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Technical Note

A simplified approach to estimate the resultant force for the equilibrium of unstable slopes

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Abstract

This paper presents a simplified approach to estimate the resultant force, which should be provided by a retention system, for the equilibrium of unstable slopes. The results were obtained with a developed algorithm, based on limit equilibrium analyses, that assumes a two-part wedge failure mechanism. Design charts to obtain equivalent earth pressure coefficients are presented. Based on the results achieved with the developed computer code, an approximate equation to estimate the equivalent earth pressure coefficients is proposed. Given the slope angle, the backslope, the design friction angle, the height of the slope and the unit weight of the backfill, one can determine the resultant force for slope equilibrium. This simplified approach intends to provide an extension of the Coulomb earth pressure theory to the stability analyses of steep slopes and to broaden the available design charts for steep reinforced slopes with non-horizontal backslopes.

Keywords: *Earth pressure coefficients, Reinforced soil, Limit equilibrium methods, Unstable slopes.*

1. Introduction

The construction of a slope steeper than the naturally stable angle requires additional stabilising forces. These forces can the provided by a heavy facing system (such as gabions or large rocks) or by horizontal reinforcements placed in the embankment. The results herein presented can be applied to the internal design of geosynthetic reinforced steep slopes and, also, to the design of a stable facing system.

In the last decades some methods have been proposed for the internal design of geosynthetic reinforced soil structures. These methods can be grouped into three different approaches. The first approach, usually limited to reinforced soil slopes, is an extension of the classical limit equilibrium slope stability methods (methods of slices) with the inclusion of the reinforcement forces ([1], [2], [3]). The second approach is based on considerations of limit equilibrium, such as two-part wedge or logarithmic spiral analyses ([4], [5], [6], [7]). The third is a kinematic approach of limit analysis and can be performed considering a continuum medium, through the soil and reinforcement homogenization, or two separated structural components – soil and reinforcement components ([8], [9]). This paper follows the second approach.

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The horizontal resultant force due to lateral earth pressures that should be supported by the reinforcement layers or by the facing system to ensure the structure equilibrium is usually determined by limit equilibrium analyses. The failure surface associated with the maximum value of this horizontal force defines the critical surface.

The resultant force to be taken by the reinforcement layers is equal to the resultant of the assumed earth pressure distribution, considered as a function of the earth pressure coefficient. This paper presents updated results from a developed computer program, based on limit equilibrium analysis, able to calculate the resultant force for structure equilibrium under static and seismic loading [10]. Based on the results derived from the developed computer code, an approximated equation for earth pressure coefficients estimation is proposed. Simplified equations to estimate earth pressure coefficients for static and seismic design of geosynthetic reinforced structures were previously proposed by [6] but these equations are limited to horizontal backslopes.

The design of geosynthetic reinforced steep slopes is often based on design charts such those proposed by [4] and [5]. These design charts are, usually, limited to horizontal backslopes. Ghanbari and Ahmadabad [11] proposed a formulation to estimate active earth pressures for inclined walls, considering a plane failure surface, but the presented results are also limited to horizontal backslopes. The purpose of this paper is to broaden the available design charts for steep reinforced slopes with non-horizontal backslopes and, simultaneously, to simplify the evaluation of the earth pressure coefficient through the proposal of an approximate equation.

2. Limit Equilibrium Approach

The two-part wedge failure mechanism is, as mentioned before, one of the limit equilibrium approaches suitable for the evaluation of the resultant force for unstable slopes equilibrium. In the two-part wedge mechanism, the potential sliding soil mass is divided in two blocks (Fig. 1). The required force for equilibrium, P_a, is represented in Fig. 1 at the face of the structure.

Usually, the horizontal component of the earth pressure coefficient is the parameter used in the analysis of facing stability or reactive force in reinforcements, hence the required force for equilibrium force, P_a, was considered horizontal.

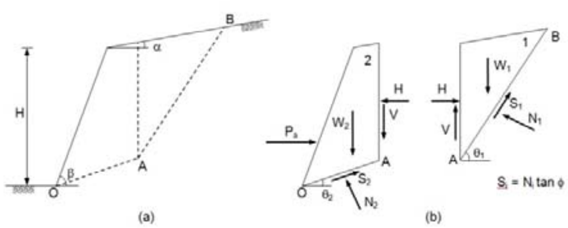


Fig. 1 Potential failure surface: (a) geometric characteristics of the slope; (b) two-part wedge failure mechanism.

A vertical inter-wedge potential failure surface was assumed. The inter-wedge force was considered by its horizontal and vertical components, H and V, related by the following equation:

$$V = \lambda H \tan\phi \tag{1}$$

where λ is the inter-wedge mobilized shear stress ratio and ϕ is the soil internal friction angle.

The effect of the direction of the inter-wedge force, expressed by λ, on the earth pressure coefficient, K_{req}, was analysed by [10]. This author found that a horizontal inter-wedge force (λ = 0) is very conservative when compared with log spiral failure mechanism. Considering this conclusion and taking into account that it was assumed the fully mobilization of the soil shear strength along the failure surfaces OA and AB (Fig. 1), the present study considers λ = 1 (soil shear strength fully mobilized in the inter-wedge vertical surface).

Obviously, the geometry of the blocks that leads to the maximum value of the resultant force for equilibrium, i.e., the geometry shown in Fig. 1, is not known. Thus a computer code was developed to find the most critical failure surface, i.e., the one that leads to the maximum horizontal force, P_a.

To find the critical failure surface the software creates a square mesh of points, with lateral side equal to the height of the structure and spacing between points equal to 1% of the mesh side. For each of these points (represented by point A in Fig. 1), several potential failure surfaces are analyzed, ranging the angle θ_1 (see Fig. 1) from θ_2 to 90º with increments of 0.1º.

The two-part wedge failure mechanism degenerates to a single wedge with a plane failure surface when this mechanism is more adverse (when the slope face is near vertical).

The equilibrium equation, on horizontal direction, of the forces acting on wedge 1 (Fig. 1), taking into consideration the relation between the horizontal and vertical components of the inter-wedge force stated by Equation (1), provides the horizontal component of the inter-wedge force, H:

$$H = \frac{W_1}{\lambda \tan\phi + \dfrac{\tan\phi \sin\theta_1 + \cos\theta_1}{\sin\theta_1 - \tan\phi\cos\theta_1}} \tag{2}$$

Known the horizontal component of the inter-wedge force, H, the required force for equilibrium, P_a, can be calculated, for each potential failure surface, by the equilibrium, on horizontal direction, of the forces acting on wedge 2, with the equation:

$$P_a = H - \frac{\tan\phi\cos\theta_2 - \sin\theta_2}{\tan\phi\sin\theta_2 + \cos\theta_2}\left(W_2 + V\right) \tag{3}$$

The bilinear failure surface to which corresponds the maximum value of P_a, is considered the critical failure surface.

The need of reinforcement or the need of the facing system to reach the slope equilibrium may be represented by an earth pressure distribution. By the similarity between critical potential failure surfaces, Terzaghi [12] demonstrated that the earth pressure distribution at the back of a wall increases, like a hydrostatic pressure, in simple proportion to depth. Based on this, Jewell [5] stated that the magnitude of the maximum reinforcement force required for equilibrium increases with the square of the slope height. Therefore, assuming that these earth pressures (or required tensile strengths for equilibrium) increase linearly with depth, the required force for equilibrium, P_a, could be expressed by:

$$P_a = \frac{1}{2} K_{req} \gamma H^2 \tag{4}$$

where K_{req} is an equivalent earth pressure coefficient, γ is the soil unit weight and H is the slope height. The results will be expressed, in sequence, by the equivalent earth pressure coefficient. This earth pressure coefficient can be used to determine the required tensile strength of the reinforcement or to design the slope facing system.

3. Results and Discussion

The results presented in this paper regards a purely frictional backfill material, with internal friction angle in the range of 20º-45º, slope angles between 40º and 90º, backslope angles in the range 0º-18.4º, zero pore water pressure ($r_u = 0$) and a competent foundation.

Fig. 2 summarizes the earth pressure coefficients as a function of the slope angle, β, and design friction angle, ϕ_d, for different backslopes: from horizontal (Fig. 2a) to a backslope angle of 18.4º (1V:3H). Note that the backslope angle was expressed in Fig. 2 by the ratio of the vertical to the horizontal distances. It should be, also, mentioned that the results presented in Fig. 2a are equivalent to those reported by [5] and [13] for zero pore water pressure ($r_u = 0$).

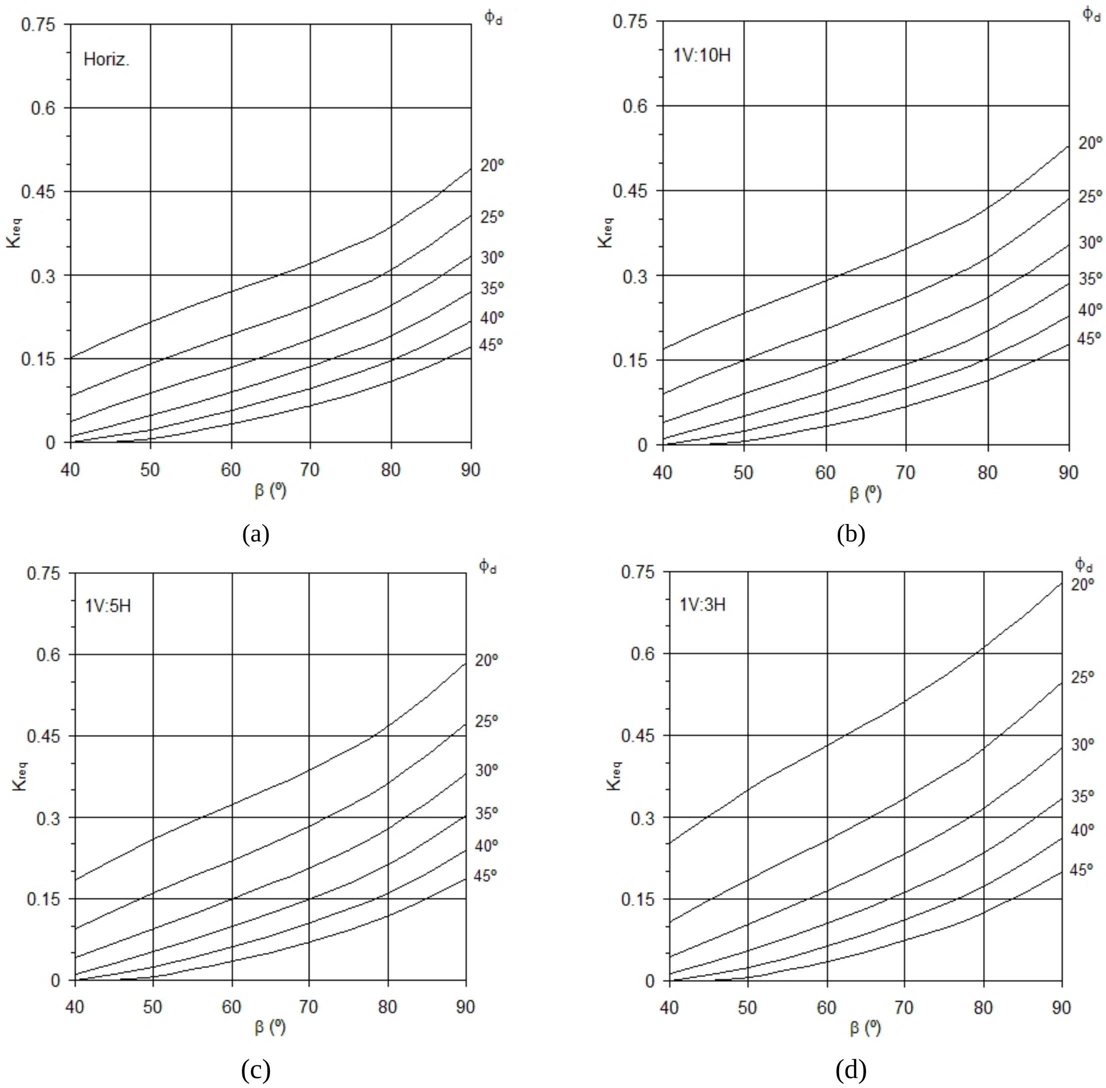


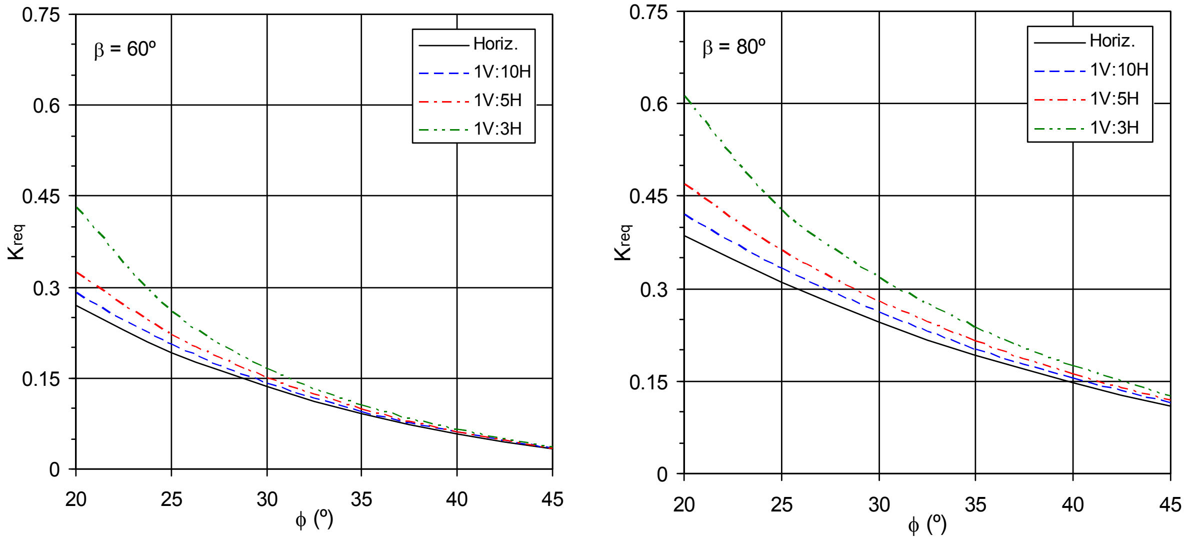
Fig. 2 Design charts for different slope and soil friction angles: (a) horizontal backslope; (b) backslope angle, α =atan (0.1); (c) α= atan (0.2); (d) α = atan (0.33).

The effect of the backslope on the required force for equilibrium seems to increase with slope angle, β. Assuming a design friction angle, ϕ_d, equal to 30º, the earth pressure coefficient, K_{req}, increases approximately 23% and 29% when the backslope ranges from horizontal to an angle of 18.4º, for β = 60º and β = 80º, respectively. These variations decrease, respectively, to 11% and 18% for a design friction angle of 40º.

The influence of the backslope angle on the required force for equilibrium, as a function of the soil friction angle, is illustrated in Fig. 3 for β = 60º and β = 80º. This figure corroborates that the effect of the backslope on the required force for equilibrium decreases with the soil friction angle. The analysis of Fig. 3 also shows that the effect of the backslope is more significant for steeper slopes.

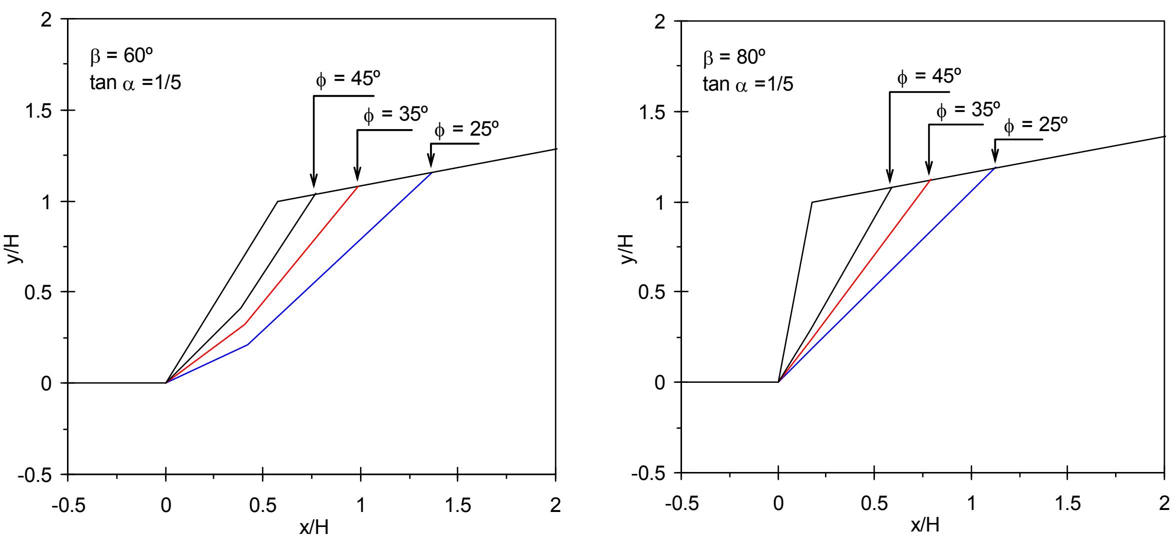
Fig. 4 illustrates the effect of soil friction angle on critical failure surfaces for embankments with slope angles of 60º (Fig. 4a) or 80º (Fig. 4b) and a backslope angle of 11.3º (1V:5H). As expected the volume of soil potentially in failure decreases with the soil friction angle. While the potential failure surfaces for β = 60º are bilinear (Fig. 4a), for the steeper slope the critical failure surfaces become planar (Fig. 4b) and coincident with those admitted in Coulomb's earth pressure theory [14].

Fig. 4a evidences that the Coulomb's earth pressure theory is not suitable to estimate the potential failure surface for an embankment with slope angle of 60º.



(a) (b)

Fig. 3 Effect of the backslope angle on the required force for equilibrium: (a) slope angle, β = 60º; (b) slope angle, β = 80º.



(a) (b)

Fig. 4 Effect of soil friction angle on critical failure surfaces for backslope angle, α = atan (0.2): (a) slope angle, β = 60º; (b) slope angle, β = 80º.

The potential failure surface for a two-part wedge failure mechanism to which corresponds the maximum value of the horizontal force for slope equilibrium (critical failure surface) and the Coulomb failure surface, for β = 60º and soil friction angle of 35º are illustrated in Fig. 5. The potential failure surface derived from the equation proposed by [11] (for the angle of the failure wedge) is also represented in Fig. 5. For static conditions, the study presented by [11] gives coincident results (potential failure surfaces and earth pressure coefficients) with those achieved by the Coulomb's earth pressure theory [14].

Fig. 5 includes the comparison between the earth pressure coefficient obtained by the developed code and the value estimated by the Coulomb's theory. As it will be demonstrated in Section 4, when the slope angle, β, deviates from 90º, Coulomb theory should not be used to estimate the value of K_{req}, since it does not correspond to the maximum value of the horizontal force to reach the slope equilibrium.

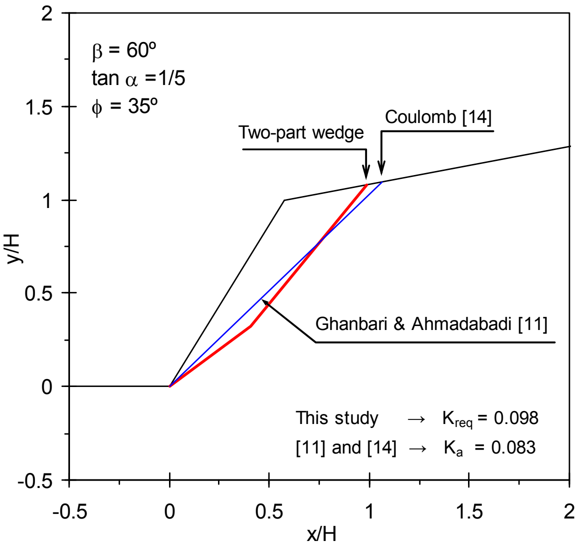


Fig. 5 Comparison of the critical failure surface achieved by the two-part wedge mechanism with those admitted by other authors, β = 60º ; α = atan (0.2).

4. Simplified Approach

In the particular case of geosynthetic reinforced structures, the design is usually performed using design charts ([4], [13]). To avoid the use of design charts and the need of extrapolation for values not included on them, and thus, to simplify the evaluation of the earth pressure coefficient, an approximate equation suitable for non-horizontal backslopes is proposed. Vieira et al. [6] presented a similar approach, limited to horizontal backslopes, considering logarithmic spiral potential failure surfaces.

Under the Coulomb's earth pressure theory [14], the active earth pressure coefficient for a structure with slope angle, β, backslope angle, α, backfill internal friction angle, ϕ, and friction angle between the soil and the wall, δ, (Fig. 6) may be calculated by the equation:

$$K_a^{Coulomb} = \frac{\sin^2(\beta-\phi)}{\sin^2\beta\,\sin(\beta+\delta)\left[1+\sqrt{\dfrac{\sin(\phi-\alpha)\sin(\phi+\delta)}{\sin(\beta-\alpha)\sin(\beta+\delta)}}\right]^2} \quad (5)$$

The reinforcement layers are usually placed horizontally, then they should support the horizontal earth thrust (or horizontal force for slope equilibrium). Assuming the angle δ (Fig. 6) equal to (90º-β), Equation (5) may be rewritten as:

$$K_{req}^{Coulomb} = \left[\frac{\sin(\beta-\phi)}{\sin\beta\left(1+\sqrt{\dfrac{\sin(\phi-\alpha)\cos(\beta-\phi)}{\sin(\beta-\alpha)}}\right)}\right]^2 \quad (6)$$

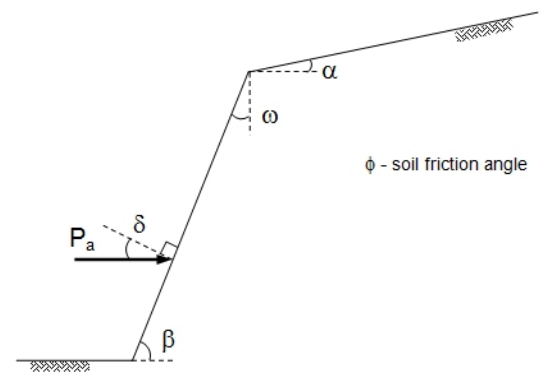


Fig. 6 Definition of the angles in the earth pressure coefficient equations (adapted from [6]).

As mentioned before, when the slope angle, β, is not close to 90º, Equation (6) is not suitable to estimate the equivalent earth coefficient, K_{req}, seeing that the values estimated by Equation (6) do not correspond to the maximum values of the horizontal earth thrust, P_a. Applying a correction factor to the last equation, depending on the slope angle, β, on the backslope angle, α, and on the backfill internal friction angle, ϕ, it is possible to estimate, by the following equation, the earth pressure coefficient, K_{req}:

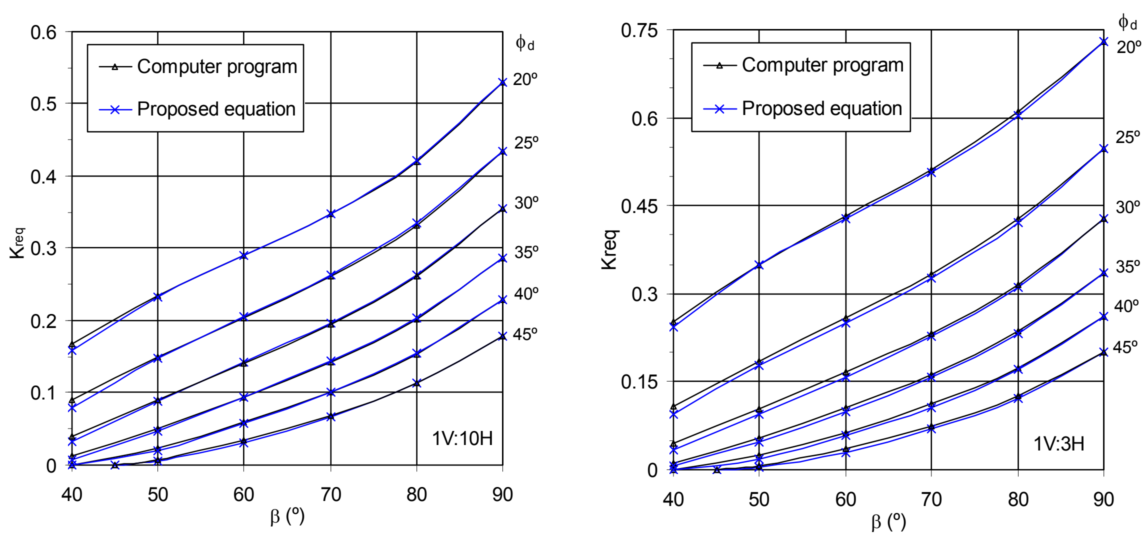
$$K_{req}^{aprox} = \left[\frac{\sin(\beta-\phi)}{\sin\beta\left(1+\sqrt{\dfrac{\sin(\phi-\alpha)\cos(\beta-\phi)}{\sin(\beta-\alpha)}}\right)}\right]^2 \times\left[1+\cos\beta\cos(\beta-\phi)\cos(\beta-\alpha)\right] \quad (7)$$

Table 1 compares the values of K_{req} obtained by the developed computer program with those estimated with Equation (7) and also with the values calculated by the Coulomb earth pressure theory. When the structure face deviates from the vertical, the value obtained with Coulomb equation does not correspond to the maximum value of the earth thrust, P_a. Indeed, Table 1 shows that when $\beta < 80º$, Coulomb earth pressure coefficients are smaller than those corresponding to the maximum value of the horizontal earth thrust.

The differences between the earth pressure coefficients obtained by the developed computer code and those estimated by Equation (7) are also illustrated in Fig. 7 for backslope angles of 5.7º (1V:10H) and 18.4º (1V:3H). From the analysis of Fig. 7 and Table 1, it may be concluded that the values obtained by Equation (7) are very close to those achieved with the computer code.

Table 1 Comparison of K_{req} values obtained by Equation (7) with those achieved with the developed software and Coulomb earth pressure theory, for backslope angle α = atan (0.2)

K_{req}	β (º)	ϕ (º)					
		20	25	30	35	40	45
Coulomb theory	50	0.201	0.121	0.069	0.036	0.015	0.003
	60	0.284	0.193	0.129	0.083	0.050	0.026
	70	0.372	0.272	0.198	0.142	0.098	0.065
	80	0.469	0.362	0.280	0.214	0.161	0.118
	90	0.584	0.472	0.380	0.304	0.241	0.188
Computer program	50	0.259	0.160	0.095	0.052	0.024	0.007
	60	0.323	0.221	0.150	0.098	0.061	0.034
	70	0.387	0.284	0.208	0.150	0.105	0.071
	80	0.469	0.362	0.280	0.214	0.161	0.118
	90	0.584	0.472	0.380	0.304	0.241	0.188
Proposed equation	50	0.254	0.155	0.090	0.046	0.019	0.004
	60	0.319	0.218	0.147	0.095	0.057	0.030
	70	0.384	0.282	0.206	0.148	0.103	0.068
	80	0.468	0.362	0.279	0.213	0.160	0.117
	90	0.584	0.472	0.380	0.304	0.241	0.188



(a) (b)

Fig. 7 Comparison of Kreq values obtained by the developed computer program and by Equation (7) for: (a) backslope angle, α = atan (0.1); (b) backslope angle, α = atan (1/3).

The comparison of the earth pressure coefficients obtained by the developed algorithm with values published in the literature was presented by [6] for horizontal backslopes. Published results that consider non-horizontal backslopes are very scarce. So, it has been decided to compare earth pressure coefficients estimated by Equation (7) with published results based on limit analyses [15, 16] and horizontal backslopes (α = 0).

Table 2 presents a comparative summary for structures with slope angle, β, equal to 45º and 60º and distinct soil internal friction angle, ϕ. The values imputed to [15] and [16] were read from the published charts, so they appear only with two decimal digits. Earth pressure coefficients estimated by Equation (7) for the steeper slope, β = 60º, are coincident to those published by [15] and [16]. For β = 45º the values estimated by Equation (7) tend to be slightly lower than those reported by [15] and [16]. However, should not be despised the difficulty of reading in charts so lower values.

Table 2 Comparison of earth pressure coefficients estimated by Equation (7) with values published in the literature (α = 0).

β (º)	ϕ (º)	Equation (7)	Nouri et al. [15]*	Michalowski [16]*
45	25	0.112	0.13	0.12
	30	0.060	0.08	0.07
	35	0.025	0.04	0.04
	40	0.006	0.01	0.01
60	25	0.198	0.20	0.20
	30	0.139	0.14	0.14
	35	0.093	0.09	0.10
	40	0.057	0.06	0.06
	45	0.031	0.03	0.03

*Values read in charts

Equation (7) is a very good and simple approach to estimate the earth pressure coefficient, K_{req}, for the design of geosynthetic reinforced soil structures or to quantify the resultant force for the equilibrium of unstable slopes. Given the slope angle, the backslope, the design friction angle, the height of the slope and the unit weight of the backfill, it is possible to quickly determine, by Equations (7) and (4), the resultant force for the slope equilibrium.

It should be noted that the proposed equation is limited to purely frictional backfill material, null pore water pressures and static loading conditions.

5. Conclusions

This paper presented a limit equilibrium approach which uses a two-part wedge failure mechanism to achieve the horizontal resultant force needed to the equilibrium of an unstable slope. This force can be supported by the facing system (such as gabions or large rocks) or geosynthetic reinforcement layers.

It was confirmed that the use of Coulomb earth pressure theory to evaluate the resultant force is only accurate for nearly vertical slopes ($\beta > 80^o$). For flatter slopes, Coulomb earth pressure coefficients is unconservative and should not be used to evaluate the resultant force needed to the structure stability.

Design charts and an approximate equation were presented. Given the slope angle, the backslope and the internal friction angle of the backfill material, it is possible to obtain an earth pressure coefficient to calculate the resultant force needed to provide a stable slope. Compared with design charts published by other authors, the results presented in this paper include the effect of the backslope. On the other hand, the proposed equation is easily introduced into a scientific calculator and therefore simplifies the evaluation of the earth pressure coefficient.

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References

[1] Wright S.G, Duncan, J.M. Limit equilibrium stability analyses for reinforced slopes, Transportation Research Record, 1991, Vol. 1330, pp. 40-46.

[2] FHWA. Design and construction of mechanically stabilized earth walls and reinforced soil slopes, Geotechnical Engineering Circular, 2010, No. 11, in FHWA-NHI-10-024, V. Elias, et al, Editors: Washington D.C.

[3] Ghanbari A, Ahmadabadi M. New analytical procedure for seismic analysis of reinforced retaining wall with cohesive-frictional backfill, Geosynthetics International, 2010, No. 6, Vol. 17, pp. 364-379.

[4] Schmertmann G.R, Chouery-Curtis V.E, Johnson R.D, Bonaparte R. Design charts for geogrid-reinforced soil slopes, Proceedings of Geosynthetics' 87, New Orleans, 1987, pp. 108-120.

[5] Jewell R.A. Revised design charts for steep reinforced slopes, Proc. Symp. Reinforced Embankments: Theory and Practice, Thomas Telford, p. 65.

[6] Vieira C.S, Lopes M.L, Caldeira L.M. Earth pressure coefficients for design of geosynthetic reinforced soil structures, Geotextiles and Geomembranes, 2011, No.5, Vol. 29, pp. 491-501.

[7] Basha B, Basudhar P. Pseudo static seismic stability analysis of reinforced soil structures, Geotechnical and Geological Engineering, 2010, No.6, Vol. 28, pp. 745-762.

[8] Michalowski R.L. Stability of uniformly reinforced slopes. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 1997, No. 6, Vol. 123, pp. 546-556.

[9] Ausilio E, Conte E, Dente G. Seismic stability analysis of reinforced slopes, Soil Dynamics and Earthquake Engineering, 2000, Vol. 19, pp. 159-172.

[10] Vieira C.S. Geosynthetic reinforced soil retaining walls and slopes, Seismic behaviour and design methodologies, PhD Thesis, Civil Engineering Department, University of Porto, 2008, 575 p.

[11] Ghanbari A, Ahmadabadi M. Active earth pressure on inclined retaining walls in static and pseudo-static conditions, International Journal of Civil Engineering, 2010, No. 2, Vol. 8, pp. 159-173.

[12] Terzaghi K. Theoretical soil mechanics, John Wiley and Sons, New York, 1943, 510 p.

[13] Jewell R.A. Soil Reinforcement with Geotextiles, London, Special publication 123 ciria, Thomas Telford, 1996, 332 p.

[14] Coulomb C.A. Essai sur une application des règles des maximis et minimis à quelques problèmes de statique relatifs à l' architecture, in Memoires de l' Academie Royale pres Divers Savants, Paris, 1776, Vol.7.

[15] Nouri H, Fakher A, Jones C.J.F.P. Evaluating the effects of the magnitude and amplification of pseudo-static acceleration on reinforced soil slopes and walls using the limit equilibrium Horizontal Slices Method, Geotextiles and Geomembranes, 2008, No. 3, Vol. 26, pp. 263-278.

[16] Michalowski R.L. Soil reinforcement for seismic design of geotechnical structures, Computers and Geotechnics, 1998, Vol. 23, pp. 1-17.