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# Optimization of high-performance concrete structures by variable neighborhood search

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***Abstract***

*This paper describes a methodology in designing high-performance concrete for simply supported beams, using a hybrid optimization strategy based on a variable neighborhood search threshold acceptance algorithm. Three strategies have been applied to discrete optimization of reinforced concrete beams: Variable Neighborhood Descent (VND), Reduced Neighborhood Search (RNS) and Basic Variable Neighborhood Search (BVNS). The problem includes 14 variables: two geometrical; one material type; one mix design; and 10 variables for the reinforcement setups. The algorithms are applied to two objective functions: the economic cost and the embedded $CO_2$ emissions. Firstly, this paper presents the application of these three different optimization strategies, which are evaluated by fitting the set of solutions obtained to a three-parameter Weibull distribution function. The Variable Neighborhood Descent with Threshold Accepting acceptance strategy algorithm (VND-TA) results as the most reliable method. Finally, the study presents a parametric study of the span length from 10 to 20 m in which it can be concluded that economic and ecological beams show a good parabolic correlation with the span length.*

**Keywords:** *Structural optimization, Reinforced concrete, Sustainable construction, $CO_2$ emission, High-performance concrete, Heuristics.*

## 1. Introduction

In the field of structural design, the traditional goals of engineers were the design of a safe and economical structure. Nowadays, there is a growing concern for sustainability, which leads to a change in the accounting of resource consumption. In this context, the objective of structural design becomes to optimize the consumption of materials not only from an economic point of view, but also environmental. It is worth to note that concrete is the most widely used material on Earth [1]. Portland cement, the principal hydraulic binder used in modern concrete, is responsible for large emissions of carbon dioxide ($CO_2$). Worldwide, the cement industry alone is estimated to be responsible for about 7% of all $CO_2$ generated [2]. Therefore, it seems vital to include design criteria to minimize the embedded $CO_2$ emissions in reinforced concrete (RC) structures. In recent years, there have been numerous studies to reduce greenhouse gas emissions resulting from concrete construction. Foremost is the increasing use of cementitious materials, especially those that are by-products of industrial processes (such as fly ash, ground granulated blast furnace slag, and silica fume), that can serve as partial substitutes for Portland cement [3,4]. Other studies analyze the substitution of various recycled materials in aggregate, by using materials like recycled concrete aggregate, scrap tires and plastics. By this way, they reduce the need to extract virgin aggregates [5]. Another way of optimizing the consumption of materials is the use of high performance materials, which are proposed to reduce cross sections and the volume of concrete. They are also intended to increase the durability of concrete structures, to minimize the maintenance needs of the concrete construction and to limit the amount of non-renewable special repair materials for maintaining the concrete.

This study deals with the discrete optimization, in terms of economic cost and $CO_2$ emissions, of different mix designs of high-performance concrete (HPC). The mix designs considered in this study have been taken from different publications on technology of HPC. These studies are

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dedicated to various properties of fresh mix or hardened concrete, and unfortunately provide only partial information of the actual composition of the mix. Furthermore, it is important to note that in this study each component is described using only one term: its quantity in the concrete mixture. Each of these terms represents a variety of information (e.g. cement can be powdered to various degrees of finenesses and composed of several different chemical compositions). Taking into account these limitations, experimental data from 22 different sources have been considered [6-27] and compressive strength of HPC has been expressed as a function of six input features: cement (kg/m$^3$), water (kg/m$^3$), coarse aggregate (kg/m$^3$), fine aggregate (kg/m$^3$), superplasticizer (kg/m$^3$) and silica fume (kg/m$^3$).

The structural optimization problem can be accomplished using either exact or heuristic methods. Exact methods are traditional and well-established approaches, usually based on the calculation of optimal solutions following iterative techniques of linear programming [28,29]. The objective function in exact methods must be differentiable or continuous or the reasonable region must be convex. This requirement indicates that the efficiency of exact methods is limited to problems with a few design variables. Sarma and Adeli [30] review the application of non-heuristic structural concrete cost optimization in different types of concrete structures.

The second main category of optimization methods is heuristic methods, whose recent development is linked to the evolution of artificial intelligence procedures. This category includes a large number of heuristic search algorithms based on iterations in which the objective function is evaluated and the structural constraints are checked. The first studies on heuristic structural optimization [31,32] were applied to steel structures to reduce the weight of the structure. Concerning RC structures, early applications include a pioneering optimization of RC beams by Coello et al. [33], and the application of genetic algorithms to prestressed concrete beams by Leite and Topping [34]. Recently, heuristic methods have been reported on different types of RC structure optimization: retaining walls [35,36], frames [37-40], and so on.

The aim of this study is the economic and environmental optimization of simply supported HPC reinforced beams. The method followed in this research consisted in the development of an evaluation computer module where dimensions, materials and steel reinforcement were taken as variables. This module computes the economic cost and the $CO_2$ emissions of a solution and checks all the relevant limit states. The heuristic model used to optimize the beams is a hybrid multistart strategy based on a Variable Neighborhood Search (VNS) with Threshold Accepting (TA). This strategy has already been broadly and successfully applied in the engineering field, mainly in location and scheduling problems; however, few applications have been developed in structural design optimization [41].

## 2. Optimization problem definition

The problem of structural concrete optimization proposed in this study consists of a single-objective optimization of either the embedded $CO_2$ or the total cost of the structure. Therefore, it deals with the minimization of one of the two objective functions: structure cost ($F_1$, defined by Eq. (1)) or $CO_2$ emissions ($F_2$, defined by Eq. (2)); satisfying the constraints of Eq. (3).

$$F_1(x_1, x_2, ..., x_n) = \sum_{i=1}^{r} p_i \cdot m_i \tag{1}$$

$$F_2(x_1, x_2, ..., x_n) = \sum_{i=1}^{r} e_i \cdot m_i \tag{2}$$

$$g_j(x_1, x_2, ..., x_n) \le 0 \tag{3}$$

In the expressions above, note that $x_1, x_2, ..., x_n$ are the design variables of the problem. The objective functions considered in Eq. (1) and Eq. (2) are defined as the sum of the unit costs –prices ($p_i$) for economical optimization or $CO_2$ unit emissions ($e_i$) for the embedded $CO_2$ optimization– multiplied by the measurements of construction units ($m_i$) such as concrete, steel, formwork, etc. Unit prices and $CO_2$ unit emissions considered for the problem optimization are given in Table 1 and are obtained from the BEDEC PR/PCT ITEC materials database of the Catalonia Institute of Construction Technology [42] in November 2011.

Constraints in Eq. (3) include the serviceability and ultimate limit states (SLSs and ULSs) as well as the geometric and constructability constraints of the problem. In this study, only feasible solutions have been considered and therefore, penalty functions have not been applied. Given the variables of the present problem, the measurement and cost evaluation of a particular solution is straightforward.

### 2.1. Variables

A total of 14 variables define a solution for the simply supported HSC beam (Fig. 1 and Fig. 2). These variables define the geometry, the type of steel grade, the passive reinforcement of the beam and the HPC mix design. The first two variables are geometric and they correspond to the height ($h$) and the width ($b$) of the cross section. The next 10 variables correspond to the definition of a passive reinforcement setup. Each reinforcement setting is defined by two variables: number of bars ($n$) and diameter ($Ø$). The longitudinal upper reinforcement is displayed throughout the beam length ($n_1$,

**Table 1** Basic prices and $CO_2$ emission considered in the analysis

| Unit | Cost (euros) | $CO_2$ emissions (kg) |
|---|---|---|
| kg of cement | 0.11 | 0.83 |
| dm$^3$ of water | 1.19 | 0.29 |
| kg of superplasticizer | 1.27 | 13.73 |
| kg of coarse aggregate | 0.02 | 0.01 |
| kg of fine aggregate | 0.02 | 0.01 |
| kg of silica fume | 0.94 | 1.05 |
| m2 of formwork | 27.34 | 2.96 |
| kg of steel B-400-S | 1.19 | 3.03 |
| kg of steel B-500-S | 1.21 | 3.03 |

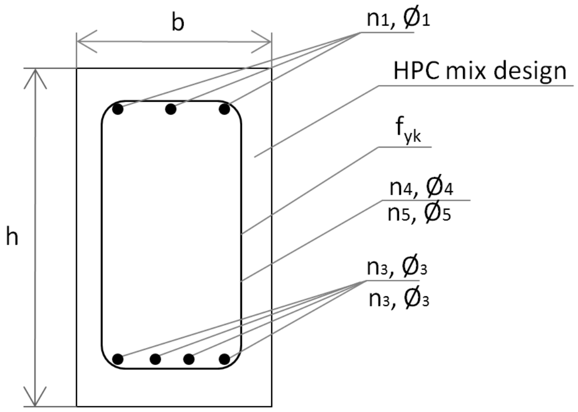


**Fig. 1** Design variables of the simply supported HPC beam: cross section

$Ø_1$). Flexural bars include two reinforcement systems: a continuous lower reinforcement which covers all the beam length ($n_2$, $Ø_2$); and a central reinforcement displayed in the central part of the beam ($n_3$, $Ø_3$). Four variables define the design of the shear reinforcement: the central reinforcement ($n_5$, $Ø_5$); and the reinforcement to be displayed in a length of L/3 near the supports ($n_4$, $Ø_4$). The next variable relate to the steel grade ($f_{yk}$), which can vary between B-400-S and B-500-S. The last variable is the HPC mix design. A set of 120 mix designs is considered in this study, considering 20 possible HPC mix design for each concrete grade value (55, 60, 70, 80, 90 and 100 MPa). Each mix design option is a set of seven values: the concrete grade and the amount of cement, water, superplasticizer, coarse aggregate, fine aggregate and silica fume for each $m^3$ of HPC.

The set of combinations of values of the 14 variables may be defined as the solution space. This space is practically unlimited due to what is known as combinatorial explosion. Each vector comprised of 14 variables defines a solution having an economic cost following Eq. (1) and a $CO_2$ emission following Eq. (2). Solutions that satisfy the constraints of Eq. (3) will be feasible solutions. Those that do not satisfy all constraints will be unfeasible solutions.

### 2.2. Parameters

The parameters are all those magnitudes taken as data and therefore, remain constant in the optimization process. They are divided into geometric, loading, cost and durability data. Table 2 shows the parameter values considered in the optimization research.

### 2.3. Structural constraints

Once the 14 variables defining a beam solution are chosen, then geometry, materials and passive reinforcement are fully defined. Considering these 14 variables that define a particular solution, the structural evaluation module verifies the feasibility of the beam solution. This module calculates the stress envelopes of the structure and checks all the restrictions considered in Eq. (3) that the structure must satisfy: all the serviceability and ultimate limit states (SLS and ULS), as well as the geometric and constructability constraints. The critical sections of the simply supported beam are verified in accordance with the requirements of the Spanish Standard EHE-08 [43]. The verifications considered in the ULS include service and ultimate flexure and ultimate shear. Moreover, both flexural and shear minimum amounts of reinforcement as well as the geometrical minimum, are also taken into account. The SLS verifications check that the cracking width does not exceed the limitation for the existing durability conditions.

## 3. Variable neighborhood search

The heuristic method used in this work is an optimization strategic method based on a variable neighborhood search strategy (VNS). The VNS strategy was originally proposed by Mladenovic and Hansen [44]. The VNS method is based on three observations: (i) a local optimum with respect to one neighborhood structure is not necessarily a local optimum for another neighborhood structure; (ii) a global optimum is a local optimum with respect to all possible neighborhood structures; and (iii) for many problems local minima with respect to one or several neighborhoods are relatively close to each other. Unlike many other metaheuristics, the basic schemes of VNS are simple and require few decisions: the number and types of neighborhoods; the order of their use in

**Table 2** Parameters considered in optimization

| Parameter | Value |
|---|---|
| Beam span ($L$) | 15 m |
| Permanent distributed load ($g$) | 20 kN/m |
| Variable distributed load ($q$) | 10 kN/m |
| Moving load ($Q$) | 4 kN |
| Control level during construction | Normal |
| Control level of materials | Normal |
| Exposure class | IIa (high humidity) |

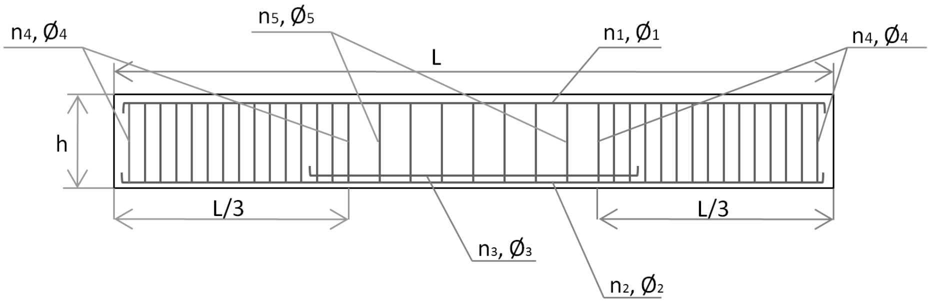


**Fig. 2** Design variables of the simply supported HPC beam: longitudinal view

the search, the strategy for changing the neighborhoods, the local search methods and the stop criterion [44]. Let us denote with $N_k$, ($k=1,\dots, k_{max}$), a finite set of pre-selected neighborhood structures, and with $N_k(x)$ the set of solutions in the $k^{th}$ neighborhood of $x$. The search of solutions in the neighborhoods can be done in three different ways: deterministic, stochastic and both deterministic and stochastic.

*3.1. Reduced variable neighborhood search*

The reduced variable neighborhood search (RVNS henceforth) method uses a stochastic way in the change of neighborhoods. In RVNS, solutions from the pre-selected neighborhoods $N_k(x)$ are chosen at random, without being followed by descend. Rather, the values of these points are compared with that of the incumbent as updating takes place in case of necessary improvement. RVNS is useful in very large instances, for which local search is costly. The steps of this method are:

*Initialization.* Select the set of neighborhood structures $N_k$, $k=1,\dots, k_{max}$, that will be used in the search; find an initial solution $x$.

Repeat the following until the stopping condition is met:

Set $k \leftarrow 1$

Until $k = k_{max}$, repeat the following steps:

*Shaking.* Generate a point $x$' at random from the $k^{th}$ neighborhood of x ($x' \in N_k(x)$);

*Move or not.* If the solution thus obtained $x$' is better than $x$, set $x \leftarrow x'$; otherwise, set $k \leftarrow k + 1$.

*3.2. Variable neighborhood descent*

The Variable Neighborhood Descent (VND henceforth) method is obtained if the change of neighborhoods is performed in a deterministic way. Steps of the VND are:

*Initialization.* Select the set of neighborhood structures $N_k$, $k$=1,…,$k_{max}$, that will be used in the search; find an initial solution $x$.

Repeat the following until no improvement is obtained.

Set $k \leftarrow 1$

Until $k = k_{max}$, repeat the following steps:

Exploration of the neighborhood. Find the best neighbor $x$' of $x$.

*Move or not.* If the solution thus obtained $x$' is better than $x$, set $x \leftarrow x'$; otherwise, set $k \leftarrow k + 1$.

*3.3. Basic variable neighborhood search (BVNS)*

The Basic VNS (BNVS) method combines deterministic and stochastic changes of the neighborhood. Steps of the BNVS are:

*Initialization.* Select the set of neighborhood structures $N_k$, $k$=1,…,$k_{max}$, that will be used in the search; find an initial solution $x$; choose the stopping criterion.

Repeat the following until the stopping condition is met:

Set $k \leftarrow 1$

Until $k = k_{max}$, repeat the following steps:

*Shaking.* Generate a point $x$' at random from the $k^{th}$ neighborhood of $x$ ($x' \in N_k(x)$);

*Local search.* Apply any local search method with $x$' as initial solution; denote with $x$'' the so obtained local optimum;

*Move or not.* If this local optimum is better than the incumbent, move there ($x \leftarrow x''$), and continue the search with $N_1$($k \leftarrow 1$); otherwise, set $k \leftarrow k + 1$.

*3.4. Threshold Accepting (TA)*

In this study, the local search used in the VNS methods is the Threshold Accepting method (TA henceforth), which was originally proposed by Dueck and Scheuer [45]. The VNS-TA strategy combines the diversification approach of exploring distant neighborhoods (VNS) with the intensification given in the improvement phase (TA). The VND and BVNS method for the local search used was the TA and will be called, henceforth, VND-TA and BVNS-TA. The TA algorithm starts with an initial randomly generated working solution, $x$, and an initial high threshold value for accepting solutions. TA changes the solution by a move. The new current solution, $x$', is accepted if it improves the value of the objective function or when the value increment (cost or $CO_2$ emission, depending on the objective function that is being optimized) is smaller than the current threshold value. Again, the current solution is checked against structural constraints and if it is feasible, it is adopted as the new working solution $x$. The stop criterion considered in this algorithm has been set as the total amount of iterations and the initial threshold has been adjusted following the method proposed by Medina [46]. In this problem, the TA strategy was calibrated with chains of 12,000 iterations for cost optimization and 24,000 iterations for $CO_2$ emissions. An initial threshold of 80 € in cost optimization and 160 kg $CO_2$ in environmental optimization are considered.

## 4. Results of the heuristics

The three optimization methodologies used in this study (VND-TA, RVNS and BVNS-TA) were applied to the simply supported HPC beam described above with the span length varying between 10 and 20 m in steps of 1 m. The pre-selected neighborhood structure considered to be $N_k$, $k$ = 1,8,2,9,3,10,4,11,5,12,6,13,7,14. This sequence of neighborhood structure allows exploration with alternating changes in many variables (extending the search space of solutions) with minor modifications. It is worth noting that, since the optimization methods need an initial solution (randomly generated), every test is repeated 1,000 times to study the influence of the initial solution on the results. The optimization problem was coded in Fortran 95 with a Silverfrost Plato 4.3.0 compiler. The running time of one execution of the 1,000 iterations of each algorithm was, on average, 20 minutes, on a PC AMD Phenom II X6 1055T Processor 2.80 GHz.

*4.1. Statistical description results*

This section describes the statistical properties of the sample of solutions obtained with 1,000 runs of the algorithms (RVNS, VND-TA and BVNS-TA) with cost and $CO_2$ emissions optimization. Table 3 and Table 4 show the mean and minimum cost and $CO_2$ emissions results for cost optimization and emissions optimization, respectively.

It can be seen that results of RVNS method have an average standard deviation with regard to the mean value of 25% in the cost optimization and of 29% in $CO_2$ emissions optimization. It may be concluded that this method is very sensitive to the selection of the initial solution. It can also be seen that VND-TA results have a deviation with regard to the mean value of 11% in cost optimization and 14% in $CO_2$ emissions optimization. This method is less sensitive to the selection of the initial solution than the RVNS. Regarding BVNS-TA method, the results have even a lower deviation with respect to the mean of 6% in cost optimization and of 12% in $CO_2$

**Table 3** Statistical description of 1,000 results with RVNS, VND-TA and BVNS-TA methods. Cost optimization

| | | L (m) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| RVNS | Mean | 2,332 | 2,681 | 3,004 | 3,379 | 3,790 | 4,252 | 4,681 | 5,210 | 5,759 | 6,365 | 6,903 |
| | σ | 683 | 732 | 784 | 880 | 960 | 1,038 | 1,142 | 1,241 | 1,366 | 1,455 | 1,520 |
| | Min | 1,151 | 1,309 | 1,492 | 1,694 | 1,933 | 2,308 | 2,494 | 2,746 | 2,942 | 3,479 | 4,032 |
| | Max | 5,849 | 5,560 | 5,945 | 7,485 | 7,646 | 8,644 | 10,729 | 10,823 | 12,039 | 13,283 | 13,005 |
| | C.I* | ±42,34 | ±45,4 | ±48,61 | ±54,53 | ±59,48 | ±64,32 | ±70,77 | ±76,94 | ±84,67 | ±90,17 | ±94,22 |
| | Skewness | 10.8 | 9.3 | 8.2 | 9.6 | 9.9 | 10.2 | 14.0 | 13.0 | 11.9 | 11.3 | 9.6 |
| | Kurtosis | 5.0 | 2.7 | 0.7 | 4.0 | 3.9 | 4.5 | 12.6 | 9.8 | 8.0 | 8.1 | 3.0 |
| VND-TA | Mean | 1,065 | 1,220 | 1,373 | 1,588 | 1,860 | 2,099 | 2,387 | 2,641 | 2,948 | 3,275 | 3,660 |
| | σ | 129 | 166 | 120 | 109 | 165 | 180 | 274 | 298 | 340 | 401 | 462 |
| | Min | 833 | 1,001 | 1,152 | 1,332 | 1,539 | 1,761 | 2,004 | 2,228 | 2,501 | 2,753 | 3,045 |
| | Max | 1,481 | 1,815 | 2,194 | 1,975 | 2,393 | 2,820 | 3,524 | 4,088 | 4,767 | 5,075 | 5,613 |
| | C.I* | ±7,97 | ±10,27 | ±7,43 | ±6,76 | ±10,2 | ±11,16 | ±17 | ±18,46 | ±21,1 | ±24,84 | ±28,66 |
| | Skewness | 5.5 | 15.0 | 19.4 | 5.1 | 8.7 | 12.3 | 16.5 | 19.8 | 23.5 | 19.4 | 15.8 |
| | Kurtosis | -2.1 | 5.3 | 31.0 | -1.8 | 0.3 | 7.4 | 10.8 | 18.4 | 28.0 | 15.0 | 8.4 |
| BVNS-TA | Mean | 927 | 1,100 | 1,282 | 1,484 | 1,713 | 1,965 | 2,215 | 2,512 | 2,822 | 3,109 | 3,433 |
| | σ | 51 | 62 | 69 | 82 | 100 | 114 | 117 | 141 | 163 | 179 | 216 |
| | Min | 833 | 978 | 1,151 | 1,332 | 1,539 | 1,761 | 2,000 | 2,233 | 2,499 | 2,754 | 3,060 |
| | Max | 1,103 | 1,312 | 1,550 | 1,973 | 2,203 | 2,403 | 2,653 | 3,131 | 3,543 | 4,089 | 4,625 |
| | C.I* | ±3,14 | ±3,82 | ±4,31 | ±5,05 | ±6,18 | ±7,06 | ±7,22 | ±8,72 | ±10,1 | ±11,07 | ±13,39 |
| | Skewness | 5.8 | 7.0 | 8.5 | 11.4 | 8.6 | 5.6 | 8.7 | 10.9 | 11.1 | 16.3 | 15.7 |
| | Kurtosis | -1.4 | -0.5 | 3.6 | 11.5 | 3.5 | -2.0 | 0.3 | 6.3 | 7.0 | 19.2 | 18.3 |

(*) Confidence Interval for a 0.05 level of significance

**Table 4** Statistical description of 1,000 results with RVNS, VND-TA and BVNS-TA methods. $CO_2$ emissions optimization

| | | L (m) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| RVNS | Mean | 5,570 | 6,274 | 7,011 | 7,797 | 8,809 | 9,776 | 10,740 | 10,740 | 12,854 | 14,171 | 15,456 |
| | σ | 1,910 | 1,991 | 2,154 | 2,318 | 2,588 | 2,806 | 3,051 | 3,051 | 3,478 | 3,743 | 3,903 |
| | Min | 2,219 | 2,730 | 2,614 | 3,443 | 3,318 | 4,592 | 5,040 | 5,040 | 5,141 | 6,465 | 7,109 |
| | Max | 13,140 | 14,241 | 16,428 | 18,474 | 18,771 | 21,339 | 22,528 | 22,528 | 27,716 | 30,606 | 29,530 |
| | C.I* | ±118.4 | ±123.4 | ±133.5 | ±143.7 | ±160.4 | ±173.9 | ±189.1 | ±189.1 | ±215.5 | ±232.0 | ±241.9 |
| | Skewness | 10.4 | 9.6 | 9.7 | 9.2 | 9.2 | 11.3 | 11.5 | 11.5 | 9.6 | 9.9 | 8.8 |
| | Kurtosis | 2.7 | 1.7 | 3.2 | 3.0 | 2.7 | 6.7 | 7.2 | 7.2 | 3.5 | 4.0 | 1.4 |
| VND-TA | Mean | 2,037 | 2,382 | 2,683 | 3,196 | 3,755 | 4,254 | 4,866 | 5,506 | 6,258 | 6,837 | 7,623 |
| | σ | 320 | 325 | 352 | 417 | 495 | 531 | 624 | 727 | 905 | 971 | 1,115 |
| | Min | 1,344 | 1,594 | 1,930 | 2,239 | 2,606 | 3,003 | 3,436 | 3,788 | 4,297 | 4,914 | 5,488 |
| | Max | 3,696 | 3,737 | 3,918 | 4,606 | 6,194 | 6,785 | 8,138 | 8,977 | 9,908 | 10,601 | 12,073 |
| | C.I* | ±19.9 | ±20.1 | ±21.8 | ±25.9 | ±30.7 | ±32.9 | ±38.7 | ±45.1 | ±56.1 | ±60.2 | ±69.1 |
| | Skewness | 10.1 | 7.6 | 3.8 | 2.7 | 8.9 | 8.4 | 11.8 | 10.0 | 12.1 | 11.4 | 11.6 |
| | Kurtosis | 9.8 | 6.2 | -3.4 | -1.1 | 10.5 | 9.2 | 13.7 | 9.2 | 7.5 | 5.4 | 5.2 |
| BVNS-TA | Mean | 1,858 | 2,195 | 2,611 | 3,048 | 3,530 | 4,071 | 4,595 | 5,238 | 5,865 | 6,568 | 7,342 |
| | σ | 244 | 286 | 328 | 378 | 419 | 509 | 557 | 681 | 756 | 810 | 927 |
| | Min | 1,344 | 1,598 | 1,920 | 2,239 | 2,573 | 2,955 | 3,379 | 3,798 | 4,263 | 4,786 | 5,341 |
| | Max | 2,461 | 2,997 | 3,822 | 4,215 | 4,805 | 5,758 | 6,175 | 6,966 | 8,326 | 9,565 | 11,235 |
| | C.I* | ±15.1 | ±17.7 | ±20.4 | ±23.5 | ±26.0 | ±31.6 | ±34.5 | ±42.2 | ±46.8 | ±50.2 | ±57.5 |
| | Skewness | 3.7 | 5.3 | 4.5 | 5.4 | 4.7 | 6.8 | 5.2 | 5.7 | 6.7 | 9.1 | 8.3 |
| | Kurtosis | -5.2 | -4.5 | -3.2 | -2.8 | -2.5 | -3.0 | -3.9 | -3.3 | -1.8 | 2.4 | 2.4 |

(*) Confidence Interval for a 0.05 level of significance emissions optimization. The main characteristics of the optimal solution obtained with the different algorithms for a span length of 15 m are given in Table 5.

In order to evaluate the viability of using HPC instead of conventional concrete, the authors carried out a process of economic optimization of a beam of the same features as those outlined in paragraph 2, but using conventional reinforced concrete ($f_{ck}$ between 25 and 50 MPa) instead of HPC. This comparison reveals that the use of HPC decreases by more than 15% the cost of the most economical beam. Thus, the increased cost resulting from the use of HPC is offset by its better mechanical performance, proving to be a more efficient material than conventional concrete.

### *4.2. Adequacy of the heuristics*

The adequacy of the three methods is evaluated on the basis of their adjustment to a three-parameter Weibull distribution [47]. This type of fitting has already been applied to cost estimation of concrete structures [48]. It is based on the fact that the results of each computer run of the three heuristics applied in this study (RVNS, VND-TA and BVNS-TA) are extreme values in terms of cost and $CO_2$ obtained after a large number of solutions. It is important to note that even if the beam analyzed in this study has a high but a finite number of possible solutions, we suppose that the population space approximates a continuous space because the number of variables is large and is represented in an almost continuous manner.

Some preliminary tests have been applied to the set of data before the fitting to a three-parameter Weibull distribution. Firstly, Table 6 and Table 7 show that the samples have a positive asymmetric distribution and, in most of the cases, they present a leptokurtic distribution. Secondly, by computing the Kolmogorov-Smirnov and Chi-squared [49] statistics with a 0.05 level of significance, it has been verified that there is no reason to reject the Weibull hypothesis. Each set of 1,000 optimal solutions (in terms of cost and $CO_2$ emissions) found through the three different methods (RVNS, VND-TA and BNVS-TA) has been compared with a three-parameter Weibull distribution. Thirdly, the independence of each set of solutions has been tested by running a Wald-Wolfowitz test. Taking this consideration into account, Tables 6 and 7 show the three-parameter Weibull distribution results, where $\gamma$ is the location parameter, $\eta$ is the scale parameter, $\beta$ is the shape parameter and $\rho$ is the correlation coefficient.

In view of the results, we can conclude that in all cases, the set of solutions fit well using a three-Weibull parameter distribution, as the minimum correlation coefficient obtained is 0.977. The fit to the Weibull distribution gives us, for each optimization problem, the theoretical minimum value to which the method converges. This value is called location parameter ($\gamma$). For this reason, it has been considered that a method is more efficient than another if its fit to a Weibull distribution has a lower value of the location parameter. In view of the results (Table 6 and Table 7), we can note that, if the overall optimization process is considered (cost and $CO_2$ emissions optimization), VND-TA method provides better solutions in 15 of the 22 optimization processes.

## 5. Parametric study

This section describes the parametric study of practical simply supported beams in HPC concrete. The parameter considered is the span length of the beam ($L$, see Fig. 2). The considered values of L amount to a total of eleven lengths varying from 10 to 20 m in steps of 1 m. Eleven beam types were investigated, adding a total of 11,000 computer runs:

**Table 5** Main characteristics of the optimal HPC beams with span length 15 m

| | RVNS | | VND-TA | | BVNS-TA | |
|---|---|---|---|---|---|---|
| Objective | Cost | $CO_2$ | Cost | $CO_2$ | Cost | $CO_2$ |
| Height (m) | 0.75 | 0.90 | 0.75 | 0.95 | 0.75 | 0.85 |
| Width (m) | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| As1 | 3Ø12 | 3Ø10 | 7Ø 6 | 11 Ø 6 | 7Ø 6 | 9Ø6 |
| As2 | 10Ø25 | 8Ø25 | 9Ø25 | 7 Ø 25 | 9Ø25 | 12Ø20 |
| As3 | 9Ø20 | 9Ø 6 | 11Ø10 | 6 Ø 12 | 11Ø10 | 4Ø16 |
| As4 | 5Ø10 | 4Ø 8 | 4Ø 6 | 4Ø6 | 4Ø6 | 4Ø6 |
| As5 | 6Ø25 | 9Ø 6 | 4Ø 6 | 4Ø6 | 4Ø6 | 4Ø6 |
| Mix design | 79 | 105 | 49 | 48 | 48 | 49 |
| $f_{ck}$ (MPa) | 55 | 70 | 80 | 90 | 90 | 90 |
| $f_{yk}$ (MPa) | 400 | 400 | 500 | 500 | 500 | 500 |

where:
As1: Upper reinforcement
As2: Lower continuous reinforcement
As3: Lower central reinforcement
As4: Shear reinforcement in the *L*/3 length near the supports (stirrups per meter)
As5: Shear reinforcement in the *L*/3 central length of the beam (stirrups per meter)

**Table 6** Parameters of the Weibull distribution of the optimal HPC beams. Cost optimization

| | | L (m) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| **RVNS** | Min | 1,151 | 1,309 | 1,492 | 1,694 | 1,933 | 2,308 | 2,494 | 2,746 | 2,942 | 3,479 | 4,032 |
| | $\gamma$ | 1,132 | 1,293 | 1,484 | 1,662 | 1,924 | 2,295 | 2,467 | 2,701 | 2,923 | 3,419 | 4,002 |
| | $\beta$ | 1.838 | 1.999 | 2.032 | 2.096 | 2.076 | 1.999 | 2.203 | 2.303 | 2.340 | 2.257 | 2.042 |
| | $\eta$ | 1,352 | 1,568 | 1,720 | 1,938 | 2,111 | 2,295 | 2,493 | 2,825 | 3,201 | 3,317 | 3,273 |
| | $\rho$ | 0.998 | 0.997 | 0.995 | 0.998 | 0.993 | 0.995 | 0.992 | 0.993 | 0.987 | 0.997 | 0.997 |
| **VND-TA** | Min | 833 | 1,001 | 1,152 | 1,332 | 1,539 | 1,761 | 2,004 | 2,228 | 2,501 | 2,753 | 3,045 |
| | $\gamma$ | 799 | 991 | 1,130 | 1,309 | 1,507 | 1,718 | 1,994 | 2,199 | 2,461 | 2,735 | 3,025 |
| | $\beta$ | 2.153 | 1.434 | 2.424 | 2.772 | 2.292 | 2.373 | 1.516 | 1.712 | 1.724 | 1.540 | 1.373 |
| | $\eta$ | 300.9 | 253.1 | 273.0 | 314.0 | 400.5 | 429.7 | 437.3 | 492.5 | 539.6 | 592.9 | 705.3 |
| | $\rho$ | 0.992 | 0.983 | 0.982 | 0.984 | 0.985 | 0.987 | 0.989 | 0.982 | 0.977 | 0.988 | 0.984 |
| **BVNS-TA** | Min | 833 | 978 | 1,151 | 1,332 | 1,539 | 1,761 | 2,000 | 2,233 | 2,499 | 2,754 | 3,060 |
| | $\gamma$ | 826 | 969 | 1,135 | 1,319 | 1,515 | 1,745 | 1,990 | 2,221 | 2,484 | 2,737 | 3,039 |
| | $\beta$ | 2.066 | 2.315 | 2.310 | 2.211 | 2.143 | 1.983 | 2.028 | 2.302 | 2.298 | 2.473 | 2.111 |
| | $\eta$ | 114.8 | 148.0 | 166.4 | 185.5 | 224.1 | 249.1 | 255.3 | 328.0 | 380.8 | 418.3 | 442.5 |
| | $\rho$ | 0.986 | 0.993 | 0.993 | 0.992 | 0.985 | 0.985 | 0.986 | 0.991 | 0.990 | 0.987 | 0.984 |

Where:
$\gamma$: Location parameter of the Weibull distribution
$\beta$: Shape parameter of the Weibull distribution
$\eta$: Scale parameter of the Weibull distribution
$\rho$: Correlation coefficient to Weibull distribution

**Table 7** Parameters of the Weibull distribution of the optimal HPC beams. $CO_2$ emissions optimization

| | | L (m) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| **RVNS** | Min | 2,219 | 2,730 | 2,614 | 3,443 | 3,318 | 4,592 | 5,040 | 5,040 | 5,141 | 6,465 | 7,109 |
| | $\gamma$ | 2,162 | 2,679 | 2,597 | 3,341 | 3,254 | 4,430 | 4,911 | 4,599 | 5,061 | 6,410 | 7,048 |
| | $\beta$ | 1.887 | 1.893 | 2.244 | 2.035 | 2.392 | 2.078 | 2.087 | 2.534 | 2.531 | 2.295 | 2.399 |
| | $\eta$ | 3,843 | 4,056 | 4,995 | 5,033 | 6,262 | 6,024 | 6,569 | 8,107 | 8,780 | 8,760 | 9,486 |
| | $\rho$ | 0.997 | 0.997 | 0.984 | 0.999 | 0.992 | 0.999 | 0.998 | 0.985 | 0.987 | 0.990 | 0.986 |
| **VND-TA** | Min | 1,344 | 1,594 | 1,930 | 2,239 | 2,606 | 3,003 | 3,436 | 3,788 | 4,297 | 4,914 | 5,488 |
| | $\gamma$ | 1,270 | 1,521 | 1,863 | 2,048 | 2,427 | 2,878 | 3,309 | 3,732 | 4,264 | 4,825 | 5,443 |
| | $\beta$ | 2.678 | 2.984 | 2.452 | 3.010 | 3.061 | 2.919 | 2.875 | 2.791 | 2.572 | 2.349 | 2.159 |
| | $\eta$ | 864 | 965 | 928 | 1,285 | 1,484 | 1,543 | 1,745 | 1,990 | 2,241 | 2,264 | 2,462 |
| | $\rho$ | 0.991 | 0.993 | 0.987 | 0.996 | 0.995 | 0.996 | 0.993 | 0.993 | 0.985 | 0.994 | 0.988 |
| **BVNS-TA** | Min | 1,344 | 1,598 | 1,920 | 2,239 | 2,573 | 2,955 | 3,379 | 3,798 | 4,263 | 4,786 | 5,341 |
| | $\gamma$ | 1,283 | 1,532 | 1,824 | 2,137 | 2,487 | 2,935 | 3,313 | 3,684 | 4,134 | 4,700 | 5,231 |
| | $\beta$ | 2.501 | 2.502 | 2.556 | 2.590 | 2.655 | 2.380 | 2.461 | 2.430 | 2.490 | 2.480 | 2.448 |
| | $\eta$ | 649 | 749 | 1,824 | 1,028 | 1,178 | 1,289 | 1,450 | 1,759 | 1,954 | 2,167 | 2,390 |
| | $\rho$ | 0.987 | 0.982 | 0.987 | 0.986 | 0.988 | 0.980 | 0.987 | 0.986 | 0.987 | 0.980 | 0.988 |

Where:
γ: Location parameter of the Weibull distribution
β: Shape parameter of the Weibull distribution
η: Scale parameter of the Weibull distribution
ρ: Correlation coefficient to Weibull distribution

each computer test is performed 1,000 times in order to obtain statistics of the results. As provided in Section 4.2, the VND-TA method is the most efficient algorithm and thus is used in the parametric study.

Fig. 3 and Fig. 4 show the minimum cost and $CO_2$ variation of the 11 HPC reinforced beams investigated. The results obtained have a good parabolic variation in terms of the span length: Cost (€) = $7.22*L^2 + 5.10*L + 59.68$ with a regression coefficient of $R^2 = 0.9998$ and $CO_2$ (kg) = $17.15*L^2 - 105.24*L + 696.71$ with a regression coefficient of $R^2 = 0.9992$.

It is also necessary to determine whether the characteristics of the cost-optimized beams and the emission-optimized beams are similar. Thus, Table 8 shows the characteristics of emission-optimized beams and the ratio of the characteristics between $CO_2$ and cost-optimized beams. In all cases and in both optimization processes, optimized beams have the minimum width, which had been set at 0.30 m for constructability constraints. In addition, there is a trend in the use of concretes of high strength (between 70 and 90 MPa). However, the concrete with the greatest compressive strength available in the optimization process (100 MPa) is not considered in any of the optimal solutions and therefore, it can be concluded that its greatest strength is not worth in terms of cost and $CO_2$ emissions.

The steel grade used in optimal beams is, in all cases, the greatest yield strength (500 MPa). Therefore, the greater mechanical resistance in steel has similar costs and $CO_2$ emissions. Even if the steel grade of both optimal processes is the same, the total steel consumption of optimal beams changes depending on the objective function.

Fig. 5 and Fig. 6 show the relationship between the steel

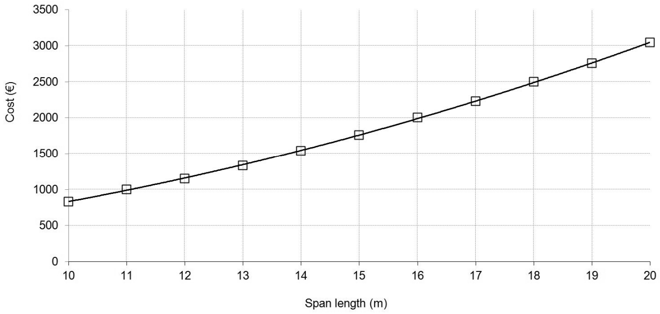


**Fig. 3** Variation of minimum cost for the 11 beams with VND-TA method

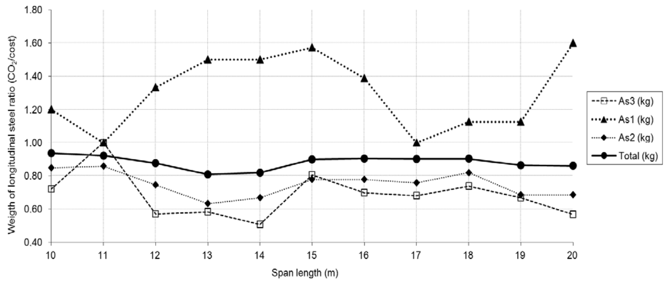


**Fig. 5** Weight of longitudinal and total steel ratio ($CO_2$/cost)

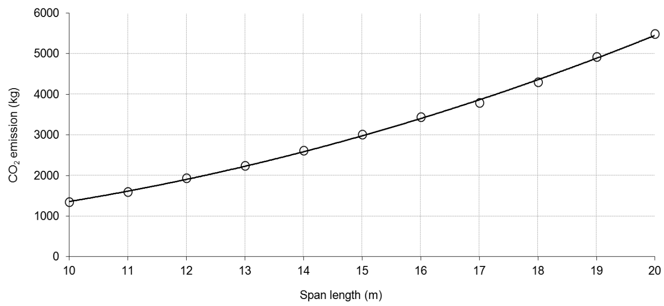


**Fig. 4** Variation of minimum $CO_2$ emission for the beams with VND-TA method

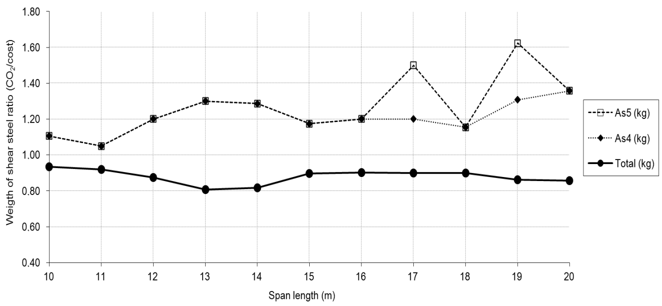


**Fig. 6** Weight of shear and total steel ratio ($CO_2$/cost)

**Table 8** $CO_2$ emissions and cost-optimized beams characteristics

| | | L (m) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| Height (m) | a | 0.65 | 0.65 | 0.80 | 0.90 | 0.95 | 0.95 | 1.10 | 1.10 | 1.10 | 1.30 | 1.50 |
| | b | 1.18 | 1.08 | 1.33 | 1.50 | 1.46 | 1.27 | 1.29 | 1.29 | 1.22 | 1.44 | 1.50 |
| Width (m) | a | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| | b | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| As1 (kg) | a | 14.55 | 15.88 | 22.95 | 27.81 | 29.81 | 38.88 | 41.73 | 39.78 | 42.00 | 53.07 | 74.30 |
| | b | 1.20 | 1.00 | 1.33 | 1.50 | 1.50 | 1.57 | 1.39 | 1.00 | 1.13 | 1.13 | 1.60 |
| As2 (kg) | a | 194.7 | 250.6 | 271.1 | 291.6 | 362.2 | 449.5 | 476.5 | 544.8 | 682.0 | 654.3 | 686.4 |
| | b | 0.85 | 0.86 | 0.75 | 0.63 | 0.67 | 0.78 | 0.78 | 0.76 | 0.82 | 0.68 | 0.69 |
| As3 (kg) | a | 8.0 | 12.8 | 11.0 | 16.2 | 17.3 | 23.2 | 21.3 | 29.3 | 30.8 | 38.0 | 33.9 |
| | b | 0.72 | 1.00 | 0.57 | 0.58 | 0.51 | 0.80 | 0.70 | 0.68 | 0.74 | 0.67 | 0.57 |
| As4 (kg) | a | 62.9 | 67.1 | 83.9 | 98.6 | 107.8 | 115.9 | 137.8 | 143.8 | 152.8 | 183.4 | 212.6 |
| | b | 1.11 | 1.05 | 1.20 | 1.30 | 1.29 | 1.17 | 1.20 | 1.20 | 1.15 | 1.31 | 1.36 |
| As5 (kg) | a | 62.9 | 67.1 | 83.9 | 98.6 | 107.8 | 115.9 | 137.8 | 179.8 | 152.8 | 227.5 | 212.6 |
| | b | 1.11 | 1.05 | 1.20 | 1.30 | 1.29 | 1.17 | 1.20 | 1.50 | 1.15 | 1.62 | 1.36 |
| $f_{ck}$ (MPa) | a | 90 | 90 | 90 | 90 | 90 | 90 | 80 | 90 | 90 | 90 | 70 |
| | b | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.89 | 1.00 | 1.00 | 1.00 | 0.78 |
| $f_{yk}$ (MPa) | a | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 |
| | b | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

(a) $CO_2$ emission-optimized beams characteristics
(b) Ratio between $CO_2$ and cost-optimized beams characteristics
As1: Upper reinforcement
As2: Lower continuous reinforcement
As3: Lower central reinforcement
As4: Shear reinforcement in the *L*/3 length near the supports
As5: Shear reinforcement in the *L*/3 central length of the beam

weight ratio between the ecological beams and the economic ones. In view of the results, it can be noted that, generally, ecological beams present a higher amount of steel in the upper and shear reinforcement. Nevertheless, regarding the total weight of steel consumption, economic beams require on average a 12% less of steel.

Regarding concrete consumption ratio in Fig. 7, the $CO_2$ emission-optimized beams needed, on average, 33% more concrete than cost-optimized ones. Table 9 shows the mix design obtained in both optimization processes. It is important to note that in 90% of the cases, the mix design of the optimal beams in terms of cost and $CO_2$ emissions is the same. This mix design is characterized by a low relation water/cement of 0.26, no silica fume addition and approximately 5 kg/m3 of superplasticizer.

## 6. Conclusions

In the light of the results obtained in this study, the following conclusions may be derived:

- The proposed VND-TA algorithm is an efficient procedure for the optimum design in terms of cost and $CO_2$ emissions of HPC beams. BVNS-TA method provides less effective solutions and the RVNS method is very sensitive to the selection of the initial solution and therefore, it is less suitable for this heuristic optimization.
- The use of HPC concrete in the optimization process has led to a reduction of more than 15% of the cost of the beams. This represents an important economic saving from the use of more efficient materials, and therefore, HPC can be considered a more sustainable material than the conventional concrete.
- The set of optimal solutions found using the different optimization methods (RVNS, VND-TA and BVNS-TA) for the cost and $CO_2$ emission optimization are samples of independent values that the three-parameter Weibull distribution fits well.
- From the parametric analysis, it can be concluded that economic and ecological beams show a good parabolic correlation with the span length.
- The optimized beams always use steel with the greatest yield strength (500 MPa) and high levels of concrete grades (80 or 90 MPa).
- Optimal beams in terms of cost and $CO_2$ emissions have a mix design with low relation water/cement, no silica fume and an addition of superplastizicer lower than the average value used in HPC mix design (4.8 kg/m$^3$ instead of 10.5 kg/m$^3$).

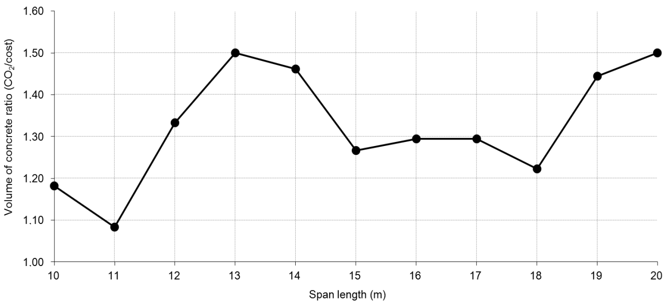


**Fig. 7** Volume of concrete ratio ($CO_2$/cost)

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**Table 9** Mix design characteristics of the optimal HPC beams

| | | L (m) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| Mix design | a | 49 | 49 | 49 | 14 | 49 | 49 | 50 | 49 | 49 | 49 | 115 |
| | b | 49 | 49 | 49 | 49 | 49 | 49 | 49 | 49 | 49 | 49 | 49 |
| $f_{ck}$ (MPa) | a | 90 | 90 | 90 | 90 | 90 | 90 | 80 | 90 | 90 | 90 | 70 |
| | b | 90 | 90 | 90 | 90 | 90 | 90 | 90 | 90 | 90 | 90 | 90 |
| Cement (kg/m$^3$) | a | 382 | 382 | 382 | 429 | 382 | 382 | 382 | 382 | 382 | 382 | 400 |
| | b | 382 | 382 | 382 | 382 | 382 | 382 | 382 | 382 | 382 | 382 | 382 |
| Water (kg/m$^3$) | a | 99 | 99 | 99 | 150 | 99 | 99 | 99 | 99 | 99 | 99 | 140 |
| | b | 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 |
| Coarse aggregate (kg/m$^3$) | a | 539 | 539 | 539 | 1089 | 539 | 539 | 539 | 539 | 539 | 539 | 1196 |
| | b | 539 | 539 | 539 | 539 | 539 | 539 | 539 | 539 | 539 | 539 | 539 |
| Fine aggregate (kg/m$^3$) | a | 359 | 359 | 359 | 741 | 359 | 359 | 359 | 359 | 359 | 359 | 744 |
| | b | 359 | 359 | 359 | 359 | 359 | 359 | 359 | 359 | 359 | 359 | 359 |
| Superplasticizer (kg/m$^3$) | a | 4.8 | 4.8 | 4.8 | 7.3 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 5.6 |
| | b | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 |
| Silica fume (kg/m$^3$) | a | 0 | 0 | 0 | 45 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | b | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(a) $CO_2$ emission-optimized beams characteristics

(b) Cost-optimized beams characteristics

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