International Journal of Civil Engineering

Water-Hydraulic Structure

Estimation of maximum scour depth downstream of horizontal and adverse stilling basins using a semi-theoretical approach

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Received: June 2013, Revised: July 2013, Accepted: October 2013

Abstract

Because of the complexity of the physical processes in the vicinity of the hydraulic structures due to the separation of the flow, traditional methods for for prediction of maximum scour depth downstream of hydraulic structures are mostly based on empirical approaches. Hence, only a few theoretical works have been reported to study this phenomenon. The present paper describes a new approach based on the momentum principles to estimate the maximum local scour depth downstream of a submerged sluice gate flowing over horizontal or adverse stilling basin. A control volume of the fluid in the equilibrium state of the scour hole was considered and based on momentum principles, some equations are derived to estimate the scour depth at equilibrium state. To verify the proposed equations, large numbers of experiments were planned and conducted under wide range of characteristic parameters such as, incoming Froude number, sediment size, tailwater depth, length and slope of the apron. It was found that the proposed equations fall in a good agreement with experimental results. It was also observed that, in the case of horizontal apron, a specific tailwater depth exists with which the local scour depth attains a minimum value. However, in the case of adverse basins when the tailwater depth takes a specific value, the maximum depth of the scour hole reaches to its maximum and then decreases to a constant value as the tailwater depth increases. This critical tailwater depth was formulated using a semi-theoretical equation.

Keywords: *Local scour, Momentum principle, Sluice gate, Submerged jet, Adverse Stilling basin.*

1. Introduction

Local scour problem downstream of hydraulic structures is a serious problem which endanger the stability of such structures leading to undermine the foundation. Hence, the geometrical characteristics of scour hole is too significant to be considered by designers of hydraulic structures such as, stilling basins and sluice gates. The presence of large scale turbulence, local eddies encountered with hydraulic jump and increasing shear stress results in formation of the scour hole downstream of hydraulic structures. Because of the complexity of the physical process, many researchers have employed empirical approaches to study this phenomenon.

Farhoudi and Smith [1,2] studied the scour process downstream of hydraulic jump featuring the characteristic parameters defining the scour hole.

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They demonstrated that the development of local scour hole downstream of apron in the passage of time shows a certain geometrical similarity and nondimensional scour profile can be presented by a unified equation.

Hassan and Narayanan [3] studied the local scour downstream of an apron due to a submerged jet issuing from a sluice opening and developed an empirical equation for the time variation of scour depth to reach an asymptotic stage. Chatterjee et al. [4] studied the local scour downstream of an apron due to a submerged jet issuing from a sluice opening and developed an empirical equation for the time variation of scour depth to reach an asymptotic stage. Balachandar et al. [5] investigated the effect of grain size on local channel scour below a sluice gate. They have found that the tests with a mixed sand bed resulted in less scour relative to those in which a uniformly graded sand of similar grain size was used. Dey and Sarkar [6] carried out an experimental investigation on the effects of different parameters on scour depth due to submerged horizontal jets. A particular geometrical similarity of scour profiles at different times have been exhibited and expressed by a combination of two polynomials. Dey and Sarkar [7] investigated the effects of upward seepage on scour hole at the downstream an apron due to submerged jets. They reported that the maximum scour depth increases linearly with an increase in upward seepage velocity. Oliveto et al. [8] showed that the main

[DOI: 10.22068/IJCE.13.1.1]

dimensionless parameters governing the scour process are the tailwater densimetric Froude number, relative time, and relative tailwater flow depth. Many investigations have been conducted regarding the evolution of local scour in last three decades which are summarized in Table 1, where y_1 is flow depth at the *vena contracta*, ϕ is the angle of repose, D_{50} is the median sediment size, $Fr_1 = \frac{q}{\sqrt{g y_1^3}}$ is the approaching Froude number, s_s is the relative density of sediments, L_B is the length of stilling basin, y_t is the tailwater depth, u_o is the jet velocity at the gate opening, $F_o = \frac{q}{\sqrt{g(s_s - 1)D_{50}^3}}$ is the densimetric Froude number, w is the gate opening, D_{90}^* is the sedimentological diameter, and u_2 is the flow velocity at the downstream section. Despite to the several above mentioned works, it seems that the effects of some important parameters such as tailwater depth, length and slope of the stilling basin are not still well understood. Therefore, the local scour phenomenon would be one of the concerns in engineering applications. It should be noted that the mentioned relations have been proposed in certain conditions. For example, Altinbilek and Basmaki [9] and Hoffmans [10] proposed some relationships for scour evolution downstream of the submerged sluice gates flowing over waterways without any bed protections. On the other hand, Dey and Sarkar [6] derived a relationship from a set of experimental data where some relatively short aprons (maximum relative length of apron ($\frac{L_B}{w}$) was equal to 55) were employed. Hamidifar et al. [11] performed some experiments on a stilling basin with sufficient long apron. Apparently, it would be concluded that, the suggested equations by Dey and Sarkar [6] and Hamidifar et al. [11] may not correctly estimate the geometry of local scour hole where aprons with different lengths are used.

Table 1 Proposed equations to estimate the maximum equilibrium scour depth downstream of submerged sluice gates, by different authors

Researchers	Proposed relation
Altinbilek and Basmaki [9]	$\frac{y_{me}}{y_1} = \sqrt{\frac{y_1 \tan\phi}{D_{50}}}\left(\frac{Fr_1}{\sqrt{s_s - 1}}\right)^{1.5}$
Chatterjee et al. [4]	$\frac{y_{me}}{w} = 0.775\left(\frac{U_o}{\sqrt{gw}}\right)$
Hoffmans [10]	$\frac{y_{me}}{w} = \frac{50}{\left(D_{90}^*\right)^{1/3}}\left(1 - \frac{u_2}{u_0}\right)$
Dey and Sarkar [6]	$\frac{y_{me}}{w} = 2.59 F_o^{0.94}\left(\frac{L_B}{w}\right)^{-0.37}\left(\frac{y_t}{w}\right)^{0.16}\left(\frac{D_{50}}{w}\right)^{0.25}$
Hamidifar et al. [11]	$\frac{y_{me}}{w} = 0.031\left(F_o \frac{y_t}{w}\right)^{0.81}$

It is understood that any changes in channel geometry, such as adverse slope of stilling basin, would change the characteristics of hydraulic jump and therefore, its consequences at downstream of such structures. Since the adverse aprons would increase the energy loss and decrease the length of hydraulic jump, it could be recommended as an economical replacement to classical stilling basins which detailed by, McCorquodale and Mohamed [12], Pagliara and Peruginelli [13]. On the other hand the hydraulic jump on such aprons accompanies with some instabilities on hydraulic jump exiting from the basin (Rajaratnam [14], Pagliara and Peruginelli [13], Baines and Whitehead [15]). Therefore, the adverse aprons may act dual roles in evolution of scour hole downstream of such stilling basins which requires a close study to achieve some recommendations for designers.

In this research, some new regression relations are presented in those a wide range of parameters were considered. It is clear that, regression equations entirely depends on experimental data. Using the momentum equation would facilitate the derivation of some semi-theoretical relationships to explain the real phenomenon. The momentum principles were used to derive some theoretical equations to predict the maximum scour depth at presence of a horizontal and adverse stilling basin. At the end, the semi-theoretical equations are compared with those of regression relationships which shows that the semi-theoretical approach is capable to predict the specific depth affecting the characteristics of scour hole.

2. Material and Methods

To evaluate the characteristics of local scour downstream of a horizontal and adverse stilling basin, a perspex-walled flume of 4.9 m length and 0.41 m width with a recirculation flow system was employed where a submerged sluice gate, with its opening (w) of 1.4, 2, and

[DOI: 10.22068/IJCE.13.1.1]

2.5 cm, was erected at upstream of 1.18 m perspex apron. The details of experimental layouts are shown in Fig. 1. Seven bed slopes ($S_o = \tan\theta$) of 0.156, 0.0956, 0.0764, 0.0532, 0.0316, 0.0129, and 0 were used where an alluvial chamber with 0.15 m depth and 1.8 m length was constructed across the whole width of the flume downstream of the stilling basin. A sand trap was placed downstream of the alluvial bed to prevent any incidental fine sand intrusion into the recycling flow. Totally, 233 tests in a wide range of sediment size (D_{50}), relative length of stilling basin ($\frac{L_B}{w}$), bed slope ($S_o = \tan\theta$), inflow Froude number (Fr_1) and relative tailwater depth (y_t/w) were performed as is summarized in Table 2.

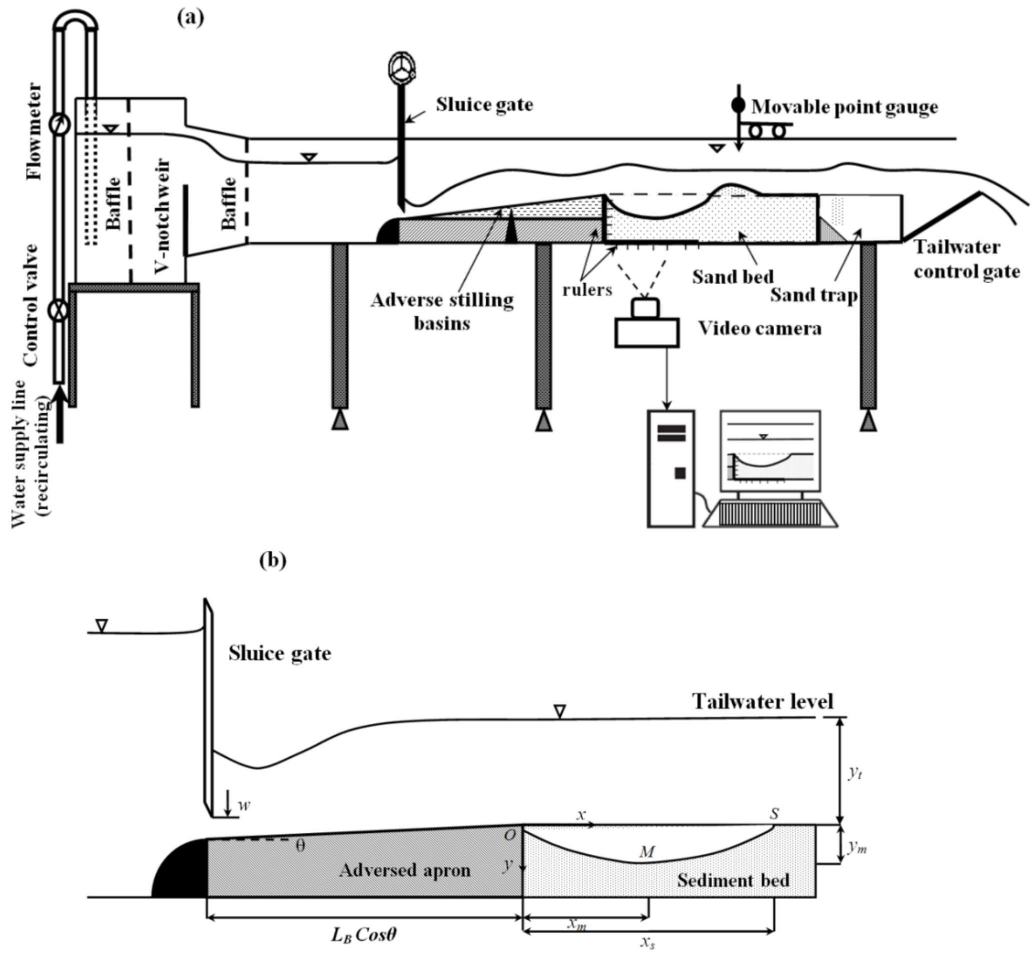

Fig. 1 Experimental layout; (a) setup, (b) Typical sketch of the scour hole

Table 2 Ranges of different parameters in this study

parameter	$D_{50}(mm)$	$S_o = -\tan\theta$	$\frac{L_B}{w}$	$\frac{y_t}{w}$	Fr_1
Range of variation	0.58(D_{141}),1.11(D_{11}),1.78(D_3)	0.156, 0.0956, 0.0764, 0.0532, 0.0316, 0.0129,0	47.2,59.05,84.3	1.74-15.19	3.78-11.14

Figure 2 shows the sediment size distribution of the three used sediments throughout the experiments. From Table 2, it would be realized that the sediments could be assumed as uniformly graded. A hinged tailgate was used for adjusting the tailwater depth. The discharge was measured by a V-notch weir which was calibrated against an electromagnetic flowmeter. The tailwater depth was measured by means of a moving point gauge with an accuracy of ±0.1 mm. In order to avoid the undesirable erosion of sediment at the beginning of each experiment, the flume was initially filled with water from the downstream section. Once the water level reached to a desired depth, the experiment was commenced. The profiles of scour holes were traced by means of digital

[DOI: 10.22068/IJCE.13.1.1]

photography technique. On the basis of the previous studies conducted by Farhoudi and Smith [1,2], each test was carried out over a period of 24 hours. Although, the equilibrium sour state was not achieved in this time length, but it was sufficient to reach a quasi equilibrium state of scouring in most of the tests. Figure 3 shows the typical time variation of scour holes for ($S_o = -0.156, Fr_1 = 8.53, D_{50} = 1.11 mm, \frac{L_B}{w} = 84.3$).

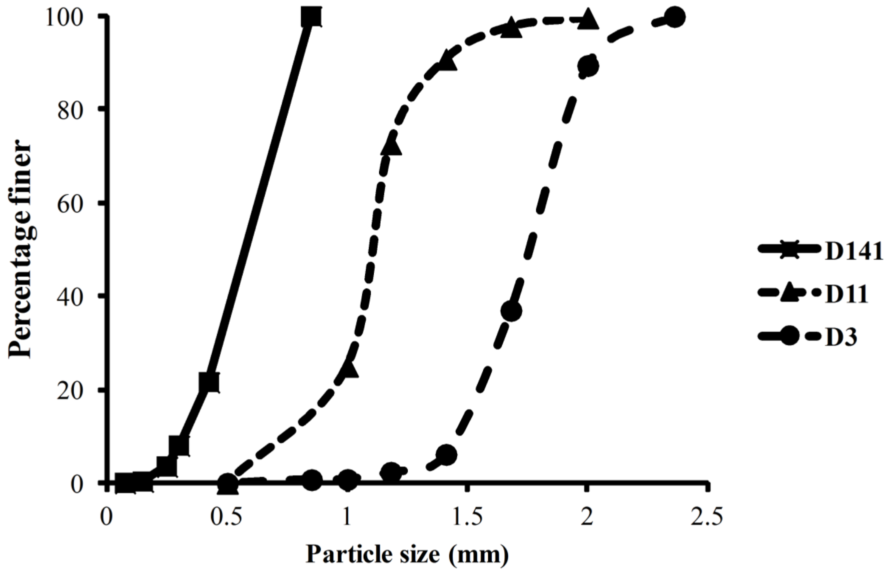

Fig. 2 Distributions of particle size

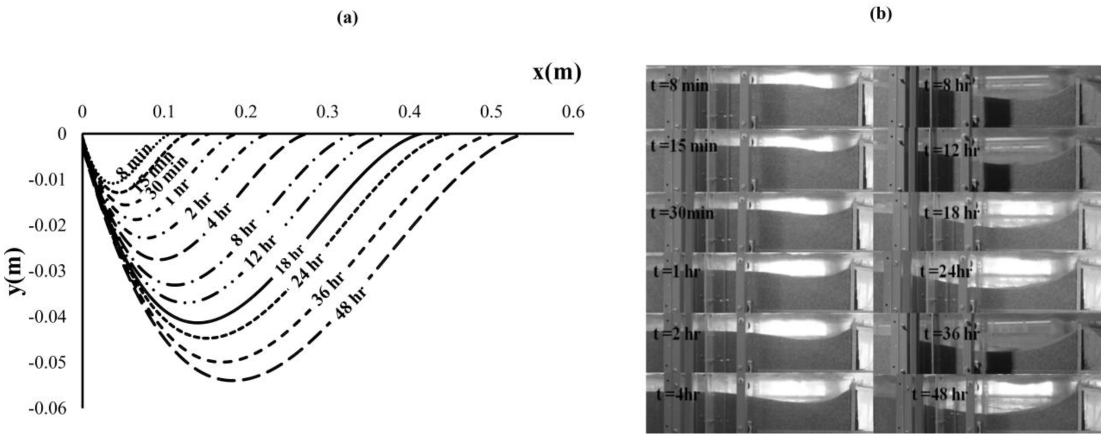

Fig. 3 Typical time variation of scour hole profiles for $S_o = -0.156, Fr_1 = 8.53, D_{50} = 1.11 mm, \frac{L_B}{w} = 84.3$

It is noteworthy to mentioned that some dunes were developed with fine sediments (i.e. $D_{50} = 0.58 mm$) which may not occur in natural flows ,as reported by Balachandar et al. [5].

According to Ali and Lim [16], the minimum channel width should not be less than ten times of gate opening. Considering the maximum gate opening as 2.5 centimeters, the selected width of flume would be acceptable since, 41/2.5=16.4>10.

It is noteworthy to mention that some uncertainties may arise as the consequences of secondary phenomena such as dune formation, unsteady flood flow, non-uniformity of sediment and time duration which is the task of future researches.

3. Results and Discussion

3.1. Maximum equilibrium depth of local scour

Maximum equilibrium scour depth at downstream of adverse stilling basin due to the submerged jet issuing from a sluice gate, can be expressed in a functional form as:

$$F\left(\rho, \nu, y_t, g, q, L_B, \tan\theta, w, D_{50}, \rho_s, \sigma_g, C_u, \tan\phi, y_{me}\right) = 0 \quad (1)$$

where ρ is the mass density of fluid, ν is the fluid kinematic viscosity, q is the flow discharge per unit

[DOI: 10.22068/IJCE.13.1.1]

width, ϕ is the repose angle of material, ρ_s is the mass density of sediments, σ_g is the geometric standard deviation of sediments, C_u is the sediments uniformly coefficient. Since the present study considers only uniformly graded sediments, Eq. (1) could be simplified as:

$$F(\rho, \nu, y_t, g, q, L_B, \tan\theta, w, D_{50}, \rho_s, \tan\phi, y_{me}) = 0 \qquad (2)$$

Using Buckingham Theorem, and taking the maximum equilibrium scour depth (y_{me}) as a dependent parameter the following equation is obtained:

$$\frac{y_{me}}{w} = \eta(\tan\theta)^{\zeta_1}\left(\frac{y_t}{w}\right)^{\zeta_2}\left(\frac{L_B}{w}\right)^{\zeta_3}(Fr_1)^{\zeta_4}\left(\frac{D_{50}}{w}\right)^{\zeta_5} R_e^{\,\zeta_6} \qquad (3)$$

Since the out coming flow from the stilling basin was turbulent, therefore R_e could be relaxed. In Eq. (3), η and $\zeta_i; i=1-6$ have to be determined experimentally in equilibrium conditions. Figure 4 shows the typical time dependent maximum scour depth (y_m), the distance of maximum scour depth location from the end of rigid apron (x_m), and the length of scour hole (x_S) along with the observation after 54 hours. It would be concluded that $\frac{x_{m(t=24hr)}}{x_{m(t=54hr)}} = 0.863$, $\frac{y_{m(t=24hr)}}{y_{m(t=54hr)}} = 0.875$ and $\frac{x_{s(t=24hr)}}{x_{s(t=54hr)}} = 0.846$. These figures dictate that the scour hole reaches to an quasi-equilibrium stage after 24 hours leaving the scour process to continue with removal of fine grains out of the hole. In this quasi-equilibrium stage, the maximum depth of scour hole becomes approximately constant, while the scour profile continues to change. On the other hand, the time required to reach the equilibrium stage, depends on the flow characteristics (e.g. approaching Froude number and tailwater depth), flow geometry and sediment size. Therefore, occurrence a complete equilibrium phase after 24 hours may not be a realistic assumption. Figure 4-b shows the rate of the extension of the maximum scour depth. It is seen that the scour process is very rapid at the beginning while within the first five hours, the rate of maximum scour depth considerably decreases. So, it seems that 24 hours is plausible considering the laboratory limitations(what do you want to say?!!). Balachandar et al. [5] reported that the equilibrium scour profiles were not attained even after 96 hours. Considering the above findings, the following relation was obtained from experimental data after 24 hours to estimate the maximum quasi-equilibrium scour depth as:

$$\frac{y_{me}}{w} = 0.38(\tan\theta)^{-0.022}\left(\frac{y_t}{w}\right)^{-0.964}\left(\frac{L_B}{w}\right)^{-0.695}(Fr_1)^{1.465}\left(\frac{D_{50}}{w}\right)^{-0.418} \qquad (4)$$

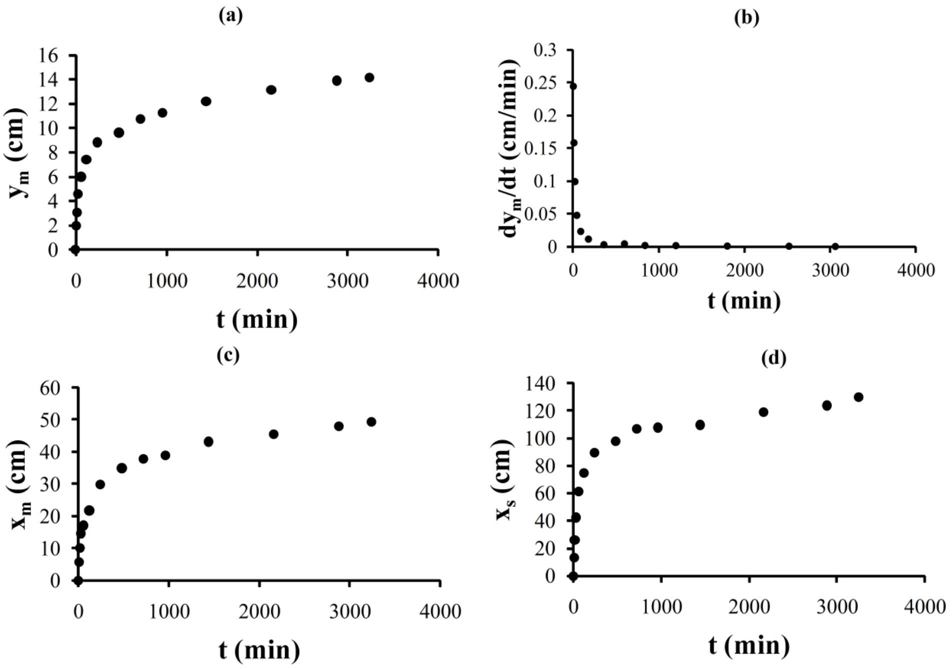

Fig. 4 (a), (c), (d) Time variation of scour hole dimensions for 54 hr test, (b) Time rate of maximum scour depth

The regression coefficient (R^2) between the experimental observations and the computed values was found to be 0.92 (standard error (SE)=0.25), indicating that Eq. (4) satisfactorily agrees with the experimental data. It should be noted that the Eq.4 is valid for the range of $0.0129<\tan\theta<0.156$, $1.74<y_t/w<10.5$, $4.21<Fr_1<11.14$, $47.24<L_B/w<84.3$, $0.023<D_{50}/w<0.127$. Eq. (4) demonstrates that, the maximum depth of scour increases

[DOI: 10.22068/IJCE.13.1.1]

with incoming Froude number while, decreases with sediment size, length and slope of stilling basin. It can be also concluded that, the maximum scour depth downstream of adverse stilling basin decreases with tailwater depth which contradicts with the report Dey and Sarkar [6] for horizontal stilling basin (see Table 1). From the experimental records downstream of horizontal rigid apron (zero slope), the following expression was revealed:

$$\frac{y_{me}}{w} = 0.732\left(\frac{y_t}{w}\right)^{0.980}\left(\frac{L_B}{w}\right)^{-0.532}(Fr_1)^{0.482}\left(\frac{D_{50}}{w}\right)^{-0.245} \quad (5)$$

Equation (5) can be used for estimating the maximum scour depth downstream of horizontal stilling basin with (R^2 =0.881) and (SE =0.451) in the range of $5<y_t/w<15.19$, $3.78<Fr_1<10.52$, $47.24<L_B/w<84.3$, $0.023<D_{50}/w<0.127$. It can be seen that the maximum scour depth downstream of horizontal bed protection increases with the tailwater depth which is in agreement with the equation previously reported by Dey and Sarkar [6].

3.2. Local scour downstream of horizontal stilling basins under submerged flow

Figure 7-a shows the scour hole in the equilibrium state due to the submerged jet issuing from the sluice gate *upstream a horizontal bed protection.* The forces on a control volume, between the gate and downstream section, are as follow (see Fig. 7-b):

- The force due to the hydrostatic pressure distribution adjacent to the gate $F_{P1} = \gamma b \frac{y_3^2}{2}$,
- The force due to the hydrostatic pressure distribution at the downstream section $F_{P2} = \gamma b \frac{y_t^2}{2}$,
- Weight of water in control volume $W = \gamma b \forall_{C.V.}$, in which γ is the specific weight of water, b is the width of gate, $\forall_{C.V.}$ is the volume of fluid, y_3 and y_t are the flow depths as shown in Fig. 7-a.
- The drag force acting on the bed of scour hole
- $F_D = \frac{1}{2}\rho C_D\left(\frac{\pi D^2}{4}\right)\left(u_f^2\right)$,

  in which C_D is the drag coefficient, D is the particle size, u_f is the flow velocity at the center of particle. It should be noted that the lift force depends on the same variables as the drag force and therefore the effect of the lift force is automatically taken into account. The drag force in the scour hole can be defined as (Graph [17]):

$$F_D = \frac{1}{2}\rho C_D b x_{se}\left(u_b^2\right) \quad (6)$$

where x_s is the length of the scour hole and u_b is the flow velocity near the bed. It should be noted that the length of upstream face of the scour hole is relatively small compared with the downstream side (Hoffmans [10]). It was observed that the upstream face of scour hole as attains 1/3 of the maximum length of scour hole after 48 hr as shown in Fig. 3.

To define a relationship for length of scour hole, the observed dimensionless length of scour hole ($\frac{x_{se}}{w}$) was depicted against the dimensionless maximum depth of scour hole ($\frac{y_{me}}{w}$) at different slopes of adverse basin (S_o) in Figs. 5 which could be expressed in the form of $\frac{x_{se}}{w} = \psi . \frac{y_{me}}{w}$ where ψ could be defined as $\psi = \frac{x_{se}}{y_{me}}$. The values of $\psi = \frac{x_{se}}{y_{me}}$ with S_o are plotted in Fig. 6 which could be concluded as:

$$\psi = 2.107 a\tan(44.61 S_o) + 6.95 \quad (7)$$

It is clear from Eq. (7) that the values of x_{se} would be related to y_{me} as:

$$x_{se} = \psi y_{me} \quad (8)$$

The flow velocity near the bed could be related to the maximum flow velocity of submerged jet in a distance x from the gate ($u_m(x)$) as $u_b = k u_m(x)$. Hassan and Narayanan [3] presented the following equations for estimation of the maximum flow velocity in the scour hole:

$$\begin{cases} u_m(x) = e^{-0.138}\left[3.83 u_1\left(\frac{L_B}{y_1}\right)^{-0.5}\right]\left(\frac{x}{\delta_{1A}}\right)^{1.055}, 0 \prec \frac{x}{\delta_{1A}} \prec 4.28 \\ u_m(x) = 0.01\left[3.83 u_1\left(\frac{L_B}{y_1}\right)^{-0.5}\right]\left(\frac{x}{\delta_{1A}}\right)^{-0.1}, \frac{x}{\delta_{1A}} \succ 4.28 \end{cases}, \delta_{1A} = y_1\left(0.5 + 0.065\frac{L_B}{y_1}\right) \quad (9)$$

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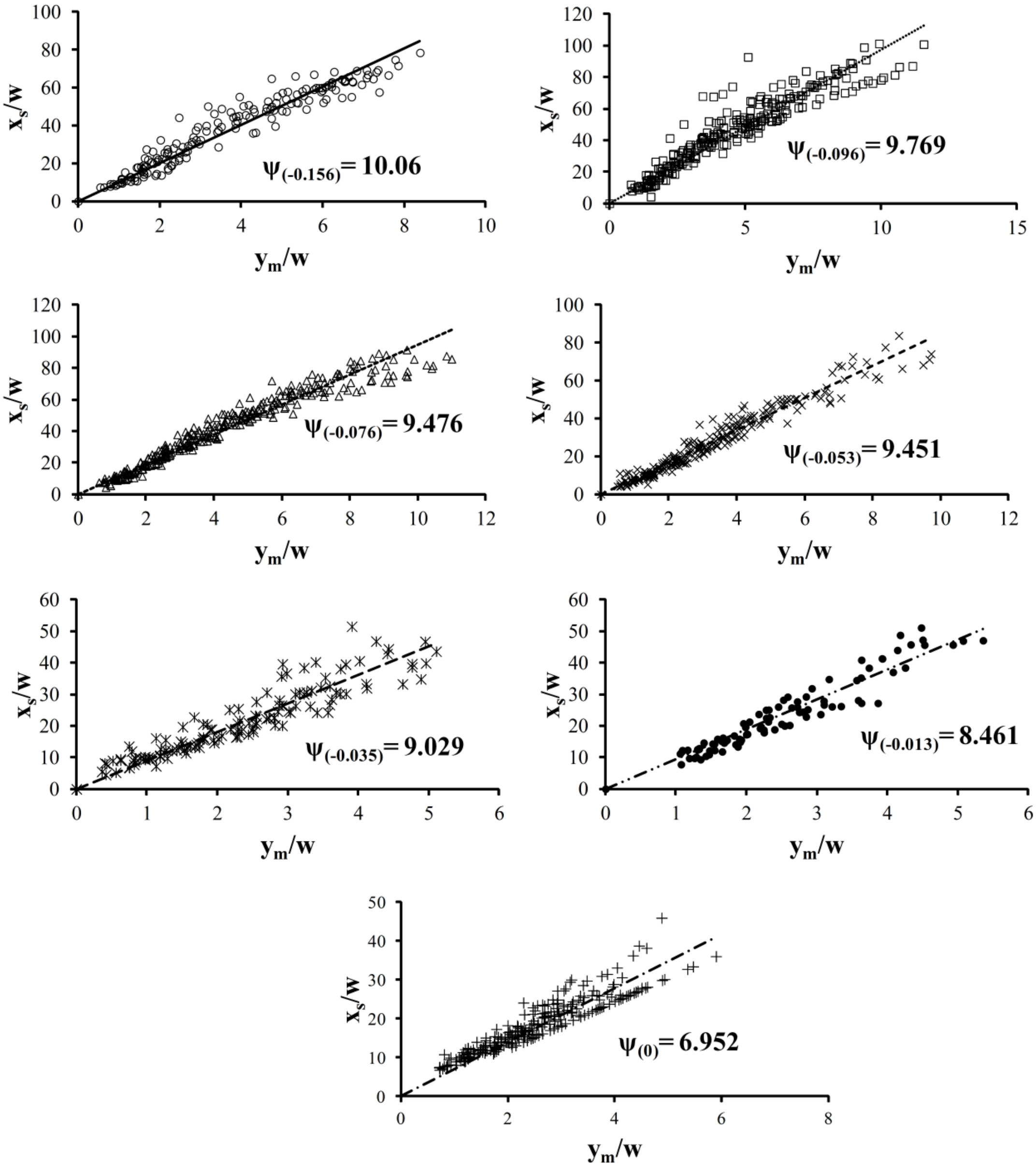

Fig. 5 Determination of ψ at different slopes of stilling basin

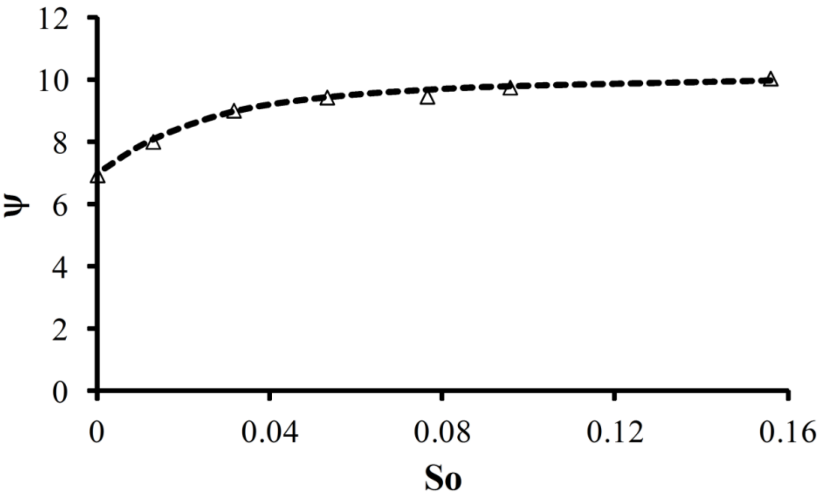

Fig. 6 Variation of ψ with S_o

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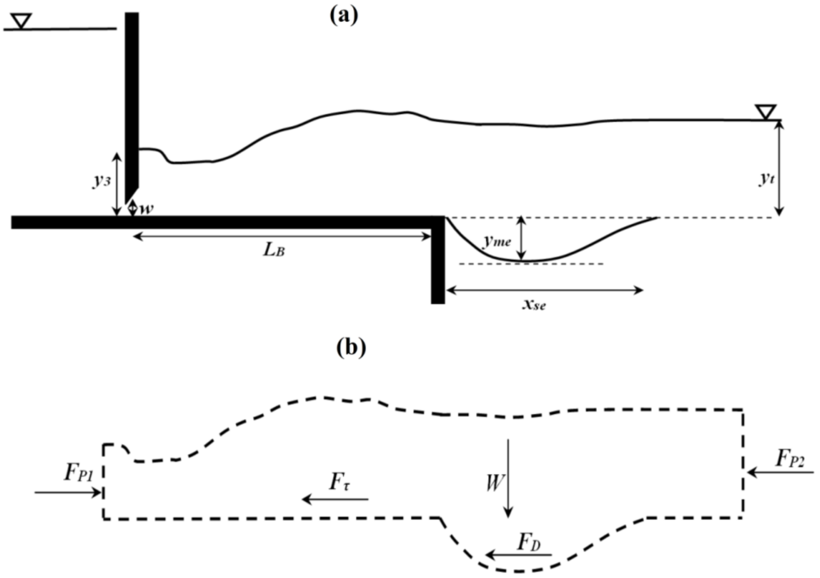

Fig. 7 (a) Scour hole downstream of submerged sluice gate with horizontal stilling basins, (b) Effective forces on the control volume after the gate

where $u_1 = \dfrac{q}{C_c w}$ is the flow velocity at the vena contracta (C_c is the contraction coefficient). Consequently,

$$u_b = k_1 u_1 \rightarrow F_D = \frac{1}{2}\rho C_D b y_{me} \psi \left(k_1^{\,2}\right)\left(u_1^2\right) \tag{10}$$

The force due to the bed shear stress (F_τ): Ead and Rajaratnam [18] reported a direct measurement of shear stress distribution due to the submerged jet issuing from a gate opening in shallow tailwater conditions. They presented the following equation to estimate the shear force at any distance (x) from the gate ($F_{\tau x}$):

$$\frac{F_{\tau_x}}{M_0} = -0.663\times10^{-9}\left(\frac{x}{w}\right)^4 + 0.272\times10^{-6}\left(\frac{x}{w}\right)^3 - 0.412\times10^{-4}\left(\frac{x}{w}\right)^2 + 0.320\times10^{-2}\left(\frac{x}{w}\right) \tag{11}$$

where $M_0 = \rho b q V_0 = \rho b \dfrac{q^2}{w}$. Integrating Eq. (11) gives the total bed shear force along the stilling basin as:

$$F_{\tau(x=L_B)} = \rho b \frac{q^2}{w}\int_{x=0}^{x=L_B}\left[\begin{array}{l} -0.663\times10^{-9}\left(\dfrac{x}{w}\right)^4 + 0.272\times10^{-6}\left(\dfrac{x}{w}\right)^3 - 0.412\times10^{-4}\left(\dfrac{x}{w}\right)^2 \\ +0.320\times10^{-2}\left(\dfrac{x}{w}\right)\end{array}\right]dx = \rho b \frac{q^2}{w} I \tag{12}$$

where I is given as:

$$I = -0.133\times10^{-9}\left(\frac{L_B}{w}\right)^5 + 0.068\times10^{-6}\left(\frac{L_B}{w}\right)^4 - 0.138\times10^{-4}\left(\frac{L_B}{w}\right)^3 + 0.160\times10^{-2}\left(\frac{L_B}{w}\right)^2 \tag{13}$$

The momentum equation between the sections adjacent to the gate and that of tailwater could be expressed as:

$$\sum F_x = \rho Q \Delta V_x \rightarrow F_{P1} - F_{P2} - F_D - F_\tau = \rho Q\left(V_2 - V_1\right) = \rho b q^2\left(\frac{y_1 - y_t}{y_1 y_t}\right) \tag{14}$$

Introducing the foregoing relationships in Eq. (14) and assuming $F_{P1} \approx F_{P2}$, a semi-theoretical equation could be obtained to estimate the maximum equilibrium depth of scour as:

$$y_{me} = \frac{K C_c^{\,2} w}{\psi}\left(\frac{y_t - C_c w}{C_c y_t} - I\right) \tag{15}$$

[DOI: 10.22068/IJCE.13.1.1]

where K is $K = \frac{2}{C_D\left(k_1^2\right)}$. It is noteworthy to mention that the flow discharge and the characteristics of the sediment are not included in Eq. (15). Using the Buckingham Theorem, the unknown K could be defined as $K = f\left(q, w, D_{50}, S_s\right) = f\left(F_o\right)$. Using the experimental results, the K values are plotted against F_0 in Fig. 8 which could be expressed as:

$$K = 2.402 F_0^{0.779} \qquad (16)$$

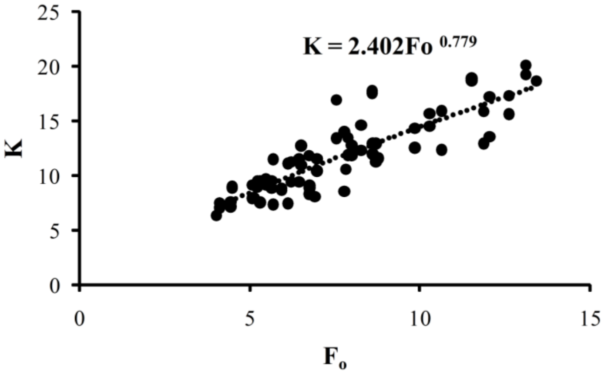

Fig. 8 Variation of K with densimetric Froude number

Finally a comparison is made between Eq. (15) and other researches related to horizontal aprons and results are presented in Figure 9 together with Table 3. The ratio of the calculated maximum local scour depth to its experimental value ($r = \frac{\left(y_{me}/w\right)_{cal}}{\left(y_{me}/w\right)_{\exp}}$), was used to evaluate the strength of each method in predicting the scour depths. Table 3 shows that the equations proposed by Altinbilek and Basmaki [9] and Hoffmans [10] may overestimate the maximum local scour depths. Whereas, the proposed equations by Hamidifar et al. [11] and Chatterjee et al. [4], relating to a sufficient length of hydraulic jump, would underestimate the maximum local scour depth in certain cases. It can be observed from Fig. 9 and Table 3 that Eqs. (5) and(15) would precisely predict the values of scour depth in which the length of stilling basin is one of the effective parameter. The regression relationships and their parameters are extremely dependent on the range of experimental data and sensitive to any changes in the experiment conditions. While the momentum approach would give a more general form of the phenomenon.

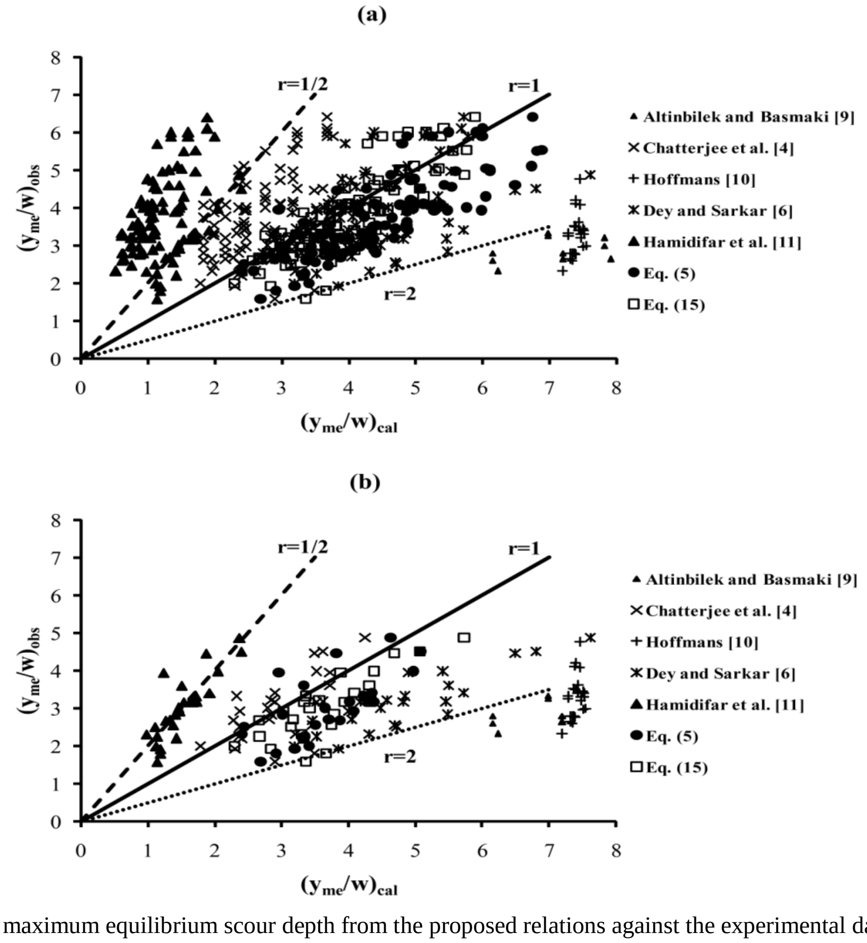

Fig. 9 Comparison of the maximum equilibrium scour depth from the proposed relations against the experimental data from this research for (a) $\frac{L_B}{w} = 47.2, 59, 84.3$, (b) $\frac{L_B}{w} = 84.3$

[DOI: 10.22068/IJCE.13.1.1]

Table 3 Comparison between proposed equations based on experimental data from this study

Proposed method	*r>1.5*	*0.9<r<1.1*	*0.5<r<1.5*	*r<0.5*
Eq. (5)	9.18	18.37	90.82	0
Eq. (15)	1.02	55.10	98.98	0
Dey and Sarkar [6]	13.26	39.80	86.74	0
Hamidifar et al. [11]	0	0	15.31	84.69
Chatterjee et al. [4]	2.04	15.31	92.86	5.10
Altinbilek and Basmaki [9]	100	0	0	0
Hoffmans [10]	97.96	0	2.04	0

3.3. Local scour downstream of adverse stilling basins under submerged flow

The equilibrium scour hole downstream of the adverse stilling basin is shown in Figure 10-a. Neglecting the shear force on the adverse stilling basin, the forces acting on the control volume are (see Fig. 10-b):

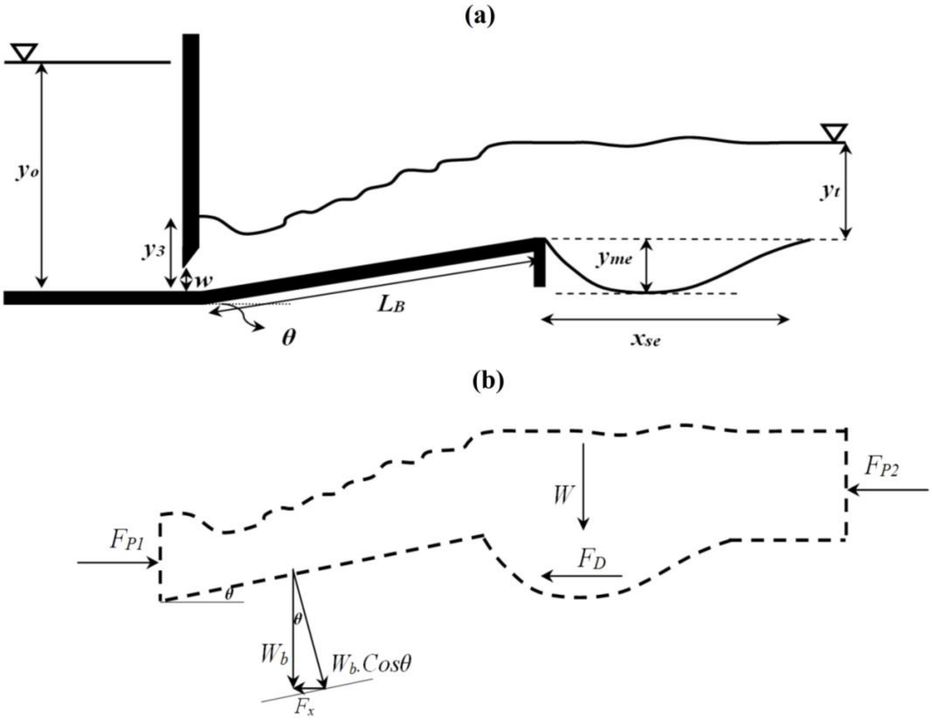

Fig. 10 (a) Scour hole downstream of submerged sluice gate followed by an adverse slope stilling basins, (b) Effective forces on the control volume downstream of the sluice gate

- The pressure force immediate to the gate; $F_{P1} = \frac{\gamma b}{2} y_3^2 Cos^3\theta$,
- The pressure force at the tailwater section; $F_{P2} = \frac{\gamma b}{2} y_t^2$,
- The horizontal force along the stilling basin;

$$F_x = W_b Sin\theta.Cos\theta = \kappa \frac{\gamma b L_B}{2}\left(y_3 + y_t\right) Sin\theta.Cos^2\theta \qquad (17)$$

The drag force acting in scour hole;

$$F_D = \frac{1}{2}\rho C_D'' b y_{me} \psi\left(k''^2\right)\left(u_1^2\right) \xrightarrow{\xi = c_D'' k''^2} F_D = \frac{1}{2}\xi \rho b y_{me} \psi\left(u_1^2\right) \qquad (18)$$

where y_3 is the flow depth immediately after the gate, θ is the bed angle, W_b is the weight of water over the adverse stilling basin, and κ is the ratio of water weight in the control volume to that is determined by assuming a linear profile for hydraulic jump. Pagliara and Peruginelli [13], from their experimental work, concluded that the κ values are neither dependent on bed slope nor on approaching Froude number and attains a constant value of $\kappa = 1.06$. Assuming $\kappa = 1$, The momentum equation in the horizontal direction may be written as:

$$\sum F_x = \left(F_{P1x}\right) - \left(F_{p2} + F_x + F_{P2} + F_D\right) = \rho b q\left(V_2 - V_{1x}\right) \qquad (19)$$

where $V_{1x} = V_1.Cos\theta = \frac{q}{y_1}.Cos\theta$, $V_2 = \frac{q}{y_t}$.

Introducing foregoing relationships in Eq. (19), results in a theoretical equation to estimate the maximum

[DOI: 10.22068/IJCE.13.1.1]

equilibrium local scour depth (y_{me}) as a function of relative submergence ($s=\frac{y_3}{y_1}$), relative tailwater depth ($s'=\frac{y_t}{y_1}$), dimensionless length of stilling basin (L_B/y_1), angle of bed slope (θ), and approaching Froude number ($Fr_1=\frac{q}{\sqrt{gy_1^3 Cos\theta}}$) as:

$$\frac{y_{me}}{y_1}=\frac{1}{\xi\psi Fr_1^2 Cos\theta}\left[s^2 Cos^3\theta+2Fr_1^2 Cos\theta\left(1-\frac{1}{s'}\right)-s'^2-\frac{L_B}{y_1}Sin\theta Cos^2\theta\left(s+s'\right)\right] \quad (20)$$

Applying the energy equation before and after the gate and neglecting the energy losses, the relative submergence could be obtained as $s=\frac{1}{C_c a_r}-\frac{Fr_1^2 Cos\theta}{2}$, where $a_r=\frac{w}{y_o}$. As a result:

$$\frac{y_{me}}{y_1}=\frac{1}{\xi\psi Fr_1^2 Cos\theta}\left[\left(\frac{1}{C_c a_r}-\frac{Fr_1^2 Cos\theta}{2}\right)^2 Cos^3\theta+2Fr_1^2 Cos\theta\left(1-\frac{1}{s'}\right)-s'^2-\frac{L_B}{y_1}Sin\theta Cos^2\theta\left(\frac{1}{C_c a_r}-\frac{Fr_1^2 Cos\theta}{2}+s'\right)\right] \quad (21)$$

For the hydraulic jump on the horizontal stilling basin ($\theta=0$), Eq. (21) reduces to:

$$\frac{y_{me}}{y_1}=\frac{1}{\xi\psi Fr_1^2}\left[\left(\frac{1}{C_c a_r}-\frac{Fr_1^2}{2}\right)^2+2Fr_1^2\left(1-\frac{1}{s'}\right)-s'^2\right] \quad (22)$$

which can be compared with Eq. (15). It is noteworthy to mention that, in Eqs. (20),(21) and (22) the physical characteristics of sediment is not included . Using Buckingham Theorem, the unknown parameter $\frac{1}{\xi}$ could be expressed as $\frac{1}{\xi}=f\left(D_{50}^*\right)$. Using the observed experimental data, the function of $f\left(D_{50}^*\right)$ could be determined by the following equation (see Fig. 11-a):

$$f\left(D_{50}^*\right)=1.08-0.005D_{50}^*+0.00008\left(D_{50}^*\right)^2 \quad (23)$$

Figure 11-b compares the accuracy of Eqs. (4),(5),(15)and(21) with experimental data. Although the above semi-theoretical equation and the proposed regression relation (Eq. 4) have a similar predictability, but one could realize that the semi-theoretical equation has a more general form.

3.4. *Effects of different parameters on the maximum equilibrium scour depth*

The proposed semi-theoretical equation can be used to understand the effects of the geometry of stilling basin, the approaching Froude number and the tailwater depth on maximum depth of scour. Eq. (21) shows that the maximum scour depth decreases with $\frac{L_B Sin\theta}{y_1}$. At the end of the adverse stilling basins, the streamlines incline toward the water surface and then impinge toward the bed. As the length or bed slope of stilling basin increases, the excess shear stress decreases and the position of impinging point moves downstream of scour hole Which results in a reduced maximum scour depth downstream of the adverse stilling basins ,as shown in Fig. 12. Figures. 11-b and 13-a show the effect of tailwater depth on the maximum scour depth for $S_o=0.096$ and $S_o=0.032$, respectively. For certain values of the densimetric Froude number ($F_o=\frac{q}{\sqrt{g(s_s-1)D_{50}^3}}$), length, and slope of stilling basin, a critical value for tailwater depth $\left(y_t\right)_\alpha$ exists which corresponds to a maximum value of equilibrium scour depth. It is shown that the scour depth increases with the tailwater depth reaching a maximum value where after decreases to attain a constant value. Figure 14 depicts the variations of the maximum scour depth as a function of the tailwater depth downstream of the horizontal stilling basins for different apron lengths and densimetric Froude numbers. It is shown that a certain value of the tailwater depth $\left(y_t\right)_\beta$ exists corresponding to a minimum value of the equilibrium scour depth which was previously reported by Dey and Sarkar [6]. This may be attributed to instability of the hydraulic jump on the adverse slope stilling basins. McCorquodale and Mohamed [12] reported that this type of jump is difficult to establish at Froude numbers lower than 9 and requires continuous tailwater adjustment to maintain a stationary position, at Froude numbers lower than about 4. The instability of the hydraulic jump over the adverse stilling basins could be balanced by the acting forces on the stilling basins. The hydraulic jump over the adverse slopes would stand on stilling basin if the repelled forces (pressure force at the upstream side) are less than

[DOI: 10.22068/IJCE.13.1.1]

the counteracting forces (pressure force at the tailwater section and the horizontal bed force along the stilling basin). At low tailwater depths, the hydrodynamic force, after the gate, would be dominant. Consequently, the hydraulic jump moves downstream and results in a considerable erosion. As the tailwater depth increases, the pressure force at the end of the stilling basin increases. Therefore, the maximum scour depth decreases to reach a constant value. It is expected that this trend can be explained by the proposed semi-theoretical equation . Differentiating Eq. (21) with respect to the relative tailwater depth ($s' = \frac{y_t}{y_1}$) and assuming that at critical tailwater depth , $\frac{d}{ds'}\left(\frac{y_m}{y_1}\right) = 0$, one could achieve:

$$\left(\frac{y_t}{y_1}\right)_\alpha = -\frac{L_B}{6y_1} Sin\theta Cos^2\theta + \sqrt[3]{q+\sqrt{D}} + \sqrt[3]{q-\sqrt{D}} \qquad (24)$$

where D is:

$$D = p^3 + q^2, p = -\frac{\left(\frac{L_B}{2y_1} Sin\theta Cos^2\theta\right)^2}{9}, q = \frac{27Fr_1^2 Cos\theta - 2\left(\frac{L_B}{2y_1} Sin\theta Cos^2\theta\right)^2}{54} \qquad (25)$$

Figure 15 depicts the effects of the stilling basin's length and bed slope on the critical tailwater depth using Eq. (24) for $Fr_1 = 9$. It is shown that the critical tailwater depth decreases with the bed slope and increases with the length of stilling basin. It should be mentioned that the regression relationship (Eq. 4) cannot predict the two folds variations of scour depth with tailwater depth. Consequently, the semi-theoretical equation is recommended to be used for prediction of maximum scour depth.

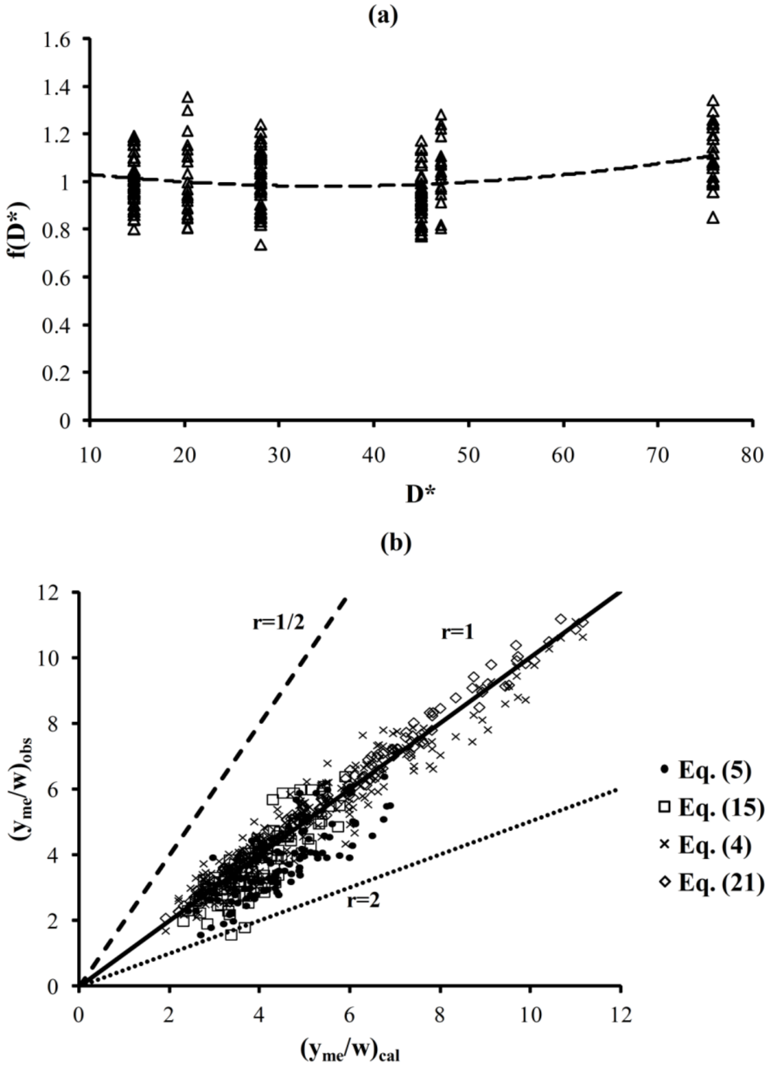

Figure 11 (a) $f\left(D^*_{50}\right)$ as function of D^*_{50}, (b) Comparison accuracy of proposed equations

[DOI: 10.22068/IJCE.13.1.1]

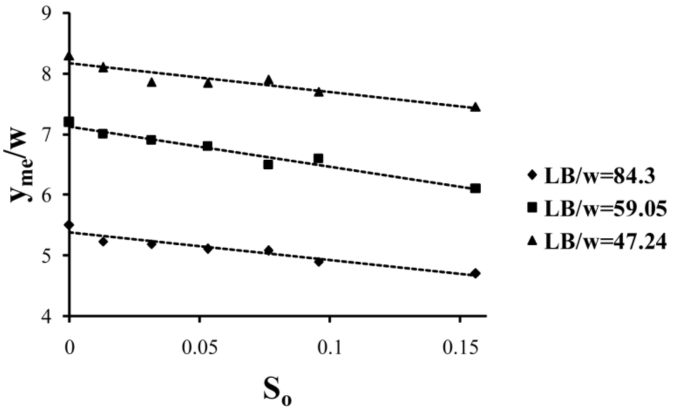

Fig. 12 Effects of slope and length of stilling basin on the maximum equilibrium local scour depth ($F_0 = 12, \frac{y_t}{w} = 6$)

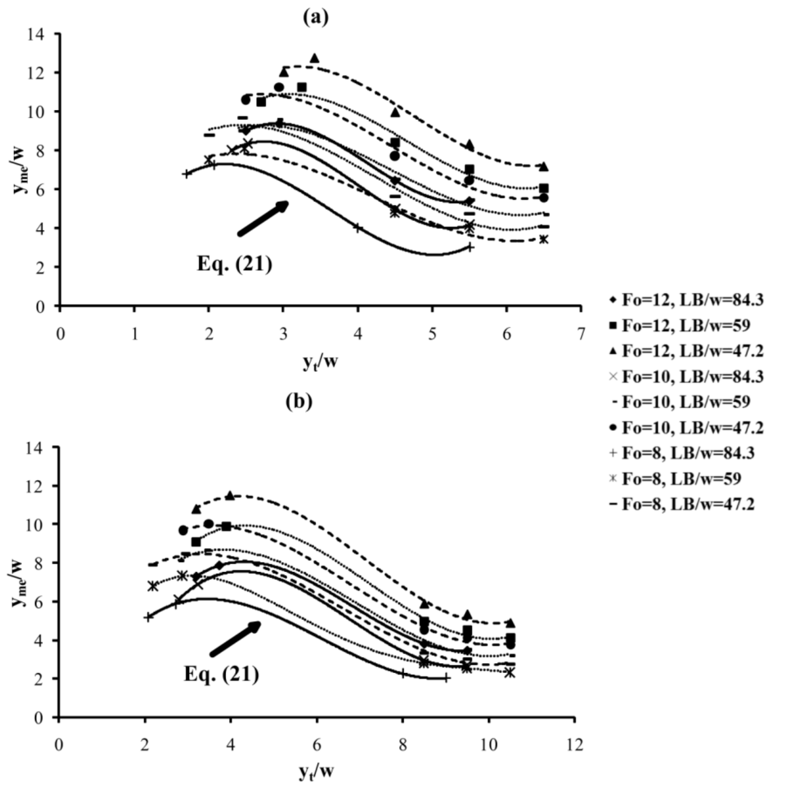

Fig. 13 Effect of tailwater depth on the maximum sour depth for (a) $S_o = 0.096$, (b) $S_o = 0.032$

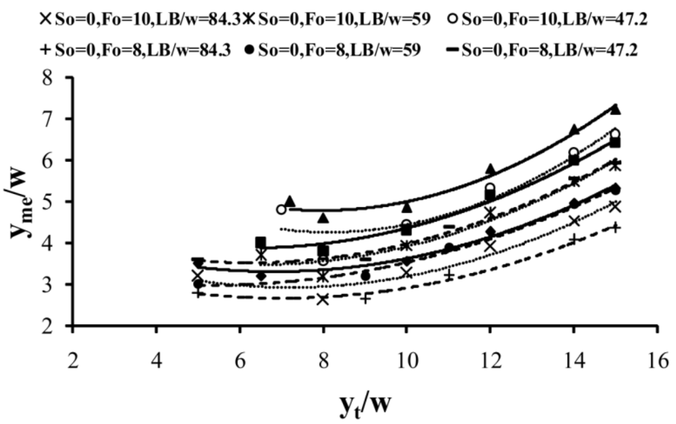

Fig. 14 Effect of tailwater depth on the maximum sour depth for horizontal stilling basin

[DOI: 10.22068/IJCE.13.1.1]

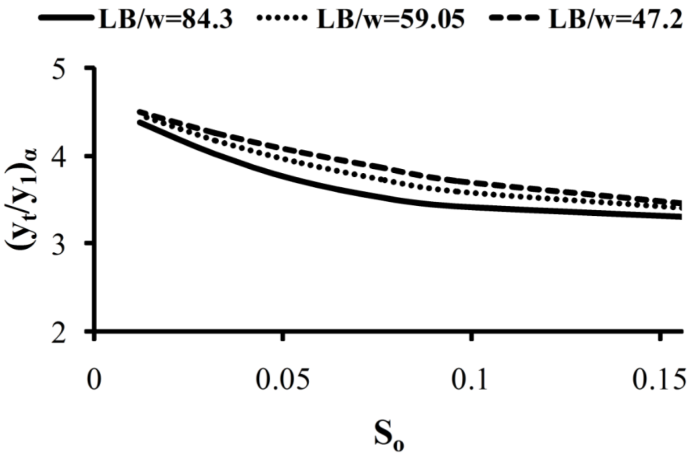

Fig. 15 Effects of slope and length of stilling basin on the critical tailwater depth

4. Conclusions

Using the momentum principle, some theoretical equations were derived to estimate the maximum depth of scour downstream of horizontal and adverse stilling basin. The proposed equations were verified by experimental observations. The proposed semi-theoretical equations showed an acceptable accuracy. The regression relationships and encountered parameters are extremely dependent on the range of experimental data and vary with any changes in the experimental conditions. In the case of adverse stilling basins, it was shown that the scour depth increases with tailwater depth reaching a maximum value where after decreases to attain a constant value. Also, it was observed that the maximum depth of the scour hole decreases as the length and slope of stilling basin increases. The proposed semi-theoretical equation can be used to predict the critical tailwater depth. In present work, a semi-theoretical method based on momentum principle was presented to predict semi-maximum depth of scour hole downstream of horizontal and adverse stilling basins which forms the core of the work. Since the achievement has a theoretical basis, the proposed relationship could reflect the rate of influences induced by different parameters on the phenomenon. Therefore, the results would be adapted to downstream of any hydraulic structures.

Notation

a_r	Relative gate opening ($a_r = \frac{w}{y_o}$)
b	Gate width
C_c	Contraction coefficient
C_D	Drag coefficient
C_u	Uniformly coefficient
D^*	Sedimentological diameter
D_{50}	50% finer sediment size
F_D	Drag force in the scour hole
F_o	Densimetric Froude number
F_{P_1}	Force due to hydrostatic pressure distribution after the gate
F_{P_2}	Force due to hydrostatic pressure distribution at the tailwater section
Fr_1	Approaching Froude number
F_τ	Force due to the bed shear stress
g	Acceleration due to the gravity
L_B	Length of stilling basin
q	Flow discharge per unit width of the rectangular basin
S_o	Slope of stilling basin
s	Relative submergence of the sluice gate ($s = \frac{y_3}{y_1}$)
s_s	Relative density of sediments
s'	Relative tailwater depth ($s' = \frac{y_t}{y_1}$)
u_o	Jet velocity at the gate opening
w	Gate opening
W	Weight of water
W_b	Weight of water on the adverse stilling basin
x_m	Horizontal distance of maximum scour depth from the stilling basin at time t
x_{me}	Horizontal distance of maximum equilibrium scour depth from the stilling basin
x_s	Length of scour hole at time t
x_{se}	Length of scour hole at equilibrium stage
y_o	Flow depth at upstream of the gate
y_1	Flow depth at the *vena contracta*
y_3	Flow depth immediately after the gate
y_m	Maximum scour depth at time t
y_{me}	Maximum equilibrium local scour depth
y_t	Tailwater depth
$(y_t)_\alpha$	Critical tailwater depth at the presence adverse stilling basin

[DOI: 10.22068/IJCE.13.1.1]

$(y_t)_\beta$	Critical tailwater depth at the presence horizontal stilling basin
θ	Angle of the bed slope
ρ	Mass density of fluid
ρ_s	Mass density of sediments
ϕ	Angle of repose
ν	Fluid kinematic viscosity
γ	Specific weight of water
σ_g	Geometric standard deviation

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[DOI: 10.22068/IJCE.13.1.1]