1. Introduction

Every year, a great deal of financial damages and casualties occur during earthquakes due to the sliding of slopes in various parts of the world. Iran is a mountainous country located on a seismic zone, where many buildings are constructed on natural slopes. Most of these slopes include weathered rock or soil mass on inclined bedrock.

Seismic Slope stability analyses mainly consist of pseudo statistic methods, dynamic analysis and simplified methods, which are based on estimation of an index displacement.

Engineers have used the pseudo static approach to analyze the seismic stability of earth structures since the 1920s (i.e. [1]). This method involves the computation of minimum factor of safety against sliding, considering the earthquake effects by consideration of horizontal and vertical forces. These horizontal and vertical forces are usually expressed as the product of a horizontal and/or a vertical seismic coefficient and the weight of the sliding mass. The horizontal pseudo static force decreases the factor of safety by reducing the resisting force and increasing the driving force. The vertical pseudo static force typically has less influence on the factor of safety since it affects positively (or negatively) both the driving and resisting forces and for this reason, many engineers ignore this. The factor of safety of a slope critically depends on the value of the horizontal seismic coefficient $k_h$.

In pseudo-statistic analysis, like other limit state methods, a safety factor is introduced, but no information is obtained about the deformation of the slope. Since the performance of a slope after an earthquake is dependant on its deformation during the earthquake, analyses that deal with these deformations may show a better index of the slope performance.

The most accurate method of stability and displacement evaluation may be considered as the time history dynamic analysis by numerical methods. In these methods, the permanent strains in each element accumulated during the earthquake and consequently the permanent deformation of the slope is calculated. Various constitute models are used in dynamic analyses; however these analyses, even in cases of
using the simplest models, may be complex and time-consuming.

Newmark [2] pioneered a simplified computation procedure in 1965 using acceleration data. The steps of Newmark procedure, which are used in this research, are:

- Identifying a critical potential sliding mass (block) and calculation the yield acceleration coefficient ($k_y$).
- Determining the time history of acceleration, $HEA(t)$ applied horizontally to the sliding mass through a seismic response analysis (by PLAXIS in this research).
- Calculation the seismically induced displacement of the sliding mass by numerically integrating twice the portions of $HEA(t)$ exceeding ($k_y$) to obtain permanent displacement.

Seismic analysis of natural slopes is usually assessed by simplified methods of determining the permanent displacements such as Newmark and Makdisi [3] methods. The seismic displacements obtained through these methods are considered as an index for the seismic performance. Despite the simplicity of the discussed methods, their precision is dependent on the evaluation of the acceleration time history in sliding mass.

In the last two decades, extensive investigations have been done about the application of these methods by Lin, Whitman [4], Ambrasyes and Menu [5], Jibson [6], Bray et al. [7], Kramer and Smith [8], Bray and Rathje [9], and Bray and Travasarou [10].

Bray and Rathje [8] used a procedure to develop a screen analysis for solid-waste landfills in 1998. They conducted their study on a special group of slopes, i.e. waste fill zone with typical geometry shown in Figure 1. They presented the permanent displacement by using Newmark method and by applying some selected records of U.S. earthquakes.

In this study, the development of the Bray and Rathje method for a typical geometrical model based on seismic data of Iran has been considered. In the following, a summary of Bray and Rathje method is given. Some strong ground motions of Iran have been selected and the acceleration time history of the sliding mass has been calculated. The effect of bedrock inclination on the acceleration records of the sliding mass has been assessed by employing an approach developed from Bray and Rathje method (explained in follow). Newmark method was employed to obtain the permanent seismic displacement of the selected slopes.

2. Bray and Rathje method

The method used by Bray and Rathje is a combination of one-dimensional dynamic analysis of waste fills and Newmark sliding block. The procedure normalizes the maximum horizontal equivalent acceleration ($MHEA$) in the slide mass by the product of maximum horizontal acceleration of rock ($MHA_{r}$) and a nonlinear response factor ($NRF$).

\[
T_s = \sum_{i} C_i (1/f_i)
\]

Parameter NRF accounts for nonlinear ground response effects.

The normalized acceleration is then related as shown in Figure 2 to the period of the sliding mass ($T_s$) normalized by the mean period of the input motion ($T_m$), defined as:

\[
T_m = \frac{\sum C_i (1/f_i)}{i}
\]

where $Ci$ are the square roots of the sum of the squared real and imaginary parts of the positive-frequency Fast Fourier Transform (FFT) coefficients, and $f_i$ are the discrete FFT frequencies from approximately 0.25 to 20 Hz.

The quantity $T_m$ represents the mean period of the earthquake and can be estimated from magnitude and distance. $T_s$ represents the fundamental period of the sliding mass.

In this method, the main features of earthquake including the maximum horizontal acceleration ($MHA$), mean period ($T_m$) and significant duration of shaking ($D_{5\text{-}95}$) have been employed. $D_{5\text{-}95}$ is the time between 5% and 95% of normalized Arias Intensity.

This method utilizes $MHEA$ to characterize the amplitude of shaking within the slide mass and $D_{5\text{-}95}$ to characterize the duration. Normalized displacements, defined as $U/k_{max}$, are related to $k_y/k_{max}$, as shown in Figure 3.

Bray and Rathje introduced the diagrams shown in Figures 2 and 3 and their method has been used in California as a guide.

**Fig. 1.** Geometry of slopes in Bray and Rathje studies

**Fig. 2.** Normalized maximum horizontal equivalent acceleration for base sliding versus normalized fundamental period of the waste fill (from Bray and Rathje 1998)

**Fig. 3.** Normalized base liner sliding displacement versus normalized critical acceleration (from Bray and Rathje 1998)
for assessing the earthquakes hazard.

Some of the most important features of Bray and Rathje method are as follows:

A) Study on waste fills with section Shown in Figure 1.
B) The natural period of sliding mass was calculated using the formula $T_s = 4H/V_s$. Bray recommended using this formula to calculate $T_s$ even for other slopes; $H$ is shown in Figure 1.
C) The critical slope acceleration coefficient is considered arbitrary.
D) Dynamic analyses are performed 1- dimensionally and by using the linear equation methods.

Bray and Rathje analyzed six landfills with various heights and compared the displacements obtained with the displacements measured in several landfills in Loma Prieta and North Bridge (1994) earthquakes.

These diagrams can be used in performance based design of slopes. First, $T_s$, $T_m$, and $MHAr$ need to be evaluated from location of the site in which the slope is being studied and specifying the slope geometry, then $MHEA$ is obtained using Figure 2 and finally $k_{max}$ will be obtained. The critical acceleration coefficient of the intended slope is determined through pseudo-static analysis by. Using Figure 3, the permanent displacement of the slope is determined and may be compared with allowable displacement.

3. General aspects of the present study

In this study, the development of the method used by Bray and Rathje for a typical geometrical model (shown in Figure 4) based on seismic data of Iran is considered and the effect of bedrock inclination, which was not studied in Bray and Rathje research, is investigated.

To this aim, a two dimensional model of a typical slope was considered and conducting dynamic analyses, the slope performance was studied for different geometries, strength parameters and shear wave velocities. Such a performance has been studied by assessing the record of acceleration in sliding mass (the mass above the critical sliding surface) and calculating the slope displacement using Newmark method.

Due to the limited number of strong ground motion records for a specific area in Iran, records related to the earthquake events in whole country were gathered and 14 of them related to 6 earthquakes with the largest magnitudes and accelerations were chosen for dynamic analyses.

A range of soil strength parameters and shear wave velocities were considered and the slopes were analyzed. The typical geometry of the slopes, analyzed in this study is shown in Figure 4. To do the analysis the above-mentioned slopes have been modeled as shown in Figure 5. In this model, a slope with height of $h$ and angle of $\beta$ has been located on inclined bedrock at an angle of $\theta$. Difference between the lower level of critical surface and upper side ground of the slope is considered as $H$.

In each analysis, the critical acceleration (causes the safety factor to be equal to 1) and the related critical sliding surface have been identified using pseudo-static analysis. In next step, conducting non-linear dynamic analysis, the response to each of the selected records on the critical sliding surface is identified and the maximum acceleration in sliding mass is estimated. Then, the permanent displacement under the effect of each record is calculated using Newmark method, and the diagrams of the slope permanent displacement related to the equivalent acceleration of the sliding mass is offered.

The present study can be considered as an improvement of the method used by Bray and Rathje [8]. The main differences are:

a) Bray and Rathje conducted their study on waste landfills with geometry shown in Figure 1. It is clear that, in addition to the difference between the geometry of the employed models in each study, the sliding surface in their study was predetermined. In the present study, the sliding surface is first evaluated by pseudo-static analysis and later calculations are conducted on the obtained sliding surface. It should be noted that the bedrock surface is considered completely rough, so the friction angle between soil layer and bedrock is considered to be equal to the friction angle of the soil mass.

b) In Bray and Rathje studies the natural period of the sliding mass is calculated using the formula: $T_s = 4HV_s$ ($H$ has been calculated according to Figure 1) where in this study, the predominant period is identified by applying the harmonic records on the model, changing frequency and obtaining the resonant frequency. For slopes such as those employed in this study, Bray and Rathje recommended that $T_s$ should be calculated by $4H/V_s$, $H$ is shown in Figure 5. The validity of this assumption will be discussed later.

c) In Bray and Rathje method, the slope critical acceleration coefficient ($k_c$) was selected, while in this study the critical acceleration coefficients are obtained through pseudo-static analyses of the selected slopes.

d) The dynamic analyses in Bray and Rathje method were performed one-dimensionally; while the models employed in this study were analyzed two-dimensionally. The soil is considered as a Mohr-Coulomb elastoplastic material.

4. Methodology of the present research

In this section, more details of the present research are explained. Descriptions are directed to investigate the effect of bedrock inclination on slope performance.
4.1. Seismic records

Due to the limited number of available strong ground motion records in Iran for a specific area, records related to the earthquake events in whole country were studied and 14 of them related to 6 earthquakes, (Bam, Avaj, Zanijran, Deyhook, Tabas, Meymand and Ab-bar) with the largest magnitudes and accelerations were chosen for dynamic analyses [11, 12]. The features of the selected earthquakes are presented in Table 1.

4.2. Slope geometry and the soil parameters

The analyses were carried out for a range of slope geometries and shear strength parameters as follows. The slope angle and the internal friction angle (φ) were considered to be 35° and 30° respectively, γ/c and h/V_s were selected to be 45 and 0.03 sec. (h is shown in Fig. 5), the bedrock inclination angles were considered to be 0°, 5°, 10°, 15° and 20° and the bedrock was assumed to be rigid (E_s=2.55e17 kN/m²).

4.3. Critical sliding surface and critical acceleration (k_c)

Critical sliding surface and critical acceleration were determined by upper bound limit analysis method using the algorithm developed by Farzaneh and Askari [13, 14]. In this algorithm, the sliding mass is modeled by a transitional rigid block mechanism.

The critical horizontal acceleration coefficient, for which the safety factor is 1, has been identified and the coordinates of the critical surface have been specified. It should be noted that the vertical component of the earthquake is ignored.

4.4. Equivalent horizontal acceleration of the sliding mass

One of the important points in application of Newmark displacement method is consideration of the effect of slope geometry and soil strength on frequency contents of the acceleration earthquake records. In this study, the average response acceleration time history is evaluated in the critical sliding mass and employed for Newmark displacement analysis.

The PLAXIS software has been employed for dynamic analyses. A typical model employed in PLAXIS program is shown in Figure 6.

As mentioned earlier, elasto-plastic dynamic analysis using of Mohr-Coulomb model was employed in this study. Material damping in soils is frequency independent. However, for simplicity, Rayleigh method was used to include the material damping. To ensure accuracy, the input parameters of the Rayleigh damping (coefficients α and β) were determined for the desired damping ratio by consideration of the most important frequencies of the slopes (i.e. the first and second natural frequencies (ω_1 and ω_2)). These frequencies were determined by applying a harmonic record on the considered slope model. By changing the frequency, the first and second resonant frequencies of the slope were obtained. Then, by twice use of Equation 2 for the values ω_1 and ω_2, α and β can be determined.

\[
\alpha + \beta \omega_1^2 = 2 \omega_1 \xi \quad (2-a)
\]

\[
\alpha + \beta \omega_2^2 = 2 \omega_2 \xi \quad (2-b)
\]

Damping ratio ξ was assumed to be 5% and for each model, α and β were determined separately from Eqs. 1-a and 1-b. After determining α and β, time history of acceleration for each earthquake record has been specified in 6 locations on sliding surface (Figure 7). Then, using Equation 3, the average time history of earthquake acceleration in critical sliding mass was obtained as:

<table>
<thead>
<tr>
<th>Station</th>
<th>Long.</th>
<th>Lat.</th>
<th>Date</th>
<th>P.G.A.</th>
<th>Predominant Frequency (Hz)</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deyhook</td>
<td>57.5</td>
<td>33.29</td>
<td>16/09/1978</td>
<td>410</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>Tabas</td>
<td>56.92</td>
<td>33.58</td>
<td>16/08/1978</td>
<td>897</td>
<td>5</td>
<td>4.2</td>
</tr>
<tr>
<td>Ab bar</td>
<td>48.97</td>
<td>36.92</td>
<td>20/06/1990</td>
<td>635</td>
<td>6.3</td>
<td>8.3</td>
</tr>
<tr>
<td>Meymand</td>
<td>52.75</td>
<td>28.87</td>
<td>20/06/1994</td>
<td>503</td>
<td>5.6</td>
<td>4.5</td>
</tr>
<tr>
<td>Zanjiran</td>
<td>52.62</td>
<td>29.07</td>
<td>20/06/1994</td>
<td>1006</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Avaj</td>
<td>49.22</td>
<td>35.58</td>
<td>22/06/2002</td>
<td>498</td>
<td>5</td>
<td>4.2</td>
</tr>
<tr>
<td>Bam</td>
<td>58.33</td>
<td>29</td>
<td>26/12/2003</td>
<td>989</td>
<td>5</td>
<td>4.5</td>
</tr>
</tbody>
</table>

**Table 1.** Ground motions used in this study
In this equation, \( HEA(t) \) is the equivalent horizontal acceleration at time \( t \) on the sliding block, \( HEA_i(t) \) is the equivalent horizontal acceleration at point \( i \) of the sliding surface and \( m_i \) is the mass of the material column located above the point \( i \). The maximum amount of \( HEA(t) \) was selected as \( M_{HEA} \) for the sliding mass. Figure 8 shows a sample of the response obtained from the analysis.

### 4.5. Permanent displacement

According to Newmark method, if we consider a block on a slope surface and if the earthquake generated force on the block is more than the resistance caused by the friction at the bottom of the block, the force difference will lead to the block displacement. This idea was first introduced by Newmark and signifies that during an earthquake, the sliding mass moves when the safety factor becomes less than 1. According to the calculated Newmark displacement, the performance of the slope may be evaluated. The slope permanent displacement was calculated based on Newmark method with critical acceleration obtained in Section 3.

### 5. Discussion on effect of bedrock inclination on slope seismic performance

To discuss the effect of bedrock inclination on seismic slope performance, a slope is considered for which the slope angle is 35°, the soil internal friction angle (\( \phi \)) is 30°, the ratio of height to shear wave velocity (\( h/V_s \)) equals to 0.3 sec. and \( \gamma h/c \) is 45. The inclination angle of the bedrock was changed from 0° to 20° and the stages 1 to 5 explained in previous section were applied to the model. In Figure 9, the obtained equivalent maximum acceleration in sliding mass, which was normalized to maximum acceleration of the input motion in bedrock (\( M_{HAR} \)), is shown for different bedrock inclination angles. In Figure 10 the values of the permanent displacement of the...
sliding mass \((U)\) for the different bedrock inclination angles are shown. As it is shown in Figure 9, the ratio of the equivalent maximum acceleration to the maximum acceleration in bedrock increases as the bedrock inclination angle increases up to 10°, and then it decreases. In fact, the greatest magnitude in the acceleration response was obtained for the angle of 10°. In addition, Figure 10 indicates that the permanent displacement of the sliding mass \((U)\) increases when the bedrock inclination angle increases up to 10°, and then it decreases. It is observed that this trend is compatible with changes in the ratio of equivalent maximum acceleration to in the bedrock (Figure 9).

Defining \(T_{m}\) as the mean period of the horizontal equivalent acceleration on sliding block \((\text{HEA}(t))\), the ratio of the Fundamental period of the slope \((T_s)\) to \(T_{m}\) was calculated for each bedrock inclination angle and for different records (Figure 11). As can be seen in Figure 11, the average values of \(T_s/T_m\) for the angles of 0°, 5°, 10°, 15° and 20° are 0.577, 0.674, 1.23, 1.08 and 0.89 respectively. It is seen that the ratio \(T_s/T_m\) is close to 1 at the angles of 10° and 15°. This might have led to amplification in these slopes and thus it can be the main reason for the increase in the acceleration response magnitude at angles 10° and 15°. In Figure 11 the variation of ratio of the equivalent maximum acceleration on sliding surface to the maximum acceleration in bedrock \((\text{MHEA/MHAr})\) in terms of the ratio of the predominant period of the slope to the mean period of the equivalent horizontal acceleration in sliding mass \((T_s/T_m)\) is presented. As it is observed, the greatest acceleration magnitudes happened at angles 10° to 15°.

Bray et al. used mean period of input motion in bedrock \((T_m)\) instead of \(T_{m}\). If we calculate the average of \(T_s/T_m\) for different angles 0°, 5°, 10°, 15° and 20° the results will be 1.42, 1.67, 2.24, 1.81, and 1.34 respectively. As it is observed, no comment can be given for amplification of the acceleration magnitudes by comparing these values.

It is worth mentioning that the predominant period of the slope \((T_s)\) has changed by variation of the bedrock inclination, as for the angles 0°, 5°, 10°, 15° and 20° the predominant period of the sliding mass \((T_s)\) are 0.435, 0.5, 0.667, 0.54 and 0.4 seconds respectively.

As it was mentioned earlier, Bray and Ratjhe suggested the formula \(T_s=4H/V_s\) to calculate the predominant period of the sliding mass. \(H\) in this formula is shown in Figure 5. In the model studied in this paper, we obtain a fixed value of 0.372 for \(T_s\) using the formula suggested by Bray and Ratjhe. This value is inconsistent with the values obtained through dynamic analyses of the present study. In other words, variation of the bedrock inclination affects on the predominant period and a fixed value of \(T_s=4H/V_s\) cannot be considered.

Figure 12 shows the variation of maximum normalized acceleration (ratio of maximum acceleration of the sliding mass \((\text{MHEA})\) to the maximum acceleration in bedrock \((\text{MHAr})\)), to the normalized predominant period \((T_s/T_m)\) for different bedrock inclination angles. The highest value is observed at angle 10°. In addition, Figure 13 shows the variation of normalized displacement (the ratio of the slope permanent displacement \((U)\) to the product of the significant duration of the earthquake \((D_{c, g})\) and maximum horizontal acceleration coefficient of the sliding mass \((k_{max})\)) with the normalized critical acceleration ratio (ratio of the critical horizontal acceleration coefficient \(k_y\) to the \(k_{max}\)) for different bedrock inclination angles. \(k_{max}\) equals \(\text{MHEA}/g\). Comparing this Figure with Figure 2, we observe that the results are consistent with each other.

By some additional seismic data in a site, these diagrams can be used in performance design of slopes. At first, by specifying the seismic site characteristics in which the slope is designed and clarifying the slope geometry, the values of \(T_s\), \(T_m\), and \(k_{max}\) can be determined. Next, the critical acceleration coefficient value of the intended slope will be determined by pseudo-static analysis. The slope permanent displacement can be calculated using Figure 13 and compared with the allowable displacement.

On the other hand, if we consider a value for the allowable slope displacement (the expected performance), at first, the values of \(T_s\) and \(T_m\) will be determined and then by using the Figure 12, \(\text{MHEA}\) and finally \(k_{max}\) are evaluated. According to the allowable displacement and Figure 13; \(k_y/k_{max}\) and in turn \(k_y\) will be estimated. The obtained \(k_y\) value is the
suggested pseudo static coefficient for the desired performance level. If the slope safety factor - regarding $k_h$ equals $k_v$ - be more than one, the slope performance will be satisfactory.

6. Conclusion

In this paper, the effect of bedrock inclination on slope seismic performance has been studied. The present study can be considered as an improvement of Bray and Rathje (1998) screening method. The main aspects of the current study are:

a) The sliding surface is determined by pseudo-static analysis and not predetermined.

b) The predominant period of the slope is determined by dynamic analysis.

c) The critical acceleration coefficient of the slope is obtained by pseudo-static analysis of the selected slope.

d) The dynamic analyses were performed two-dimensionally. The Soil is considered as a Mohr-Coulomb elasto-plastic material.

This study was carried out by:

- Considering a typical model of a slope on bedrock.
- Selecting some of the earthquake acceleration records of Iran, considering them as bedrock records.
- Analyzing the model and determining the equivalent acceleration records of the sliding masses.
- And calculating the slope permanent displacement by Newmark sliding block method.

The results of the calculations for the slope angle of 35°, $\phi$ of 30°, $\phi/c$ equals to 45, $h/V_s$ equals to 0.3 sec. and the bedrock inclination angles of 0°, 5°, 10°, 15° and 20° indicated that:

- The ratio of the equivalent maximum acceleration in the sliding mass to the maximum acceleration in the bedrock increased with increasing the bedrock inclination angle up to 10° and then decreased when the bedrock angle increased further. To examine the reason for this phenomenon, the average horizontal period of the equivalent acceleration on sliding block ($T_{m}$) and the natural period of sliding mass ($T_s$) were calculated and it was found out that the ratio of the above mentioned periods was close to 1 at angles 10° and 15°.
- The slope permanent displacement increased with increasing the bedrock inclination up to 10° and then it decreased.

• Neglecting the amplification caused by bedrock inclination would lead to non-real assessment of the slope seismic performance.

• The diagram of the normalized displacement in terms of the critical acceleration ratio was well - consistent with the results obtained by Bray and Rathje (1998).

• The algorithm introduced in this paper can be used to develop the diagrams required for the seismic slope performance design.

These diagrams can be used in determining the pseudo-static coefficient necessary for the expected performance of a slope.

Acknowledgements

Some parts of this paper is based upon part of a research project and a Ph.D. thesis in International Institute of Earthquake Engineering and Seismology (IIEES) of Iran. This support is acknowledged.

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