawall, and seawall located in the surf zone with 50 and 100 mm water depths in front of the wall, respectively. The results were compared with those obtained from natural beaches (with no reflection) and existing theoretical models. The main objective was to undertake a quantitative comparison of the undertow in two cases (i.e. with and without reflective conditions).Using the measured envelopes of the partially standing waves for different wave conditions in front of the seawall, the reflection coefficients of the beach were obtained within the range of 10% to 30%.

Figure 3 shows the effect of beach reflection on the measured set up across the profile for one of the wave condition used in the experiment. As can be seen, partially reflected waves have caused the breaking point to be shifted slightly offshore resulting in the reduction of both wave set down and set up. This in turn will cause a reduction in the undertow inside the surf zone, which is in agreement with the velocity measurements. It is to be noted that there is a small rise in the mean water level due to the formation of partially standing waves close to the seawall, which can affect the set up measurements in that region.

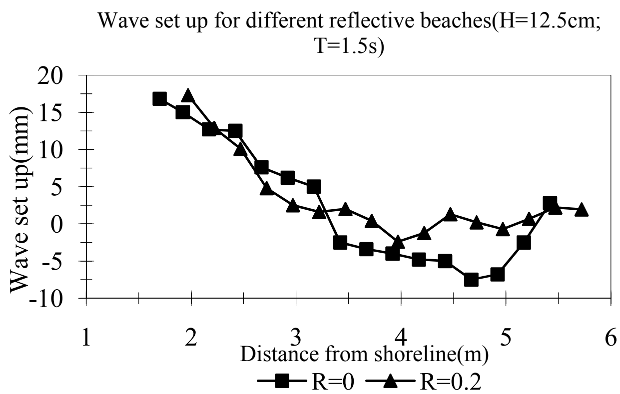


Fig. 3 Comparison between measured wave set up (and set down) across the profile for different coefficients of the beach (R) using one of the wave conditions

Figure 4 shows examples of comparison between the measured vertical distributions of undertow for non-reflective and reflective beaches for one of the wave conditions used in the experiments. The locations of the measured points were selected in such a way that they covered a wide range of inside and outside of the surf zone. As can be seen, it is clear that the undertow was reduced in front of the reflective wall and this reduction was more significant for higher reflection coefficient of the beach.

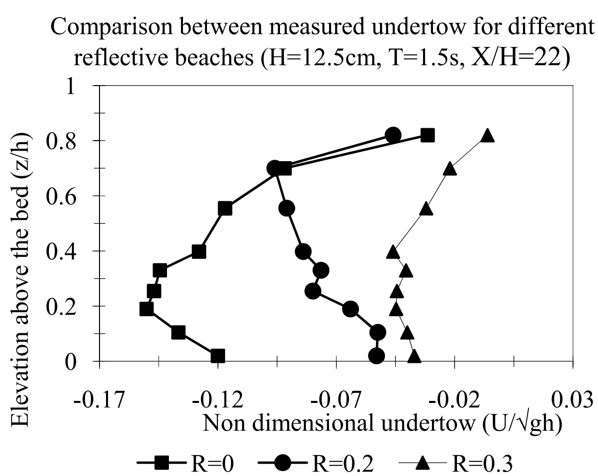


Fig. 4 Comparison between measured undertows for different reflection coefficients of the beach (R).

Figure 5 contrasts the horizontal distributions of undertow for non-reflective beach with those obtained in front of the seawall for a particular point above the bed and for one of the wave conditions. As indicated in this figure, the reduction of undertow for reflective beaches is more significant for the points inside the surf zone. As for further offshore points, because of the small magnitude for undertow, the reduction due to reflective conditions of the beach is not pronounced [15].

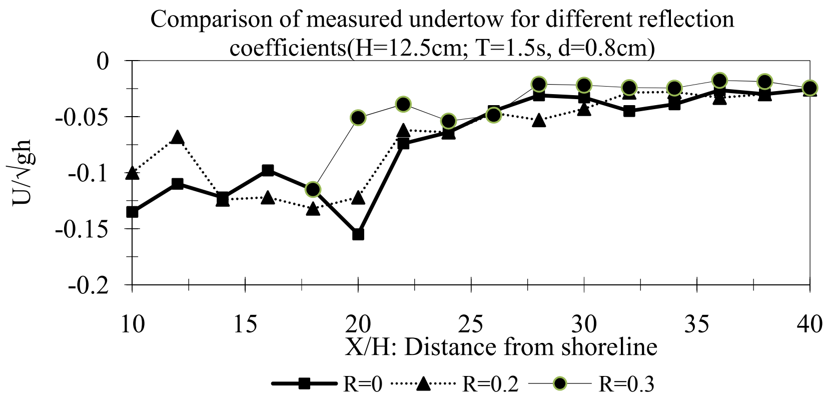
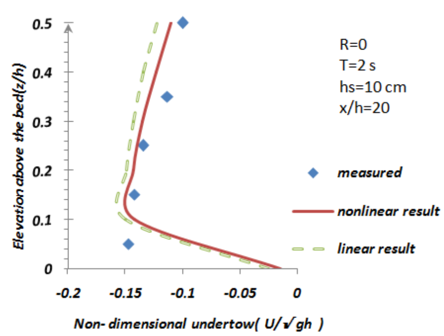


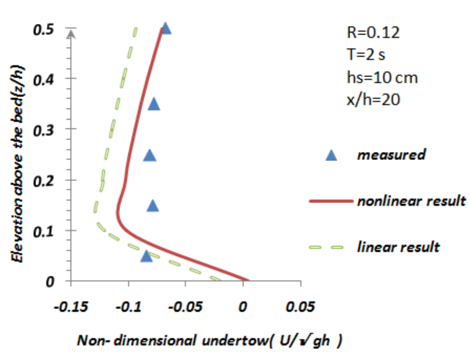
Fig. 5 Comparison between measured undertows across the profile at 0.8 cm above the bed for different reflection coefficients of the beach (R)

4. Results and Discussions

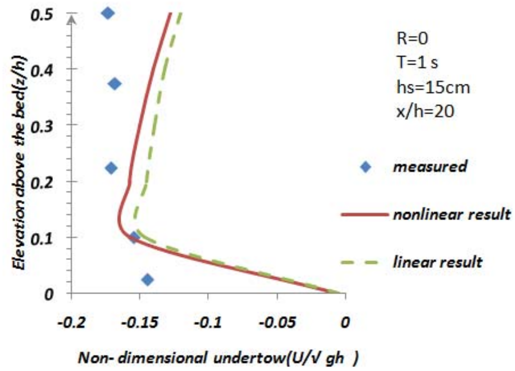
Figures 6 and 7 show examples of comparison between the estimated (linear & nonlinear) and measured undertows for different locations across the surf zone using different regular wave conditions. Calculated undertows are based on the proposed model introduced in the present work.



6.a



6.b



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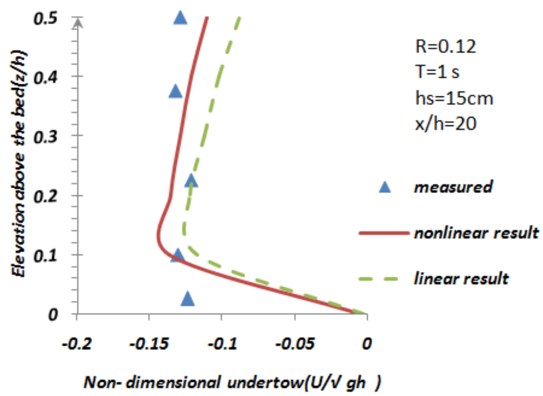
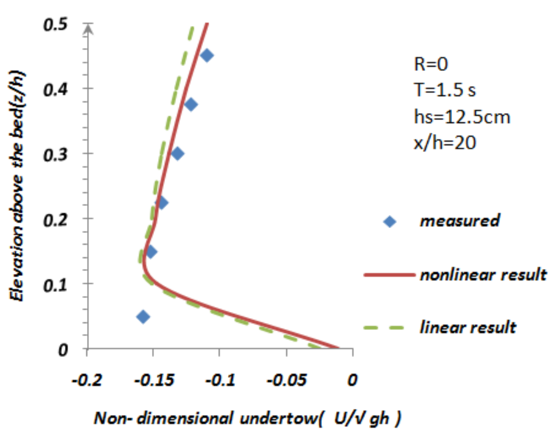
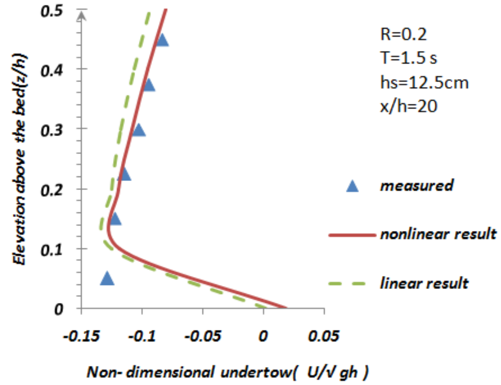


6.d

Fig. 6 (a, b, c, d). Comparison between calculated and measured undertows for different reflection coefficients of the beach (R) and different locations across the profile (X/H) using different wave conditions(hs: significant wave height).



7.a



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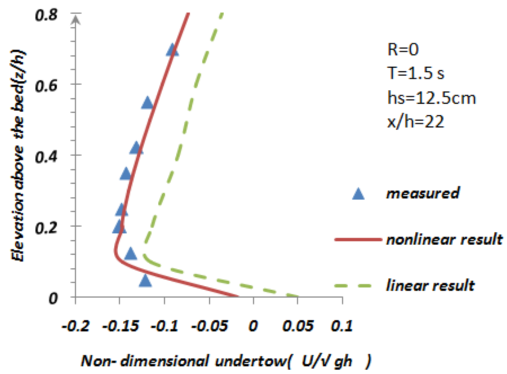

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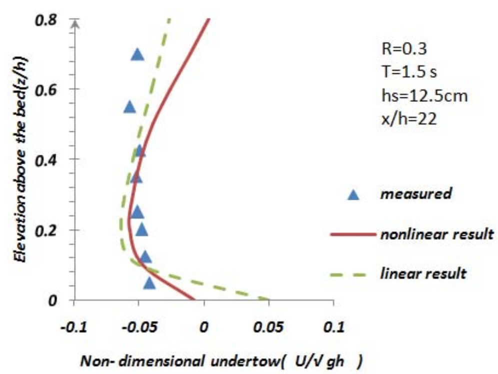


7.d

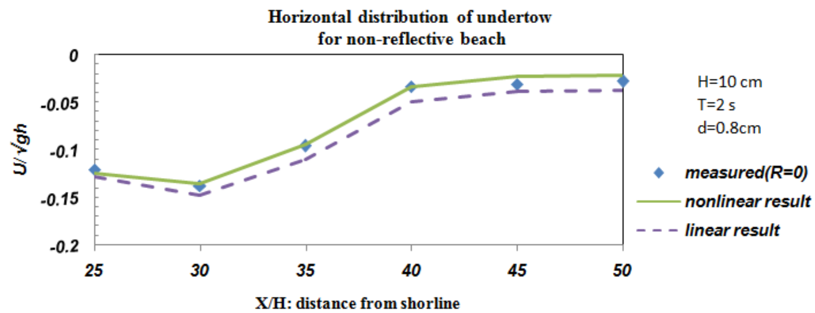
Fig. 7 (a, b, c, d). Comparison between calculated and measured undertows for different reflection coefficients of the beach (R) and different locations across the profile (X/H) using one of the wave conditions(hs: significant wave height).

Comparison between calculated and measured undertows, as indicated in these figures, shows the good agreement particularly for the locations inside the surf zone. The discrepancy for further offshore points could be attributed to the small magnitudes of undertow at those points.

It is interesting that by taking the nonlinearity effect and wave-current interaction into account the results are remarkably improved and much more accurate prediction is expected by the proposed model. It is obvious that for the large wave heights the effect of turbulence due to wave breaking is much higher. Therefore, the linear wave theory cannot predict the undertow accurately and the magnitude of the undertow for nonlinear waves is more than those predicted by linear wave theory, which are closer to the experimental results (Fig 6). In addition, a modification for the wave height by wave –current interaction is included in the model. It is to be noted that in the linear wave modeling this effect has not been taken into account, resulting in a better prediction in the case of nonlinear waves.

It has to be noted that all experimental measurement points in the Figs 6&7 are located in the surf zone. Noting to this fact that the further offshore points are closer to the breaking points, therefore due to the large turbulence in the breaking zone, the larger magnitudes for undertow are expected for nonlinear waves (Fig 7).

Comparison of the predicted and measured horizontal distribution of undertow in the surf zone for different cases of reflective beaches, as shown in Figure 8, clearly indicates the applicability of the proposed model to predict the effect of beach reflection on the mean flow velocity for different cases of regular waves. It is to be noted that the effect of turbulence and transition zone for wave breaking has not been included in the present model. Clearly, there is a need to improve the model by including the effect of turbulence and interaction between incident and reflected waves in the surf zone.



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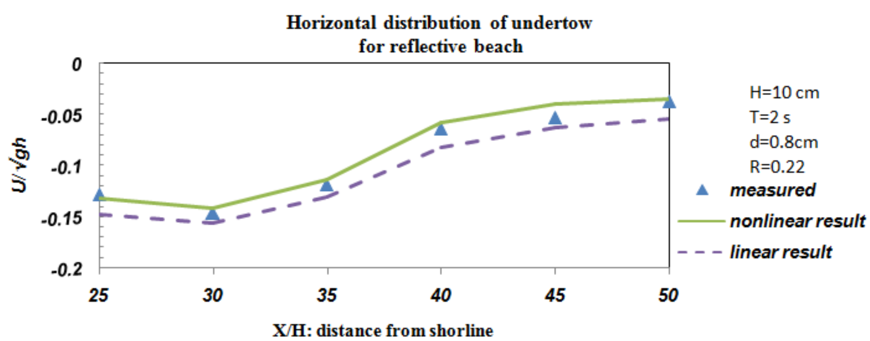

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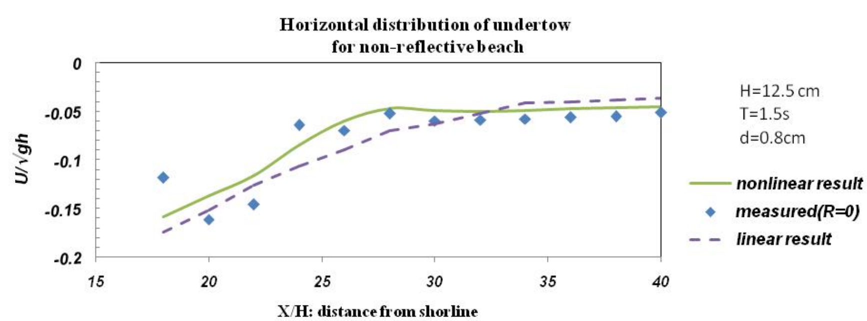

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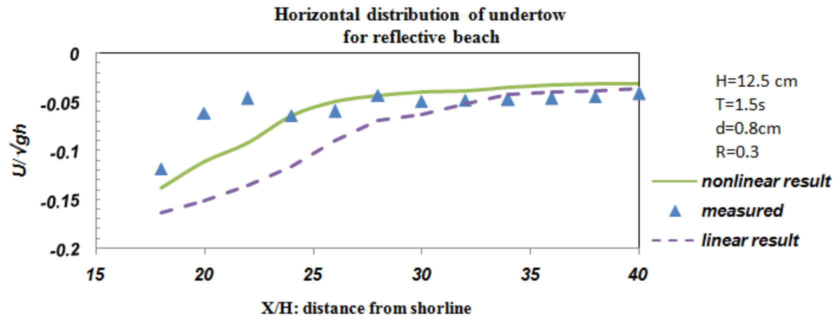

8.d

Fig. 8 (a, b, c, d). Comparison between calculated and measured undertows across the profile at 0.8 cm above the bed for different reflection coefficients of the beach (R) using different wave conditions.

It is obvious that the present model have some deficiencies in predicting undertow, particularly near the bed and close to the structure due to the mentioned effect, but using the nonlinear wave theory and wave current interaction has resulted in much better prediction for undertow velocity. Some statistical measures are given in table 2 to determine the accuracy of model.

Table 2 Model statistics and information for predicting undertow.

Figure No.	6.a	6.b	6.c	6.d	7.a	7.b	7.c	7.d
RMSE(linear)	0.0214	0.0288	0.0297	0.0279	0.0145	0.0158	0.0271	0.0198
RMSE(nonlinear)	0.0106	0.0115	0.0119	0.0112	0.0098	0.0109	0.0091	0.0105
SI (linear)	0.1343	0.1486	0.1512	0.1474	0.0975	0.0984	0.1291	0.1276
SI (nonlinear)	0.1187	0.1263	0.1289	0.1253	0.0912	0.0923	0.0905	0.1020

These statistical values are based on RMSE as root-mean squared error and SI as scatter index which are defined as follows:

$$RMSE = \sqrt{\frac{\sum (U_{calc} - U_{meas})^2}{n}} \tag{21}$$

$$SI = \sqrt{\frac{\sum ((U_{calc} - \overline{U}_{calc}) - (U_{meas} - \overline{U}_{meas}))^2}{n}} \Big/ \overline{U}_{meas} \tag{22}$$

Where: U_{calc} = the value of the calculated undertow velocity; U_{meas} = the value of the measured undertow velocity, n is the number of point; $\overline{U}_{calc}$ = the mean value of the calculated undertow velocity and $\overline{U}_{meas}$ = the mean value of the measured undertow velocity.

Looking at the results obtained from the present work, it can be seen that for the large wave height the effect of turbulence due to the wave breaking affects the predictive results. Also, close to the bed, due to the effect of boundary layer, the predicted results of model for undertow are not decisive. It is to be noted that one of the main objective of this research is considering the effect of nonlinearity of the wave on the result. It can be clearly observed from the result of the model prediction that by taking the nonlinearity of the incident wave into account, the accuracy of the prediction is much improved.

In summary, it can be stated that the main innovations of the present model are taking the nonlinearity of the incident waves, the wave-current interaction and the stokes drift induced by the asymmetry of the incident waves into account. In particular, the effects of each mentioned innovative parts of the present model are as follows:

a) The nonlinearity of the incident waves causes more complicated velocity field inside the surf zone and change the pattern of the wave breaking which in turn, will affect the hydrodynamic parts of the present model. It is to be noted that taking the effects induced by nonlinear waves can result in much better simulation of the real sea conditions and match the experimental measurements with more accuracy.

b) Taking the wave-current interaction into account can improve the level of accuracy in predicting the undertow velocities because in reality the return flow due to the wave breaking in the surf zone will affect the incident waves and modify the wave breaking location.

Taking the stokes drift in the present model can cause more on shoreward flow at the bed which consequently result in more onshore sediment transport in the surf zone. This could affect the undertow velocities and the resulting beach profile changes.

5. Conclusions

The work presented in this paper is a new modification of the originally-based model developed by Okayasu et al., (1990) and modified by Neshaei et al (2009). The main conceptual innovation of the model is taking the nonlinearity of waves, mass drift and wave-current interaction into account in estimation of undertow in the surf zone.

The presence of partially standing waves due to reflective conditions in the surf zone results in a reduction in the magnitude of the mean flow (undertow) and changes its distribution across the surf zone. The level of reflectivity of the beach is an important parameter to control the magnitude and distribution of the undertow. The results obtained from experiments and theoretical investigations show that as the reflection coefficient of a beach increases, the magnitude of undertow reduces which can affect the offshore sediment transport rate in the surf zone. This reduction is more pronounced for the inner surf zone points. The results of the current study show the effect of beach reflection on the measured set up across the profile. As can be seen, partially reflected waves have caused the breaking point to be shifted slightly offshore resulting in the reduction of both wave set down and set up. This in turn will cause a reduction in the undertow inside the surf zone, which is in agreement with the velocity measurements. The results obtained from regular wave experiments and the presented model are consistent and clearly support the conceptual elements of the proposed model to predict the undertow for reflective beaches.

In summary, comparison of the results by those obtained from the experiments clearly indicates that by taking the nonlinearity and wave-current interaction, the predictions of undertow in the surf zone are remarkably improved. It can be expected that incorporating the turbulence and vortex affected by wave breaking in the surf zone into the presented model will result in more accurate prediction which matches the experimental result.

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Nomenclatures

a = wave amplitude
c = wave celerity
d_t = water depth at wave trough
D_B = rate of energy dissipation by wave breaking
E = total energy of the wave
E_p = potential energy of wave motion
g == acceleration of gravity
h = water depth
H = wave height
H_b= wave height at the breaker point
k = wave number
L_0 = deep water wave length
U = undertow at elevation z' from the bed
U_m = mean undertow below trough level
S_{xx} = radiation stress
$(S_{xx})_r$= radiation stress for reflected wave
x = horizontal coordinate in cross-shore direction
$\tan\beta$ = bottom slope
$\overline{\eta}$ = wave set up
ρ = density of water
γ_H = ratio of wave height to water depth
σ = wave angular frequency
ν = cinematic viscosity of water
Ψ =stream function