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Transpotation

SA-IP method for congestion pricing based on level of service in urban network under fuzzy conditions

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Abstract

This paper proposes a new hybrid method namely SA-IP including simulated annealing and interior point algorithms to find the optimal congestion prices based on level of service (LOS) in order to maximize the mobility in urban network. By considering six fuzzy LOS for flows, the congestion prices of links can be derived by a bi-level fuzzy programming problem. The objective function of the upper level problem is to minimize the difference between current LOS and desired LOS of links. In this level, to find optimal prices a simulated annealing algorithm is used. The lower level problem is a fuzzy flow estimator model with fuzzy link costs. Applying a famous defuzzification function, a real-valued multi-commodity flow problem can be obtained. Then a polynomial time interior point algorithm is proposed to find the optimal solution regarding to the estimated flows. In pricing process, by imposing cost on some links with LOS F or E, users incline to use other links with better LOS and less cost. During the iterations of SA algorithm, the LOS of a lot of links gradually closes to their desired values and so the algorithm decreases the number of links with LOS worse than desirable LOS. Sioux Falls network is considered to illustrate the performance of SA-IP method on congestion pricing based on different LOS. In this pilot, after congestion pricing, the number of links with LOS D, E and F are reduced and LOS of a great number of links becomes C. Also the value of objective function improves 65.97% after toll pricing process. It is shown that the optimal congestion price for considerable network is 5 dollar and by imposing higher toll, the objective function will be worse.

Keywords: *Congestion pricing, Level of service, Meta-heuristic, Fuzzy travel time, Multi-commodity, Interior point method.*

1. Introduction

Traffic congestion in urban network is an important problem. With congested road, vehicle speed will be simultaneously up and down, the average speed will be lower, travel time will be longer, traffic delay and the total cost will increase. Therefore road users will suffer from increasing vehicle operating cost and losing more time and environment will be in worse condition due to pollution. In other words, transportation costs increase due to traffic congestion.

The cost incurred by the society as the result of the transportation and the effect of transportation include congestion cost, traffic accident cost, fuel and energy wasted.

The increase of total vehicle operating in the roads, increase the cost that be borne by society and country. To reduce such costs, especially for passengers, it can be overcome by promoting the use of public transportation, discourage unnecessary private vehicle, congestion reduction and harmonizing traffic in the entire network.

Also there are different techniques, such as increasing link capacities, enforcing traffic restrictions, increasing executive charges (user taxation), increasing parking charges, and also toll pricing [1, 2]. It is proven that the toll pricing scheme is one of the most effective methods to decrease congestion.

The general idea behind congestion pricing is to provide users an option to pay some toll for the externalities bad effects such as congestion and extra time they force on the other road users, and this can be extended to limit negative effects such as emission of pollutants, noise and accidents. Link travel time externality can be internalized only when the toll charges for each link, and also it can be decreased when the tolls are intelligently changed. To estimate the congestion cost which is the difference between perceived and actual generalized cost in traffic jam, Sugiyanto et al. [3] proposed a useful scheme in which vehicle operating cost, travel time cost, and pollution cost have been considered in based marginal cost pricing or first-best pricing (FBP). In the first-best pricing scheme, the toll charges for each link and the user equilibrium solution is driven to a system optimum. System optimum means that the maximal social welfare will be achieved. Yang [4] and Yildirim [5] applied FBP approach to attain system optimum.

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The second-best pricing (SBP) approach was also used in which just a few links must be charged. In this approach, a bi-level programming problem is recognized as to find toll levels which maximize the social surplus, see e.g. Erik [6] and Zhang [7]. In most models for in transportation planning, traffic management and operation, origin-destination (OD) matrix is a very important type of input data. It is the one of most crucial requirement in analyzing traffic equilibrium and making predictions of future traffic flows. The quality of the obtained solution of course very much depends on the quality of the OD-matrix. In SBP models, the result of lower level problem that specify flow on links is dependent on OD matrix. So, accurate estimation of OD matrix, in first level, help to determine the exact congested traffic zones and candidate links for pricing and in second level, it helps to evaluate the effect of the pricing in social welfare. Also considering up-to date information of OD matrix and variable demand provides a more accurate analysis in pricing process. Yildirim [5] and Verhoef [6] presented FBP and SBP respectively by considering variable demand.

SBP problems are non-convex and non-differentiable, so, meta-heuristic algorithms can be used for solving these problems (XU et al [8]). Yang and Zhang [9] represented a hybrid simulated annealing (SA) algorithm with random search to obtain optimal toll patterns. Also, Afandizadeh [1] and Shepherd [10] presented genetic algorithm in order to find optimal toll level and location problem.

One important drawback of methods for toll pricing is that the input parameters and behavior of users are considered as deterministic parameters, while due to uncertainty and imprecise nature of traffic network, this assumption is not true. For example the demand between origin-destination pairs may be varied in different seasons. Or travel time, travel cost and link capacity may be given under stochastic or fuzzy concepts. With uncertain condition, Sumalee and Xu [11] presented FBP for a traffic network with stochastic demand by considering risk-neutral and risk-averse users. Chen and his assistances [12] developed three stochastic multi-objective models for designing transportation network under uncertain demand. Also, Ghatee and Hashemi [13] used fuzzy rules for route choice process in fuzzy traffic assignment problem. Hu and Liu [14] defined a mathematical program to model traffic assignment under fuzzy equilibrium constraints. Also Quattrone, and Vitetta [15] applied fuzzy rule-based models to exhibit route choice behavior in case of Logit model. In location allocation problems of structure, Xu and Wei [16] presented a bi-level model under fuzzy random environment and used particle swarm optimization for optimal solution. Teodorovic and Kikuchi [17] also applied a fuzzy approximation reasoning model to choose a route between alternative routes in order to solve traffic assignment model. In addition, Teodorovic and Edara [18] studied optimal toll pricing by considering tolled and non-tolled routes with fuzzy travel time. Also Dempe and Starostina [19] modeled the toll pricing under fuzzy travel cost for a single pair of origin-destination. They transformed their fuzzy bi-level programming problem into a real-value problem applying Rommelfanger approach [20]. This paper uses the same approach too. By this approach, fuzzy lower level is transformed to linear optimization problem. In most cases, simplex algorithm is used to obtain optimal solution of linear optimization problem, but simplex algorithm maybe an exponential time algorithm. So another polynomial algorithm in category of interior point methods is used [21].

Toll pricing is applied to achieve desired traffic quality. The quality depends on the level of service (LOS) in all of the links. Highway capacity manual defines LOS as “qualitative measure that characterizes operational conditions within a traffic stream and their perception by passengers”. So the effects of LOS are considered for assessment of the lane management operations such as HOV (Highway occupancy vehicle) and HOT (Highway occupancy toll). It defines six levels of service namely A, B, C, D, E and F [22, 23]. This paper considers implementing toll strategy in a network with multiple origin- destination pairs and fuzzy travel time. By applying bi-level programming, in upper level, is tried to satisfy desired LOS for all of the links and in lower level, a fuzzy multi-commodity network flow problem is presented. Utilizing defuzzification process, fuzzy problem in lower level is transformed into a linear optimization problem. To finding optimal tolls in upper level, the hybrid SA algorithm is used and to obtain optimal flow in lower level interior point (IP) method is presented. The hybrid proposed method is entitled as SA-IP including simulated annealing and interior point algorithms.

This paper is organized as follow. The next section introduces mathematical model for fuzzy bi-level programming. In Section 3 some fuzzy transformations is presented. Section 4 includes description of interior point method for fuzzy lower level problem. Final section presents the result on some examples.

2. Mathematical Modeling And SA-IP Method

In most of congestion pricing problem, bi-level programming models are used. The upper level represents a system controller (or the decision maker) determines the tolls that optimize a given system’s performance while considering users’ path choice behavior. On the other hand, the lower level represents the users. The users will always minimize their generalized travel cost including monetary toll and travel time in their route choice behavior. The generalized travel cost in lower level problem is the monetary unit or time, so travel time can be converted into corresponding money by vot (Value of time) and vice versa. To match real condition network, in this paper, route travel time is considered as uncertain fuzzy number that lead to fuzzy lower level problem.

To solve this bi-level programming, SA-IP method is proposed. In this method, simulated annealing is used in upper level problem to find optimal toll and interior point algorithm is used to find optimal flow through links in lower level problem. In the next subsection, at first fuzzy lower level problem is presented and then, upper level problem will be explained.

2.1. Lower Level Modeling

Consider directed one-modal network $G = (V, E)$ which node set V corresponds to the junctions and the link set E corresponds to the streets. The set $M \subset V * V$ contains of origin-destination pairs. v^{od} for each $(o, d) \in M$, is the number of passengers who wish to travel between origin o and destination d. Let $L(j)$ $(E(j))$ denotes the set of all links departing from (entering to) node$j$. Variable x_e^{od} represents the number of passenger from o to d through the link e. $\tilde{t}_e$, c_e^t are fuzzy travel time and toll charge of link e, respectively. U_e is the link capacity that is given in terms of vehicle per hour. In addition, vo is vehicle capacity or vehicle occupancy, which expresses with 1, 2, 3,… passengers. Therefore $x_e = \left[\frac{\sum_{(o,d)\in M} x_e^{od}}{vo}\right]$ is the number of vehicles moving on link e. Table (1) shows some these parameters.

To assign flow to links, it is possible to state the following fuzzy multi-commodity flow problem (1) in lower level problem:

$$min \quad \psi(c^t) = \sum_{e\in E} (\tilde{t}_e . vot + c_e^t) x_e \qquad (1.a)$$

$$\sum_{e\in L(j)} x_e^{od} - \sum_{e\in E(j)} x_e^{od} = \begin{cases} v^{od} & j = o \\ 0 & \forall j \in V \setminus \{o,d\} \quad \forall (o,d) \in M \\ -v^{od} & j = d \end{cases} \qquad (1.b)$$

$$x_e = \left[\frac{\sum_{(o,d)\in M} x_e^{od}}{vo}\right] \qquad \forall e \in E \qquad (1.c)$$

$$x_e \le U_e \qquad \forall e \in E \qquad (1.d)$$

$$x_e, x_e^{od} \ge 0 \qquad \forall (o,d) \in M, \forall e \in E \qquad (1.e)$$

Table 1 Input parameters in fuzzy congestion pricing

Notation	Description
$V = \{i, j, ..\}$	Set of nodes
$E = \{e, ...\}$	Set of links
$M = \{(o, d), ...\}$	Set of origin-destination pairs
v^{od}	Demand between origin o and destination d (at passenger unit)
$L(j)$ $(E(j))$	All links departing from (entering to) node$j$
x_e^{od}	Number of passenger from o to $d$on link e
$\tilde{t}_e$	Fuzzy travel time on link e
c_e^t	Congestion price on link e
U_e	Vehicle Capacity of link e
vo	Vehicle occupancy
x_e	Number of vehicles moving on linke
vot	Value of time

In this model, (1.a) is minimization of total cost in network, (1.b) is supply-demand satisfaction for each O-D pair, (1.c) recognizes total vehicle on link e, (1.d) is capacity restriction on the links and (1,e) represents nonnegative restriction of flow variables.

2.2. Upper Level Modeling

Applying optimal congestion pricing strategy in upper level problem improves the usage of network links to meet desired LOS. This approach decreases the number of empty and congested links and makes it possible to harmonize the system. Let s_e^* and $s_e$be the desired LOS and current LOS of link e. It is tried to minimize differences between s_e^* and $s_e$forall of the congested links. According to highway capacity manual, (2) is used to compute s_e, also LOS is defined for each link using 6 categories denoted with A, B, C, D, E and F as (3).

$$s_e = x_e / U_e \qquad (2)$$

$$\begin{cases} s_e \in A \leftrightarrow & 0 <= s_e < 0.2 \\ s_e \in B \leftrightarrow & 0.2 <= s_e < 0.4 \\ s_e \in C \leftrightarrow & 0.4 <= s_e < 0.6 \\ s_e \in D \leftrightarrow & 0.6 <= s_e < 0.8 \\ s_e \in E \leftrightarrow & 0.8 <= s_e < 1 \\ s_e \in F \leftrightarrow & s_e <= 1 \quad or \quad 1 <= s_e \end{cases} \qquad (3)$$

So, the upper level problem can be defined as:

$$min \sum_{e\in E} max \{0, s_e - s_e^*\} \qquad (4.a)$$

$$s.t.$$

$$\Omega = \{c_e^t \mid c_e^t \in [c_e^l, c_e^u]\} \quad \forall e \in \acute{E} \subset E \qquad (4.b)$$

$$\tilde{x} \in \psi(c^t) \qquad (4.c)$$

In above model, (4.a) is minimizing differences between desired LOS and current LOS for congested links. Because of reasonability of tolls, their values vary between the lower bound c_e^l and the upper bound c_e^u, see (4.b). Also (4.c) expresses $\tilde{x}$ as an optimal solution from lower level problem.

Let $\acute{E}$ be candidate set of links for toll pricing. To obtain candidate links $\acute{E}$, one can run a traffic assignment algorithm in lower level problem without toll. Then links with worse LOS such as LOS E or F are candidate for toll enforcing. To find reasonable tolls, it isn't considered any toll for the following cases:

1- Origin nodes with a single output
2- Destination nodes with a single input
3- Each link located on all of paths between an OD pair

Also to decrease the social consequences of toll pricing, the total revenues of toll gathering, is minimized by imposing the following objective function as the secondary objective:

$$min \sum_{e\in \acute{E}} c_e^t . x_e \qquad (4.a')$$

For solving this problem, one can use weighted combination of these two objective functions ((4.a) and (4.a′)) for the upper level problem.

3. Defuzzification of lower level problem

For solving fuzzy optimization problems such as model (1), one of the most important ideas is defuzzification. For example it can maximize the minimum level of membership functions in order to satisfy a constants lower bound. So, the infinite objective functions are reduced into a single objective function [19], [20].

For this aim, the problem is evaluated for different cuts of fuzzy numbers. For each of these cuts, several real-value problems are obtained. The optimal values of these problems are obtained and they are used to create a membership function. By maximizing this membership function, it is possible to find an optimal solution with the greatest certainty.

In what follows, to present the details of this approach, the travel time $\tilde{t}_e$ is considered as a triangular fuzzy number $\tilde{t}_e = (t_e; \alpha_e; \beta_e)_{L-R}$, in which t_e, α_e and β_e are the center, the left spread and the right spreads, respectively. Let$\Lambda \subset [0,1]$ be a set including acceptable certainty degrees. For each $\lambda \in \Lambda$, the $\lambda - cut$ of $\tilde{t}_e$ is an interval denoted with $[t_e(\lambda, L), t_e(\lambda, R)]$ in which:

$$t_e(\lambda, L) = t_e - \alpha_e(1 - \lambda) \tag{5.a}$$
$$t_e(\lambda, R) = t_e + \beta_e(1 - \lambda) \tag{5.b}$$

Consider the following traditional linear programming problems which their objective functions are as (6.a) and (6.b) and their constraints are same as model (1).

$$LP\ (L, \lambda, c^t) = min \left\{ \psi_L(x) = \sum_{e \in E} (t_e(\lambda, L).vot + c_e^t)x_e \middle| subjected\ to\ constraints\ of\ (1) \right\} \tag{6.a}$$
$$LP\ (R, \lambda, c^t) = min \left\{ \psi_R(x) = \sum_{e \in E} (t_e(\lambda, R)vot + c_e^t)x_e \middle| subjected\ to\ constraints\ of\ (1) \right\} \tag{6.b}$$

These optimization models are the most optimistic and the most pessimistic scenarios of model (1). Then $w^*(L, \lambda)$ and $w^*(R, \lambda)$ as the lower bound of $LP\ (L, \lambda, c^t)$ and $LP\ (R, \lambda, c^t)$ can be computed by considering $c^t = c^l$ ((7.a) and (7.b)).

$$w^*(L, \lambda) = LP\ (L, \lambda, c^l) \tag{7.a}$$
$$w^*(R, \lambda) = LP\ (R, \lambda, c^l) \tag{7.b}$$

Assume $x_e(L, \lambda, c^t)$ and $x_e(R, \lambda, c^t)$ are the optimal solutions of (6.a) and (6.b), respectively. Maximal objective function values of $LP\ (L, \lambda, c^t)$ and $LP\ (R, \lambda, c^t)$ can be obtained by considering $c^t = c^u$, and evaluating $LP\ (L, \lambda, c^u)$ at the optimal solution of $LP\ (R, \lambda, c^t)$ and evaluating $LP\ (R, \lambda, c^u)$ at the optimal solution of $LP\ (L, \lambda, c^t)$ as (8.a) and (8.b).

$$w\ (L, \lambda) = \sum_{e \in E} (t_e(\lambda, L).vot + c_e^u)x_e(R, \lambda, c^t) \tag{8.a}$$
$$w\ (R, \lambda) = \sum_{e \in E} (t_e(\lambda, R).vot + c_e^u)x_e(L, \lambda, c^t) \tag{8.b}$$

Values of $w^*(L, \lambda)$, $w^*(R, \lambda)$, $w\ (L, \lambda)$ and $w\ (R, \lambda)$ are the amounts of trade-off table to define the membership functions. In other words, the membership function μ of lower level problems can be stated as follows:

$$\mu\ (L, \lambda, \omega\ ) = \begin{cases} 1 & \omega \le w^*(L, \lambda) \\ \dfrac{(w\ (L, \lambda) - \omega\ )}{(w(L, \lambda) - w^*(L, \lambda))} & w^*(L, \lambda) < \omega \le w(L, \lambda) \\ 0 & \omega > w(L, \lambda) \end{cases} \tag{9}$$

In which ω is the objective value of $LP\ (L, \lambda, c^t)$. Similarly it can be defined:

$$\mu\ (R, \lambda, \acute{\omega}) = \begin{cases} 1 & \acute{\omega} \le w^*(R, \lambda) \\ \dfrac{(w\ (R, \lambda) - \acute{\omega})}{(w(R, \lambda) - w^*(R, \lambda))} & w^*(R, \lambda) < \acute{\omega} \le w(R, \lambda) \\ 0 & \acute{\omega} > w(R, \lambda) \end{cases} \tag{10}$$

Where ω' is the objective value of $LP\ (R, \lambda, c^t)$.

In order to find the most possible solution for model (1), the minimum of these membership degrees over the feasible set of model (1), must be maximized. If γ be the lower bound for these degrees, the following problem can be solved for this aim:

$$\Gamma(c^t):\ max\, \gamma \tag{11.a}$$

s.t.

$$\frac{w\ (L, \lambda) - \omega}{w(L, \lambda) - w^*(L, \lambda)} \ge \gamma \qquad \forall \lambda \in \Lambda \tag{11.b}$$
$$\frac{w\ (R, \lambda) - \acute{\omega}}{w(R, \lambda) - w^*(R, \lambda)} \ge \gamma \qquad \forall \lambda \in \Lambda \tag{11.c}$$

Subjected to the constraints of (1). (11.d)

The optimal solution of this problem namely max-satisfying solution is denoted by x^*[24]. By considering the importance weights θ_1, θ_2 for two objective functions of the upper level problem, the following optimization problem is stated to find optimal tolls:

$$min\, \theta_1 \left(\sum_{e \in E, s_e > s_e^*} (s_e - s_e^*) \right) + \theta_2 \left(\sum_{e \in \acute{E}} c_e^t x_e \right) \tag{12.a}$$

s.t.

$$\Omega = \{ c_e^t \mid c_e^t \in [c_e^l, c_e^u] \} \quad \forall\, e \in \acute{E} \subset E \tag{12.b}$$
$$x^* \in \Gamma(c^t) \tag{12.c}$$

Thus, it is possible to use a mutual consistence method on this bi-level programming problem. In the upper level problem SA algorithm is done on model (12) to find optimal tolls. The results are set at the lower level problem (11) to find optimal flows with respect to given tolls. These results on flows are shared with the upper level problem and the tolls are adapted considering these flows. This loop is repeated up to converging.

As it is shown, fuzzy lower level problem is transmuted to a linear optimization (LO) problem as model (11.a).(11.d) by defuzzification method. Next section presents a polynomial interior point algorithm to find optimal solution for this LO problem".

4. Interior point (IP) method for LO problem

Linear optimization problem is in canonical form if it is written as:

$$min\{c^T x \ : Ax \geq b, x \geq 0, A \ is \ m.n \ matrix\}$$

with dual problem:

$$max\{b^T y \ : A^T y \leq c \,, y \geq 0\}$$

Roos et al. [21], showed that any LO problem can be solved by finding a strictly complementary solution of a specific self-dual problem that has the form:

$$min \quad \{q^t z \ : \quad Mz \geq q, \ z \geq 0\} \qquad (13)$$

In which M, r, z and q are defined as:

$$M = \begin{bmatrix} \begin{bmatrix} 0 & A & -b \\ -A^T & 0 & c \\ b^T & -c^T & 0 \end{bmatrix} & r \\ -r^T & 0 \end{bmatrix}, r = \begin{bmatrix} e_m - Ae_n + b \\ e_n + A^T e_m - c \\ 1 - b^T e_m + c^T e_n \end{bmatrix}, z = \begin{bmatrix} x \\ y \\ \rho \\ \vartheta \end{bmatrix}, q = \begin{bmatrix} 0_{n+m+1} \\ n+m+2 \end{bmatrix}$$

In this model, ρ and ϑ are nonnegative variables for satisfying interior point condition. One polynomial algorithm for (13) is Newton Step algorithm [21]. Basic steps of this algorithm are as:

Full-Newton step interior point algorithm

input:

An accuracy parameter $\epsilon > 0$

A barrier update parameter $\theta, 0 < \theta < 1$

begin

$$z = e, \quad \mu := 1$$

while$(n + m + 2)\mu \geq \epsilon$do

begin

$$\mu = (1 - \theta)\mu$$
$$z = z + \Delta z$$

end

end

in the mentioned algorithm, $\Delta z = (\mathrm{diag}(Mz + q) + \mathrm{diag}(z)M)^{-1}(\mu e - z(Mz + q))$.

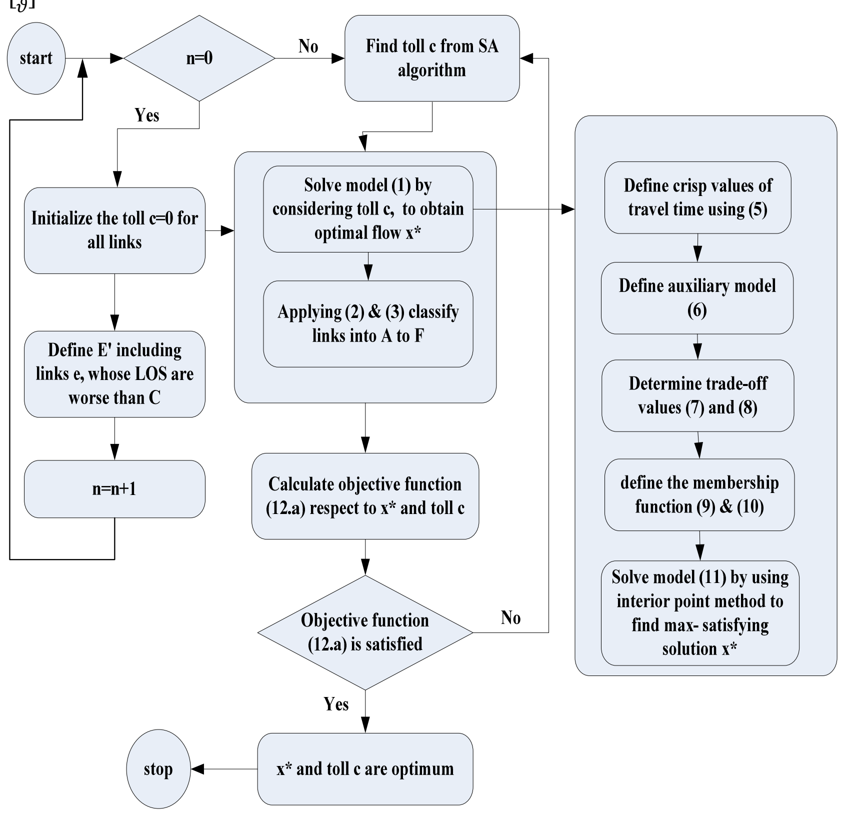

Fig.1 Flowchart of SA-IP method to find optimal tolls and flows

Fig (1) presents total process of SA-IP for obtaining optimal solution from fuzzy bi-level programming that was explained in previous sections. In Fig. (1), n is the number of iterations. In SA-IP simulated annealing which is a global optimizer is used to search the feasible space for optimal toll and also interior point algorithm is used to find optimal flow from a multi-commodity flow problem. Next section investigates the efficiency of this algorithm on some example.

5. Numerical Simulation: Sioux Falls Network

Sioux Falls network [8] is used to present the solution of the proposed algorithm. This network consists of 24 nodes and 76 links; see Fig.(2). Input parameters and O-D demand as the number of vehicle between each pair, are shown in Table (3) and Table (4), respectively (Note that origin – destination without any demand are shown with 0 values). Travel time in minute is considered as fuzzy triangular numbers. The center of fuzzy triangular time ($\tilde{t}_{max}$) is presented in Table (3) and the left and right spreads are defined as a quarter of the centers. Thus $\left(\tilde{t}_{max} - \left(\frac{1}{4}\right)\tilde{t}_{max}, \tilde{t}_{max}, \ \tilde{t}_{max} + \left(\frac{1}{4}\right)\tilde{t}_{max}\right)$ is applied as the fuzzy travel time for each link. Other parameters are shown inTable (2).

Table 2 Input parameters for example (Sioux Falls network)

Parameter	Value	Parameter	Value
c_e^l	0 (dollar)	s_e^*	0.5
c_e^u	4(dollar)	θ_1	75%
vot	1(dollar)	θ_2	25%
ϵ	0.01		

Table 3 Input parameters for Sioux Falls Network

Link Index	Capacity	$\tilde{t}_{max}$	Link Index	Capacity	$\tilde{t}_{max}$	Link Index	Capacity	$\tilde{t}_{max}$
1	3000	10	27	2500	2.3	53	2000	1.27
2	3000	2.4	28	1000	2.7	54	2000	5
3	3000	7.5	29	1500	2	55	2000	4
4	3000	2.4	30	1500	3	56	4000	8
5	3000	2.4	31	1500	1.21	57	1000	2.4
6	2000	2.4	32	2500	2	58	2000	2.54
7	2500	10	33	1500	3.82	59	2000	1.26
8	1500	2.42	34	2000	2.54	60	4000	10
9	1000	1.21	35	2500	3.75	61	1000	1.26
10	1500	2	36	2000	3.4	62	1500	1.25
11	1000	1.2	37	4000	8	63	1500	2
12	2000	4	38	4000	8	64	2200	1.5
13	1500	1.5	39	2500	2.54	65	1200	2.31
14	3000	3.4	40	2000	2.5	66	2000	1.5
15	2000	2.8	41	1500	3	67	1500	1.91
16	2500	1.27	42	1000	2.49	68	1500	2
17	2000	3.5	43	1000	2.8	69	1500	2.4
18	2000	3.5	44	1500	2	70	2000	1.24
19	2500	3.5	45	1000	2	71	2000	2.49
20	2000	3.5	46	1500	4	72	1500	2.48
21	1000	2	47	2500	2.4	73	2000	1.27
22	1000	3.5	48	1500	2	74	2500	2.54
23	1500	2	49	2000	2	75	2000	1.91
24	1000	2	50	2000	4	76	1500	1.27
25	1000	1.83	51	1500	3.5	-	-	-
26	1500	1.83	52	2000	3.2	-	--	-

Table 4 O-D demand for Sioux Falls Network

		Destination					
		1	6	10	13	18	20
Origin	1	0	1000	0	1500	0	0
	6	1000	0	1000	0	0	1000
	10	0	1000	0	1500	0	0
	13	1500	0	1500	0	1000	0
	18	0	0	0	1000	0	0
	20	0	1000	0	0	0	0

In this case, toll is enforced for links with LOS F. So, after executing lower level problem, 13 links are considered as candidate links for toll pricing. In Fig. (2) these links are shown with different color. A standard SA is used to obtain optimal tolls that basic structure is adapted from [9] and interior point method for fuzzy lower level problem. Table (5) gives the results of congestion pricing including tolls of links, max membership value in fuzzy lower level problem and improvement in upper level objective function. As it is shown, the value of objective function (total difference between desired LOS and current LOS) is improved 65.97% after toll pricing process.

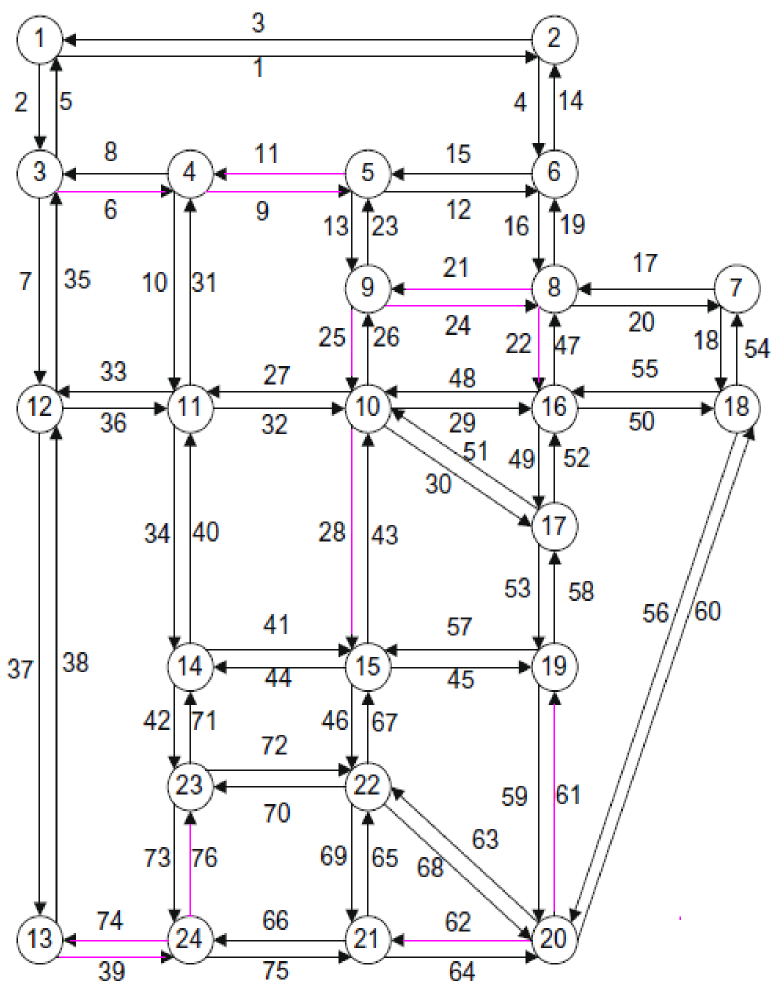

Fig. 2 Sioux Falls Network and some candidate for toll enforcing

Table 5 Link flow and LOS in Sioux Falls network before and after toll enforcing

Before tolling						After tolling							
Link	v_a	LOS_a	link	v_a	LOS_a	link	v_a	LOS_a	τ_a	Link	v_a	LOS_a	τ_a
1	0	A	39	2500	F	1	1000	B	0	39	1000	B	2.936
2	2500	E	40	1500	D	2	1500	C	0	40	0	A	0
3	0	A	41	0	A	3	1000	B	0	41	0	A	0
4	0	A	42	0	A	4	1000	B	0	42	152	A	0
5	2500	E	43	0	A	5	1500	C	0	43	0	A	0
6	2000	F	44	0	A	6	0	A	3.715	44	0	A	0
7	500	B	45	0	A	7	1500	C	0	45	0	A	0
8	1000	D	46	1000	D	8	0	A	0	46	0	A	0
9	1000	F	47	1000	C	9	0	A	0.858	47	1000	B	0
10	1000	D	48	0	A	10	0	A	0	48	0	A	0
11	1000	F	49	1000	C	11	0	A	2.964	49	1000	C	0
12	1000	C	50	0	A	12	1000	C	0	50	0	A	0
13	0	A	51	0	A	13	1000	D	0	51	0	A	0
14	0	A	52	1000	C	14	1000	B	0	52	1000	C	0
15	1000	C	53	1500	D	15	1000	C	0	53	1000	C	0
16	2000	D	54	0	A	16	1000	C	0	54	0	A	0
17	0	A	55	0	A	17	0	A	0	55	0	A	0
18	0	A	56	1000	B	18	0	A	0	56	1000	B	0
19	2000	D	57	0	A	19	1000	B	0	57	0	A	0
20	0	A	58	1000	C	20	0	A	0	58	1000	C	0
21	1000	F	59	1500	D	21	0	A	2.818	59	1000	C	0
22	1000	F	60	1000	B	22	1000	F	0	60	1000	B	0
23	0	A	61	1000	F	23	1000	D	0	61	1000	F	0.525
24	1000	F	62	1500	F	24	0	A	3.143	62	0	A	2.639
25	1000	F	63	0	A	25	1000	F	3.690	63	1000	D	0
26	1000	D	64	1000	C	26	1000	D	0	64	1000	C	0
27	0	A	65	0	A	27	1500	C	0	65	0	A	0
28	1000	F	66	1500	D	28	0	A	1.969	66	0	A	0
29	0	A	67	0	A	29	0	A	0	67	0	A	0
30	500	B	68	0	A	30	0	A	0	68	0	A	0
31	0	A	69	0	A	31	0	A	0	69	0	A	0
32	1500	C	70	1000	C	32	1500	C	0	70	1000	C	0
33	1000	D	71	1500	D	33	1347	E	0	71	0	A	0
34	0	A	72	0	A	34	153	A	0	72	0	A	0
35	1500	C	73	1000	C	35	1500	C	0	73	1153	C	0
36	0	A	74	2500	F	36	1500	D	0	74	1153	C	3.3
37	1500	B	75	1000	C	37	2847	D	0	75	1000	C	0
38	1500	B	76	1500	F	38	3000	D	0	76	0	A	3.13
Total difference between current LOS and desired LOS					9.7					3.3			
Membership value of lower level problem					1				0.998				
Improvement in objective function in pricing process: 65.97%													

Fig. (3) shows the number of links with LOS D, E and F are decreased after toll pricing. Also, in Fig. (4) the successive improvement in the values of the objective function of the upper level problem and differences between current LOS and desired ones are given.

Fig. (5) illustrates the variation of the membership degrees of the upper bound objective function with respect to the spreads of fuzzy numbers. For this examination, some fuzzy traverse time are defined that their centers are the mean values of traverse time $\tilde{t}_{max}$. The spreads of these fuzzy numbers are defined as 10% to 90% of $\tilde{t}_{max}$ .This test demonstrates that the values of reliability decreases as much as increasing the spreads values. Thus for finding a more robust solution it is necessary to use fuzzy numbers with small spreads.

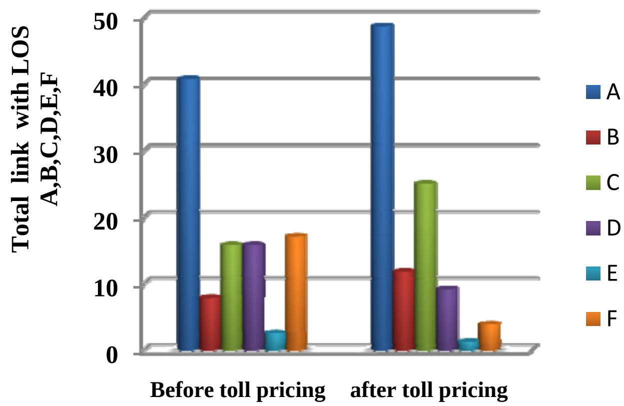

Fig. 3 Comparison between LOS of links before and after toll pricing

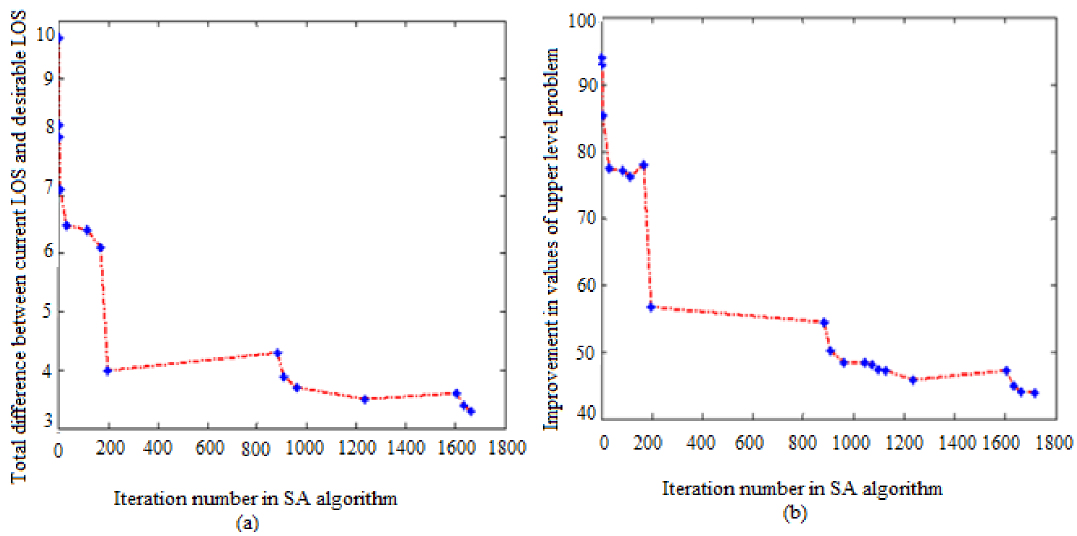

Fig. 4 (a): Total differences between desired and current LOS, (b): Improvement in the objective functions through SA algorithm.

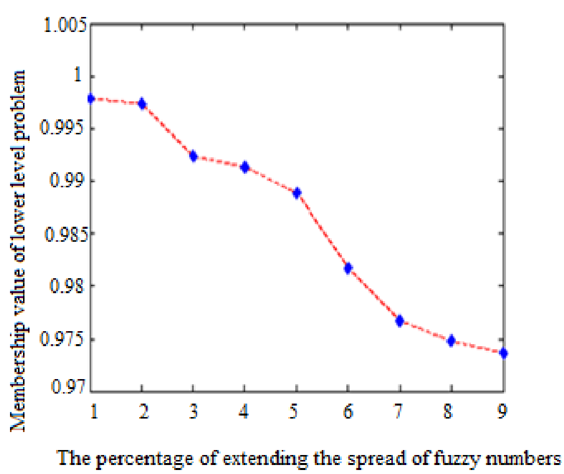

Fig. 5 Relation between membership function of lower level problem and fuzzy time interval

Fig. (6) shows the relationship between the amount of toll price and the change in LOS. Since toll values for candidate links are different with each other, so in order to examine of this relationship, the upper level of imposed toll is considered. It is shown that optimal toll for Sioux Falls network with the mentioned input is obtained when the upper level of toll is equals 5 $. For higher toll (more than 5 $) the objective function will be worse. Because, by imposing higher toll, their generalized cost (summation of travel time and toll) will be very high and users will never use them and so the flow of candidate links should be immigrated to another links. Thus, the other non-tolled links will be more congested and their LOS will be close to LOS E or F.

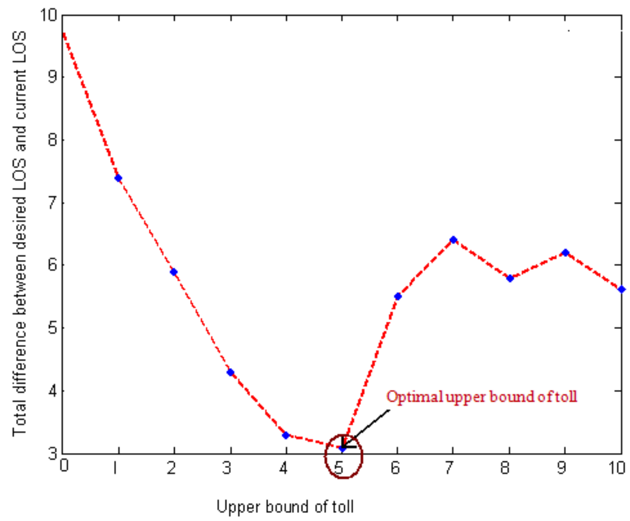

Fig. 6 Relationship between the amount of toll price and the change in level of service

6. Conclusion and Future Directions

In this paper, to guarantee some desired LOS for the network links, toll is considered on some of the congested links. So a bi-level programming problem is implemented. The upper level problem focuses on two objective functions including minimizing the total enforced toll and minimizing the differences between the link LOS and desired LOS. To find optimal solution, SA-IP method including simulated annealing and interior point algorithms is proposed. In the lower level problem, to match real condition of traffic network, fuzzy uncertain conditions are considered in travel times. So, in the lower level problem, a fuzzy multi-commodity flow problem is solved by interior point algorithm to find optimal flow with respect to imposed tolls. For solving this problem a defuzzification idea is applied in order to maximize the minimum membership value of the lower level objective function.

Simulated annealing algorithm is also used to find optimal toll in the upper level problem as well as using interior point algorithm for obtaining flow in the lower level problem. The results illustrate that SA-IP can be applied in large-scale networks. For real problems it is necessary to pursue a decomposition approach in problem which is left to the next work.

Also we try to combine meta-heuristic algorithms to obtain optimal congestion prices from a problem with multiple objective functions and to exhibit with uncertainty conditions on other parameters such as capacity and demand.

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