

# Empirical Model for Determining Rutting Parameter in Rubber Modified Bitumen

Saeed Ghaffarpour Jahromi<sup>1</sup>, Ali khodaii<sup>2</sup>

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**Abstract:** In this study an empirical model which can be used to predict the rutting parameter ( $G^*/\sin\delta$ ) for neat and powder rubber modified bitumen describes. The model was developed using 36 unique powder rubber modified bitumen combinations, rubber concentrations were varied at 5% intervals between 5 and 20%. The effects of powder rubber particle size on model accuracy were also studied; ultimately a model was produced with the capability of predicting rutting parameter values over a range of temperatures and rubber concentrations. By definition, the upper limit of the performance grade is dependent on the rutting parameter value; therefore, the relationship was also considered in terms of high end failure temperature. The Rubber Coefficient for rutting parameter ( $R_{cg}$ ) was identified as an important parameter in the estimation of rutting parameter ( $G^*/\sin\delta$ ) with the addition of powder rubber. This term is a quantitative representation of the increase typically witnessed in rutting parameter values with the addition of powder rubber. Ambient ground powder rubber exhibited higher  $R_{cg}$  values than cryogenically ground particles. Additionally, 95% confidence intervals were generated for the predictive model thus providing a range of accuracy for the model. The resulting confidence intervals were approximately  $\pm 1300$  Pa; these confidence intervals were seen to capture 92.6% of the 462 data points used. Findings from this research suggest that the differences between cryogenic and ambient powder rubber bitumen are accurately described using the  $R_{cg}$ , furthermore bitumen properties may be predicted using an empirical equation.

**Keywords:** Powder Rubber, Rubber Modified Bitumen, Rutting Parameter, Empirical Model.

## 1. Introduction

To date no model has been presented for the prediction of rutting parameter values ( $G^*/\sin\delta$ ) of powder rubber modified bitumen with varied temperatures and powder rubber concentrations. This paper proposes an empirical model which predicts the asphalt bitumen rutting parameter ( $G^*/\sin\delta$ ) values of powder rubber modified bitumen at temperatures greater than 64°C. Also, as the high temperature performance gradation of the bitumen is calculated based on rutting parameter values it was also possible to develop an empirical relationship relating failure temperature to powder rubber content. It is a common practice to evaluate bitumens using the strategic highway research program (SHRP) guidelines. The SHRP performance grading (PG) is based on the ability of bitumen to resist permanent deformation [1].

Rutting is a severe distress which is often seen in asphalt whereby elevated temperature permit undesired compaction of the pavement due to the action of wheel loads [2].

### 1.1. Powder Rubber Modified Bitumen

The addition of powder rubber to asphalt bitumen has been found to have beneficial effects on the mitigation of rutting in flexible pavements. It is well documented that the addition of powder rubber to the asphalt increases its  $G^*/\sin\delta$  values[3-5], however currently no model has been presented identifying the amount of powder rubber required to modify a given bitumen in order to achieve desired properties. Powder rubber is produced by grinding scrap tires into a fine powder; it may be added to the asphalt bitumen by the wet process or the dry process. The wet process involves the addition of powder rubber to the asphalt bitumen itself, followed by a certain amount of mixing. Typically the powder rubber used in this process is finer than that used in the dry process. The dry process involves the addition of rubber particles to the aggregate prior to the addition of the bitumen. In this case, the coarse rubber particles become part of the

<sup>1</sup> Professor associate, Department of Civil Engineering, Shahid Rejaini Teacher Training University, Tehran, Iran, E-mail: Saeed\_Ghf@Srttu.edu

<sup>2</sup> Professor associate, Department of Civil Engineering, AmirKabir University of Technology, Tehran, E-mail: Khodaii@aut.ac.ir

aggregate as opposed to a modifying agent in the bitumen itself. For the purposes of this paper only the wet process was considered.

## 1.2. Complex Shear Modulus ( $G^*$ ) and Phase Angle ( $\delta$ )

At high temperatures, the complex shear modulus ( $G^*$ ) and phase angle ( $\delta$ ) are proven indicators of the rutting susceptibility of the pavement ( $G^*/\sin\delta$ ), similarly at medium temperatures they may be used to predict fatigue cracking ( $G^*\sin\delta$ ). AASHTO TP 315 provides specifications and procedures for obtaining experimental values of the complex shear modulus and phase angle using a dynamic shear rheometer (DSR). The DSR works on the principle that asphalt bitumen performance is temperature and load duration dependent. The DSR function is based on sandwiching the bitumen between two plates, in which the lower plate is fixed and the top plate oscillates at a frequency of 10 radians per second. The oscillation is meant to simulate the shearing action corresponding to a traffic speed of about 90 km/hr (55 mph). The DSR permits the determination of the total complex shear modulus (Fig. 1) as well as its elastic (recoverable deformation) and viscous (non-recoverable deformation) components [2].

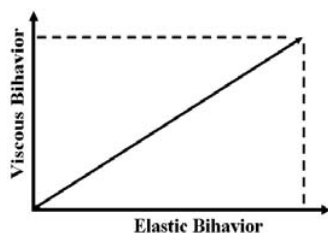


Fig. 1 Components of total complex shear modulus.

The stiffness of the powder rubber modified asphalt binder is determined using the DSR. Typically the addition of powder rubber increases the stiffness thus yielding a higher failure temperature. The change in bitumen properties occurs due to the reaction between the

rubber and the bitumen, the rubber particles tend to swell due to the absorption of aromatic oils thus producing a viscous gel [6-9].

The PG grade is determined by measuring the temperature at which the unaged asphalt bitumen's complex shear modulus divided by the sine of the phase angle ( $G^*/\sin\delta$ ) is at least 1000 Pa when measured at a frequency of 10 radian per second in accordance with AASHTO M320 (Performance Graded Asphalt Binder) [1, 2].

The main objective of the study was to establish a model for predicting the rutting parameter values of a powder rubber modified bitumen. Specifically the objectives can be identified as follows:

- Quantify the change in asphalt binder  $G^*/\sin\delta$  values when powder rubber is added.
- Quantify  $G^*/\sin\delta$  values for temperatures exceeding 64°C.
- Identify important parameters for rutting parameter ( $G^*/\sin\delta$ ) value changes with respect to rubber characteristics.
- Establishment of predictive model for determining high end failure temperature for powder rubber modified bitumen using SHRP testing procedures.

Error analysis of model and determination of accuracy.

## 2. Materials and Data Collection

Two studies were used for gathering data for model development; the first study used two bitumen sources which were blended with 4 different powder rubber sources. Both bitumen sources were PG64-22, and were each reacted for 30 minutes at 177°C at 5, 10, 15, and 20 % powder rubber by weight of bitumen. All powder rubber used in this study fell within consistent specifications for powder rubber (Table 1), two powder rubber sources utilized cryogenic grinding while the other two were obtained through ambient grinding.

Table 1 Powder Rubber Gradation.

Sieve Number	Opening size (mm)	Upper Specification (% Passing)	Lower Specification (% Passing)
No. 8	2.38	100	100
No. 10	2.0	100	100
No. 16	1.19	100	65
No. 30	0.6	100	20
No. 50	0.3	45	0
No. 200	0.075	5	0

**Table 2** Virgin Bitumen Properties.

Aging State	Parameter	Bitumen		
		A	B	C
Unaged Bitumen	Rotational Viscosity at 135°C (Pa.s)	0.703	0.43	0.472
RTFO (aged residue)	G*/Sinδ at 64°C (kPa)	2.413	1.279	1.468
		6.075	2.81	2.579
RTFO+PAV (aged residue)	G*/Sinδ at 25°C (kPa)	3352.1	4074	3573.5
	Stiffness at 12°C (MPa)	141.3	217	232.3
	m-value at 12°C	0.359	0.307	0.321

The second study was performed using three bitumen sources (Table 2); all three bitumen sources were PG 64-22, and were each reacted for 30 minutes at 177°C at 10 and 15 % powder rubber by weight of bitumen. In this case two powder rubber sources were tested; one cryogenic and one ambient ground. However, in this study the powder rubber was separated into different sizes and the effects of particle size studied, the sizes studied were 0.18, 0.425, and 0.85 mm. All testing was performed on a DSR, complex shear modulus and phase angle values were obtained for each temperature at 6°C intervals starting from 64°C until failure of the sample.

### 3. Model Development

The model was developed in two parts; first the effects of the addition of powder rubber to the virgin bitumen were studied. Once a working model was developed for the addition of the powder rubber to the virgin bitumen, then the model for temperature variation was developed. Eq. (1) illustrates the starting point for the model:

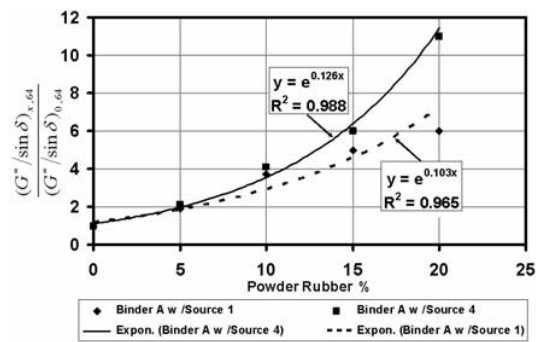
$$\left(\frac{G^*}{\sin \delta}\right)_{x,T} = \left(\frac{G^*}{\sin \delta}\right)_{0,64} \times [\text{Rubber Factor}][\text{Temperature Factor}] \quad (1)$$

Where  $(G^*/\sin \delta)_{x,T}$  is the complex shear modulus divided by the sine of the phase angle with rubber content  $x$  at temperature  $T$ , and  $(G^*/\sin \delta)_{0,64}$  is the neat bitumen rutting parameter value at 64°C. The Rubber Factor and Temperature Factor components of the model are discussed next.

#### 3.1. Rubber Factor

The rubber coefficient for rutting parameter

$(R_{cg})$  was introduced as a means of quantifying the unique effects of powder rubber on the specific bitumen; it is defined as the exponential coefficient by which the powder rubber concentration is multiplied when values of powder rubber modified bitumen are normalized with the neat bitumen of the same source. It can be seen in Fig. 2 that the normalized values follow an exponential trend with increasing powder rubber concentrations.



**Fig. 2** Determination of Rubber Coefficient for  $G^*/\sin \delta$ .

This procedure yielded high  $R$  squared values when used to describe the change in behavior of the bitumen with respect to powder rubber concentration.  $R_{cg}$  values were calculated for all powder rubber -Bitumen combinations, consistent with Fig. 2 the relationship between powder rubber concentration and rutting parameter was seen to be adequately presented by Eq. (2).

$$\frac{\left(\frac{G^*}{\sin \delta}\right)_{x,64}}{\left(\frac{G^*}{\sin \delta}\right)_{0,64}} = e^{R_{cg} x} \quad (2)$$

Where  $(G^*/\sin \delta)_{x,64}/(G^*/\sin \delta)_{0,64}$  is the ratio of rutting parameter values at 64°C between 0 and

$x\%$  is powder rubber by weight of bitumen,  $R_{cg}$  is the rubber coefficient for rutting parameter and  $x$  is the powder rubber concentration by weight of bitumen. Eq. (2) outlines the exponential growth witnessed by powder rubber modified bitumen with regards to  $G^*/\sin\delta$  values. Therefore, at a temperature of  $64^\circ\text{C}$  the rutting susceptibility of a bitumen with varying powder rubber concentrations may be predicted using Eq. (2).

### 3.2. Temperature Factor

In order to vary the rutting parameter behavior with respect to temperature an inverse log relationship was seen to have the best correlation when the data was analyzed with the Statistical Analysis System (SAS). In this case, the bitumen rutting parameter value was normalized (i.e., the  $G^*/\sin\delta$  value at  $64^\circ\text{C}$  was used as the reference). The Arrhenius equation, shown below, was used as the starting point of the temperature factor component of the model:

$$\left(\frac{G^*}{\sin\delta}\right)_T = Ae^{\frac{E_a}{RT}} \quad (3)$$

Where  $(G^*/\sin\delta)_T$  is given at the desired temperature  $T$  ( $> 64^\circ\text{C}$ ),  $T$  is temperature in degrees Kelvin,  $A$  is a constant,  $E_a$  is the activation energy, and  $R$  is the universal gas constant ( $8.314 \text{ J/mol}\cdot\text{K}$ ). As shown in Equation (4), the model was normalized by allowing the  $G^*/\sin\delta$  value to be predicted at a desired temperature. The resulting equation is:

$$\left(\frac{G^*}{\sin\delta}\right)_T = Ae^{\phi T} \quad (4)$$

Where  $K$  and  $\phi$  are constants and  $T$  is temperature in degrees Celsius, in this equation the  $\phi$  term also accounts for the conversion from degrees Kelvin to degrees Celsius.

### 3.3. The Combined Model

The rubber and temperature components were combined to produce Eq. (5):

$$\left(\frac{G^*}{\sin\delta}\right)_{x,T} = \left(\frac{G^*}{\sin\delta}\right)_{0,64} \left[e^{(R_{cg}x)}\right] \left[Ke^{(\phi T)}\right] \quad (5)$$

Where,  $(G^*/\sin\delta)_{x,T}$  is the complex shear modulus divided by the phase angle at the desired powder rubber concentration and temperature,  $(G^*/\sin\delta)_{0,64}$  is the complex shear

modulus divided by the phase angle at a powder rubber concentration of zero at  $64^\circ\text{C}$ ,  $R_{cg}$  is the Rubber coefficient for  $G^*/\sin\delta$ ,  $x$  is the concentration of powder rubber by weight of bitumen,  $K$  and  $\phi$  are constants, and  $T$  is the temperature in  $^\circ\text{C}$ . The method of nonlinear least squares was used to estimate the parameters  $K$  and  $\phi$ . All calculations were performed using the nonlinear modeling procedure in SAS.

### 3.4. High End Failure Temperature Model

Failure temperature of a bitumen is defined as the temperature at which the rutting parameter value of the unaged bitumen falls below  $1000 \text{ Pa}$  [1]. Therefore, by rearranging the preceding relationship and solving for temperature when the  $G^*/\sin\delta$  is set to  $1000$  it is possible to predict the high temperature failure temperature of the bitumen, doing so yields Eq. (6):

$$\frac{\left(\frac{G^*}{\sin\delta}\right)_{x,T}}{\left(\frac{G^*}{\sin\delta}\right)_{0,64} \left[e^{(R_{cg}x)}\right]} = \left[Ke^{(\phi T)}\right] \quad (6)$$

Eq. (7) is produced by taking the natural logarithm of both sides,

$$\ln \left[ \frac{\left(\frac{G^*}{\sin\delta}\right)_{x,T}}{\left(\frac{G^*}{\sin\delta}\right)_{0,64} \left[e^{(R_{cg}x)}\right]} \right] = \left[ \ln(K) + \ln(e^{(\phi T)}) \right] \quad (7)$$

Solving for  $T$  yields Eq.(8),

$$\frac{\ln \left[ \frac{\left(\frac{G^*}{\sin\delta}\right)_{x,T}}{\left(\frac{G^*}{\sin\delta}\right)_{0,64} \left[e^{(R_{cg}x)}\right]} \right] - \ln(K)}{\phi} = T \quad (8)$$

Therefore, as shown in Eq. (9), the failure temperature may be determined by substituting in  $(G^*/\sin\delta)_{x,T} = 1000 \text{ Pa}$ .

$$FT = \frac{\ln \left[ \frac{\left(\frac{G^*}{\sin\delta}\right)_{x,T}}{\left(\frac{G^*}{\sin\delta}\right)_{0,64} \left[e^{(R_{cg}x)}\right]} \right] - \ln(K)}{\phi} \quad (9)$$

Where,  $FT$  is the failure temperature (high

end),  $(G^*/\sin\delta)_{0,64}$  is the complex shear modulus divided by the phase angle at a powder rubber concentration of zero at 64°C,  $R_{cg}$  is the rubber coefficient for  $G^*/\sin\delta$ , and  $x$  is the concentration of powder rubber by weight of bitumen.

#### 4. Results

In this section the experimental results are compared to the predicted values using the rutting parameter model and the high end failure temperature model.

Of interest in this section was the ability of the 95% confidence interval to capture the experimental values without creating excessively large intervals. Also presented in this section are the various  $R_{cg}$  values, these values are compared by base bitumen, grinding procedure, and powder rubber source. For this study the  $R$  squared values was not used as a judge of the suitability of the model as this parameter does not include any reference to the variability accounted for by the model, instead the 95% confidence interval was used.

##### 4.1. Rutting Parameter Model

Using *SAS*, the values for  $K$  and  $\phi$  were determined to be 837.8 and -0.1047; the resulting estimated relationship is given by Eq. (10):

$$\left(\frac{G^*}{\sin \delta}\right)_{x,T} = \left(\frac{G^*}{\sin \delta}\right)_{0,64} \left[e^{(R_{cg}x)}\right] [837.8e^{-0.1047T}] \quad (10)$$

The standard error of  $K$  and  $\phi$  were found to be 90.2 and 0.002. Eq. (9) was used to predict values of  $G^*/\sin\delta$  values for an array of bitumen and temperature combinations. Upper and lower prediction intervals, were also estimated from Eq. (9).

The predicted values and the prediction intervals were plotted along with the actual values. Whenever the actual values fell within the prediction intervals, the predictions based on the model were considered to be accurate. Analysis of the model showed that overall 92.6% of the time the actual values fell within the prediction intervals.

Fig. 3 illustrates the increase in  $G^*/\sin\delta$  values as powder rubber increases; also this figure illustrates the accuracy of the model in

predicting the powder rubber modified bitumen properties. Experimental results show that as powder rubber concentration is increased, the  $G^*/\sin\delta$  value increases, a decrease in experimental  $G^*/\sin\delta$  values is noted as the temperature increases. At 82 and 88°C bitumens with less than 10% powder rubber by weight were seen to not withstand any loading. The average experimental values were consistently encapsulated by the 95% confidence interval; furthermore the 95% confidence interval was seen to be approximately +/- 1300 Pa.

##### 4.2. Failure Temperature Model

Eq. (11) is produced by substituting the computed values for  $K$  and  $\phi$  into Eq. (9), this relationship provides an idea of how failure temperature of powder rubber modified bitumen varies with powder rubber concentration.

$$FT = \frac{\ln\left(\frac{1000}{\left(\frac{G^*}{\sin \delta}\right)_{0,64} \left[e^{(R_{cg}x)}\right]}\right) - \ln(837.8)}{-0.1047} \quad (11)$$

A verification of the high end failure temperature model was also performed, as seen in Fig. 4 where the average values consistently fell within the 95% confidence interval. This suggests that the model derived from the combined model was accurate for the purposes of predicting high end failure temperature given only the virgin  $G^*/\sin\delta$  value and other variables (e.g., ambient/cryogenic and particle size) of the powder rubber to be used.

The accuracy of this limited model was seen to be good as the 95% confidence interval was approximately +/-3°C. Also seen in Fig. 3 is the effect of powder rubber type and surface morphology on the failure temperature of the bitumen, specifically it can be seen that the bitumens modified with ambient ground powder rubbers tended to yield higher failure temperatures.

Using the 95% confidence interval also provides a factor of safety in the prediction of the powder rubber modified bitumen failure temperature, using the lower value from the confidence interval means that the probability of calculating an inaccurate failure temperature on the high side is less than 5%.

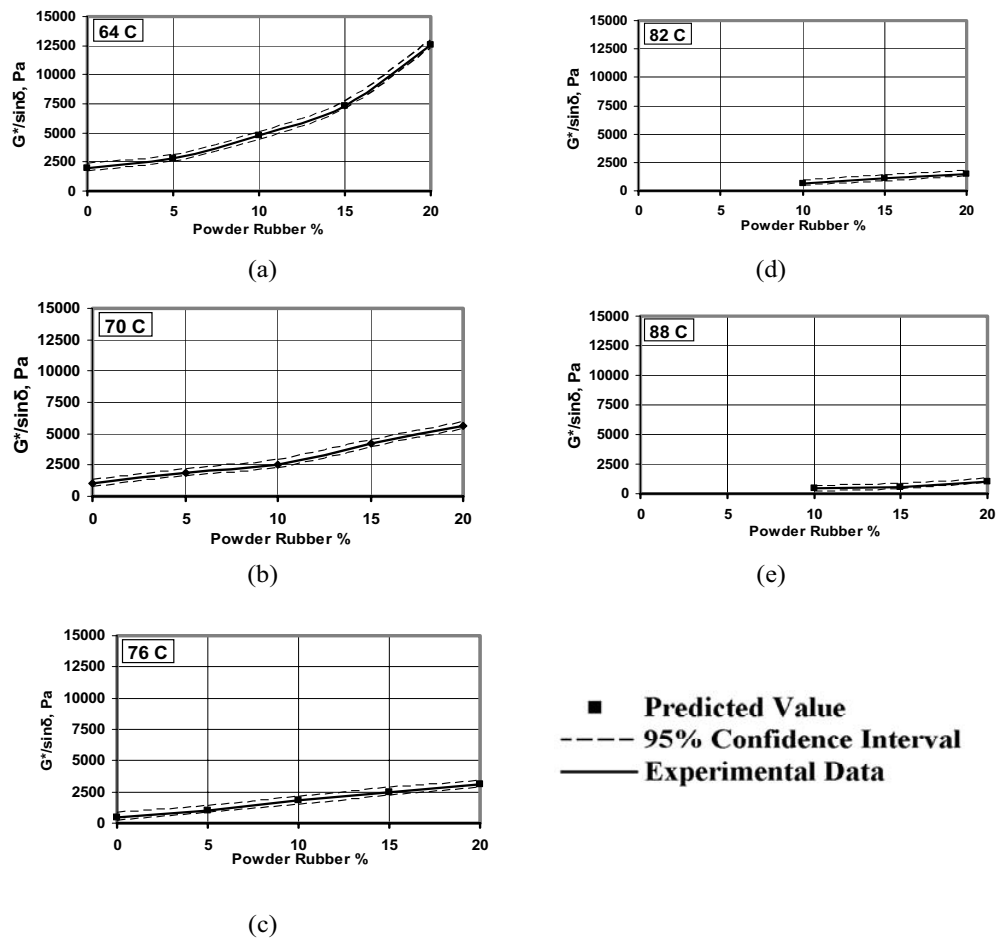


Fig. 3 Average Experimental and Predicted  $G^*/\sin\delta$  values with 95% Confidence Intervals.

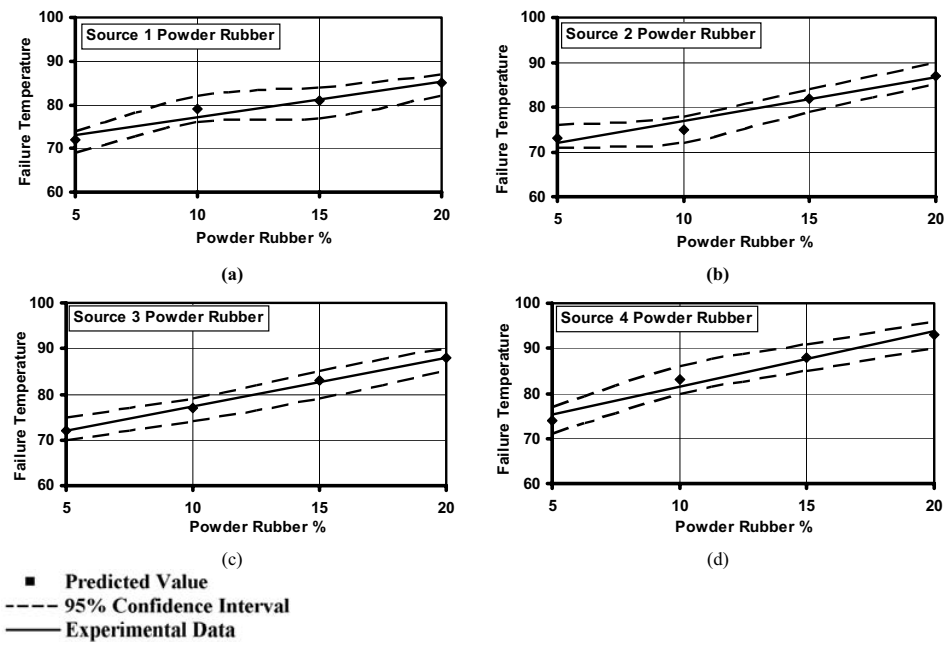


Fig. 4 Failure temperature fit.

### 4.3. Analysis of $R_{cg}$

The  $R_{cg}$  values were computed for all 462 data points, the  $R_{cg}$  were separated by bitumen source, powder rubber grinding procedure, and powder rubber source (Table 3, 4 and 5). This analysis indicated that Bitumen A yielded the highest  $R_{cg}$  values regardless of the powder rubber used to modify the bitumen. The ambient grinding procedure was seen to produce higher  $R_{cg}$  values than cryogenic grinding of the tires, while powder rubber produced from truck tires tended

to yield the highest  $R_{cg}$ . Particle size was also an important factor in determining the  $R_{cg}$  value, the analysis of the  $R_{cg}$  results showed that both ambient and cryogenically ground particles produced higher  $R_{cg}$  values as the particle size decreased. Many of these results are consistent with the literature where ambient ground rubber has been known to exert greater changes on the bitumen [10-12], while finer powder rubber particles have also been seen to produce powder rubber modified bitumen with greater rutting resistance [13].

**Table 3**  $R_{cg}$  by Binder source.

Bitumen Source	N	Mean $R_{cg}$	LSD Grouping
Shiraz (a)	202	0.102	A
Blend (b)	180	0.088	B
Tehran (c)	78	0.074	C

**Table 4**  $R_{cg}$  by Grinding Procedure.

Grinding Procedure	N	Mean $R_{cg}$	LSD Grouping
Ambient	202	0.103	A
Cryogenic	230	0.093	B

**Table 5**  $R_{cg}$  by Powder Rubber Source.

Powder Rubber Source	N	Mean $R_{cg}$	Grinding Procedure	Size	Composition	LSD Grouping
Source 4	32	0.121	Ambient	Gradation <sup>a</sup>	Truck Tire	A
Source 7	36	0.103	Ambient	0.18 mm	Passenger Car Tire	B
Source 6	36	0.102	Ambient	0.425 mm	Passenger Car Tire	BC
Source 10	36	0.100	Cryogenic	0.18 mm	Passenger Car Tire	C
Source 3	62	0.098	Ambient	Gradation <sup>a</sup>	Passenger Car Tire	D
Source 5	36	0.096	Ambient	0.85 mm	Passenger Car Tire	D
Source 1	62	0.093	Cryogenic	Gradation <sup>a</sup>	Passenger Car Tire	E
Source 2	60	0.093	Cryogenic	Gradation <sup>a</sup>	Passenger Car Tire	E
Source 9	36	0.092	Cryogenic	0.425 mm	Passenger Car Tire	E
Source 8	36	0.088	Cryogenic	0.85 mm	Passenger Car Tire	F

<sup>a</sup> Gradation corresponds to specifications provided in Table 1

## 5. Conclusions

From this limited study the following conclusions were made:

- It is possible to develop an empirical model depicting the changes in rutting parameter ( $G^*/\sin\delta$ ) values and failure temperature.
- The rubber coefficient for  $G^*/\sin\delta$ , ( $R_{cg}$ ), is an important parameter when estimating the rutting susceptibility of a powder rubber modified bitumen, analysis of the  $R_{cg}$  indicates that ambient ground powder rubber yields higher  $R_{cg}$  and therefore also higher  $G^*/\sin\delta$  values.
- The  $R_{cg}$  is also seen to be particle size dependent, whereby the finer the particle the higher the estimated values, it was also shown that powder rubber derived from truck tires yielded higher  $R_{cg}$ .
- The lower confidence interval was found to be the most conservative value, using the value corresponding to the lower end of the 85% confidence interval the probability of selecting an inaccurately high failure temperature was reduced to less than 5%. Further testing is required with other bitumen.

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