



Statistical model to predict the mechanical properties of binary and ternary blended concrete using regression analysis

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Abstract

This research paper presents the use of wasteful supplementary cementitious materials like fly ash and silica fume to conserve the cement used in concrete. The cement industry is one of the major producers of greenhouse gases and an energy user. In this study, Portland cement was used as a basic cementitious material. Fly ash and silica fume were used as the cement replacements by weight. The replacement levels of fly ash were 30%, 40% and 50%, and silica fume were 6% and 10%. The water binder ratio was kept constant as 0.4 and super plasticizer was added based on the required workability. Results of the binary and ternary concrete mixtures compressive strength, split tensile strength and flexural tensile strength were taken for study up to 90 days. Based on the experimental results of compressive strength, prediction models were developed using regression analysis and coefficients were proposed to find the split tensile strength and flexural strength of binary-ternary concrete mixtures at 28 and 90 days.

Keywords: Compressive strength, Split tensile strength, Flexural strength, Fly ash, Silica fume, Industrial waste.

1. Introduction

Over the past years, there has been increased use of cement and concrete, which has a high demand in infrastructure development. The cement industry is one of the major sources of environmental pollution. Environmental effects arising from the production of cement include air pollution by cement dust, natural resource depletion due to raw material extraction required for cement manufacturing, water pollution, noise pollution and ground vibration [1]. More importantly, the CO₂ emission from the burning of cement is one of the main causes of global warming. Therefore, reducing the cement demand is a direct solution for all these environmental problems. To save energy, resources and environment, considerable attempts are being made to find substitute materials that can be used as a partial replacement for cement. Supplementary cementitious materials (SCMs) such as fly ash (FA), silica fume (SF), ground granulated blast furnace slag, metakaolin and rice husk ash are some of the most common Portland cement (PC) replacement materials currently used in the concrete industry [2,3]. The concrete made with SCMs prevents the consumption of a

large volume of PC, which will minimize the total CO₂ emission and the cost per cubic meter of concrete [4].

Many research works are focusing on the use of SCM as a cement replacement. The selection of pozzolonic material deserves further attention due to their different properties. FA has been widely used as a cement replacement at higher levels in comparison with other natural pozzolanic materials due to its low water requirements or good workability which can be achieved. Thus, much research has focused only on high-volume fly ash (HVFA) in blended cement. However, the compressive strength of binary blended cement, such as blends of fly ash with particularly large concentrations of fly ash (>30%), are lower at early age, due to the dilution effect, and very low pozzolanic reaction [1, 5]. Researchers [6, 7] have used activators such as Na₂SO₄, CaCl₂ and CaSO₄ in order to improve the early strength of Ordinary Portland Cement concrete with FA added as a replacement.

Silica fume is very fine non-crystalline silica generated by electric arc furnaces as a by-product during the fabrication of metallic silicon or ferrosilicon alloy. The very high specific surface area combined with the high silica content accelerates the pozzolanic reaction. Silica fume is known to produce high strength concrete and is used in two different ways: as a cement replacement in order to reduce the cement content and as an additive to enhance the properties of concrete [8]. The characterization of cementitious system during the process of hydration form primary C-S-H produced by calcium silicate. Further on addition of SF, the calcium hydroxide from the original hydration product reacts with SF to form secondary C-S-H

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[9]. The SF appears to be a potential solution to the effect of FA on early age properties of concrete due to its highly reactive nature and provides a sufficient amount of CSH at an early age. Therefore, the utilization of SF together with FA provides an interesting alternative for cement [8].

This paper presents the experimental results of compressive strength, split tensile strength and flexural strength of binary-ternary combinations of cement, FA and SF. Regression analyses were performed and prediction model equations are given to predict the split tensile strength and flexural strength based on the experimental values of compressive strength.

2. Experimental Study

The materials used in this study were tested for their

Table 1 Chemical properties of PC, Class C fly ash and silica fume

Characteristics	From Test		
	PC (53 grade)	Fly ash- Class C	Silica Fume
Silica (as SiO ₂), <i>Min</i>	20.2	57.65	85.72
Calcium Oxide (Lime Content) as CaO	63.41	11.64	-
Alumina (as Al ₂ O ₃)	1.07	15.29	0.06
Iron oxide (as Fe ₂ O ₃)	1.07	6.1	0.45
Magnesia (as MgO), <i>Max</i>	1.02	0.37	-
Sulphuric Anhydride (as SO ₃), <i>Max</i>	2.02	1.82	-
Total Loss on ignition, <i>Max</i>	1.48	2.86	1.96
Total Chlorides (as Cl)	0.004	0.02	-
Sodium Oxide (as Na ₂ O)	0.35	0.44	-
Potassium Oxide (as K ₂ O)	0.95	0.04	-
Total alkalis (as Na ₂ O)	-	0.47	-
Silicon dioxide (SiO ₂) + Aluminium oxide (Al ₂ O ₃) + Iron Oxide (Fe ₂ O ₃) in % by mass, <i>Min</i>	-	79.04	-

The coarse aggregate used in this experiment was crushed gravel with a nominal maximum size of 20mm while the fine aggregate used was locally available natural river sand. The specific gravity and water absorption properties of crushed gravel and river sand are 2.65, 0.65 and 2.69, 0.95 respectively. A new generation Polycarboxylic ether (PCE) based super-plasticizer (SP) was used. The SP was a medium brown colored aqueous solution with standard specifications of ASTM C494/C 494M-11 [12]. The specific gravity and pH value of SP were 1.056 and 6.5 respectively.

2.2. Mixture proportions

The mix proportions were selected in such a way as to allow comparison of their influence on the strength

chemical and physical properties as discussed below.

2.1. Material

A commercially available PC conforming to BIS: 12269-1987 (53 grade) [10] was used in this study with a specific gravity of 3.15. The initial and final setting time was found to be 150 (min) and 265 (min) respectively. The two SCMs used were as follows; (1) FA obtained from a local power plant NLC Ltd., Neyveli (lignite base fly ash), India with a specific gravity of 2.46; and (2) SF obtained from Elkem materials conforming to ASTM C1240 [11] with a specific gravity of 2.02 and bulk density of 600 kg/m³. The oxide compositions of PC, FA and SF are summarized in Table 1.

properties of the cementitious system for different combinations of PC with FA and SF. A total of 12 concrete mixtures were designed, having a total binder content of 450 kg/m³ with a constant water/binder ratio of 0.4. For better analysis, the concrete mixtures were divided as; (1) Control mixture with only PC as a binder 450 kg/m³; (2) Five binary mixtures (three-PC+FA and two-PC+SF) with 30%, 40% and 50% FA replacement, 6% and 10% SF replacement respectively by weight of the PC for 450 kg/m³; and (3) Six ternary mixtures (PC+FA+SF) with replacement levels of FA and SF as in binary mixtures. The mixture ID are furnished in Table 2 and the mix proportions, SP content, aggregate content and SCMs content by weight used in this experimental study are summarized in Table 3 for 450 kg/m³ of total binder content.

Table 2 Mixture ID of samples

Mix ID of 450 kg/m ³	Cement (%)	Fly ash (%)	Silica fume (%)
F100	100	0	0
F906	94	0	6
F901	90	0	10
F730	70	30	0
F640	60	40	0
F550	50	50	0

F636	64	30	6
F546	54	40	6
F456	44	50	6
F631	60	30	10
F541	50	40	10
F451	40	50	10

Table 3 Mixture Proportions

Mixture ID	W/B	Cement (kg/m ³)	Fly Ash (kg/m ³)	Silica Fume (kg/m ³)	Fine Aggregate (kg/m ³)	Coarse Aggregate (kg/m ³)	Super plasticizer (kg/m ³)
F100	0.4	450	-	-	1062	708	-
F906	0.4	423	-	27	1062	708	0.04
F901	0.4	405	-	45	1062	708	0.07
F730	0.4	315	135	-	1062	708	0.07
F640	0.4	270	180	-	1062	708	0.14
F550	0.4	225	225	-	1062	708	0.18
F636	0.4	288	135	27	1062	708	0.07
F546	0.4	243	180	27	1062	708	0.16
F456	0.4	198	225	27	1062	708	0.22
F631	0.4	270	135	45	1062	708	0.11
F541	0.4	225	180	45	1062	708	0.18
F451	0.4	180	225	45	1062	708	0.29

2.3. Casting and testing of specimens

Precision in the mixing sequence and duration are very important for the production of good quality concrete. Hence the procedure for batching, mixing and placing of concrete in its moulds was done as per ASTM C192/C192M-07 [13]. It was ensured that the same homogeneity and uniformity was main-tained in all mixtures. The mixing of concrete was carried out in a rotary mixer. The aggregates were mixed initially and then the binders (PC, FA & SF) were added in dry conditions. After remixing, water was added to the dry mix, and SP was added finally to the wet mix to achieve the required workability. To evaluate the compressive strength, 100 mm x 100 mm x 100 mm cube specimens were cast; 100 mm x 200 mm cylinders were cast to determine the split tensile strength, and 100 mm x 100 mm x 500 mm prisms were cast to determine the flexural strength. These specimens were compacted in three layers on a vibrating table. The specimens which were then cured in the moulds for 24 ± 8 hrs after casting were then demolded, and immersed in a curing tank full of water (normal curing method).

Measurement of the split tensile strength of cylindrical specimens was carried out by placing the specimen with its axis horizontal between plates on the hydraulic digital compression testing machine as per ASTM C496/C496M-11 [14]. The flexural strength of the prism specimens were measured by a symmetrical two point loading set-up, with a beam support span of 400 mm tested in a hydraulic flexural testing machine as per ASTM C78/C78M-10 [15]. Similarly the concrete cube specimens were tested for compressive strength in accordance with BIS: 516-1959 [16]. The test strength value was the average of triplicate specimens. The tests for the split tensile strength and

flexural strength of the concrete samples were determined at 28 and 90 days, whereas the compressive strength values were obtained at 1, 3, 7, 28 and 90 days.

3. Results and Discussion

The results of fresh concrete test, compressive strength, split tensile strength and flexural strength of the test specimens are discussed in the following paragraphs, with different replacement levels of FA, SF and FA+SF.

3.1. Properties of fresh concrete

The properties of freshly mixed concrete, i.e., temperature and slump are given in Table 4. The addition of FA and SF affect the properties in fresh state, which were directly related to strength of hardened concrete. The slump of all concrete mixtures was greater than 65mm except for silica fume concrete for which the slump was 50mm.

Table 4 Properties of fresh concretes

Mixture ID	Temperature (°C)	Slump (mm)
F100	31	65
F906	30	55
F901	31	50
F730	28	75
F640	27	75
F550	26	80
F636	29	70
F546	28	70
F456	26	70
F631	31	65
F541	30	65
F451	28	65

3.2. Compressive strength

The compressive strength of the binary blend of PC+SF is presented in Fig 1. From the experimental results, it is observed that the replacement of cement by 6% and 10% SF increased the strength by 13% and 8% approximately on all days compared to control concrete. This may be due to the formation of secondary C-S-H bonds as well as the filler role of the particle size. Also the dosage of SF has a significant effect on the compressive strength, the concrete mixture designated F906 demonstrated a high strength of 5.5% approximately at the ages of 3, 7, 28 and 90 days compared to F901. The addition of 10% SF resulted in high strength at 1 day compared to 6% SF. This may be because the strength enhancement at very early stages of hydration (24h) is only marginal: the addition of SF merely acts as filler [17]. The filler effect increases the density in the microstructure which may lead to increase in strength.

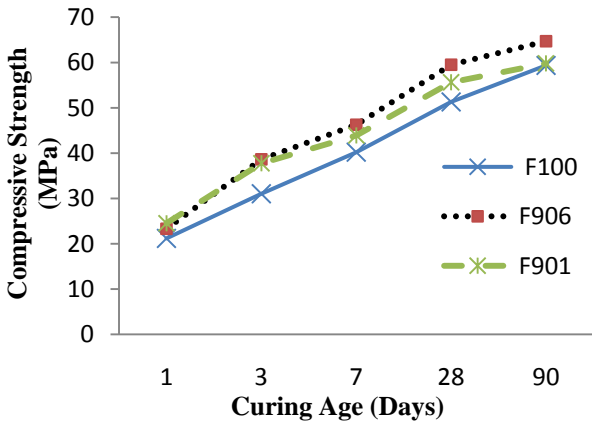


Fig. 1 Binary effects of silica fume on the compressive strength of concrete with reference to control concrete

The contribution of FA as a SCM to the concrete is very little at early ages (Fig. 2). The cube compressive strength at day 1, 3 and 7 were lower compared to control concrete based on the replacement levels of FA. The strength of F730 was lesser by 10% on an average, F640 was lesser by 18% and F550 was lesser by 30% approximately. At 28 days, the strength of F730 fly ash mix was only marginally lower by 5% on an average compared to the control mix, whereas the F640 and F550

fly ash mixtures still resulted in a 20% and 30% strength reduction. At the later age of 90 days, the FA contributes to the compressive strength. For instance, at 90 days, the F730 and F640 mix had 2% and 6% higher strength than the control mix. Still the F550 showed a negligible reduction of 3% in strength compared to the control mix. These observations are consistent with the results of Lam. L [18].

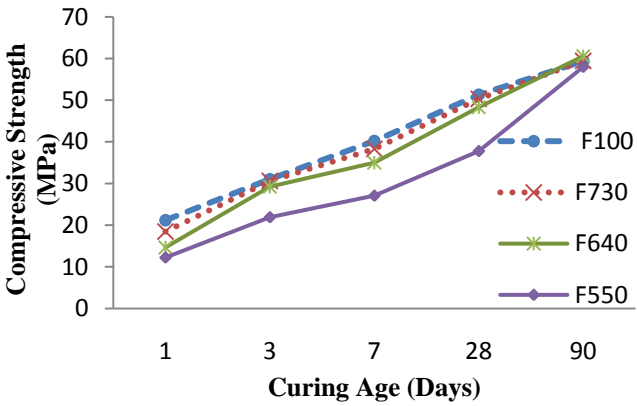


Fig. 2 Binary effects of fly ash on the compressive strength of concrete with reference to control concrete

The development of the compressive strength of the ternary system with respect to time is plotted in Fig 3. At all the initial ages of 1,3 and 7 days, the ternary mixture F636 showed 5% increased strength compared to the control mix, whereas the remaining ternary mixtures showed a decrease in compressive strength with increase in FA content, irrespective of percentage of SF present. However at 28 and 90 days, few ternary mixtures had developed strengths that were higher or partially similar to the strength of the control concrete mixture. The ternary mixtures F636 and F546 had an increased strength of 3% at 28 days and 2% at 90 days compared to control concrete. For the remaining ternary mixtures F456 and F451 containing 50% of FA, the cube compressive strength were lesser by 25% on average on all initial days up to 28 days, and at 90 days the compressive strength was lesser by 10%. This may be because of the relatively low reactivity of the FA itself, as can be seen from the 28 and 90 day strengths of the binary blends(PC+FA) with 50% of FA being lower than the control.

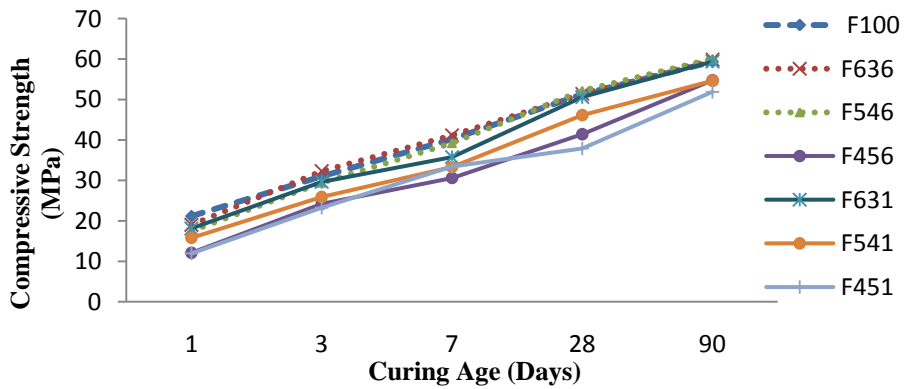


Fig. 3 Ternary effects of fly ash and silica fume on the compressive strength of concrete with reference to control concrete

3.3. Split tensile strength

The splitting tensile strength of the twelve concrete mixtures are studied and the increase in tensile strength with age for all the mixtures is shown in Fig 4. The variation in splitting tensile strength with fly ash and silica fume content is similar to that observed in the case of compressive strength. At 28 days, the splitting tensile

strength of the binary mixtures incorporating SF, F906 and F901, was 14% and 8% higher than control concrete, respectively. This pattern of strength gain continues even at 90 days. The addition of 6% SF resulted in a higher strength on all test days compared to 10% SF, and similar behaviour is observed even for the compressive strength at 28 and 90 days.

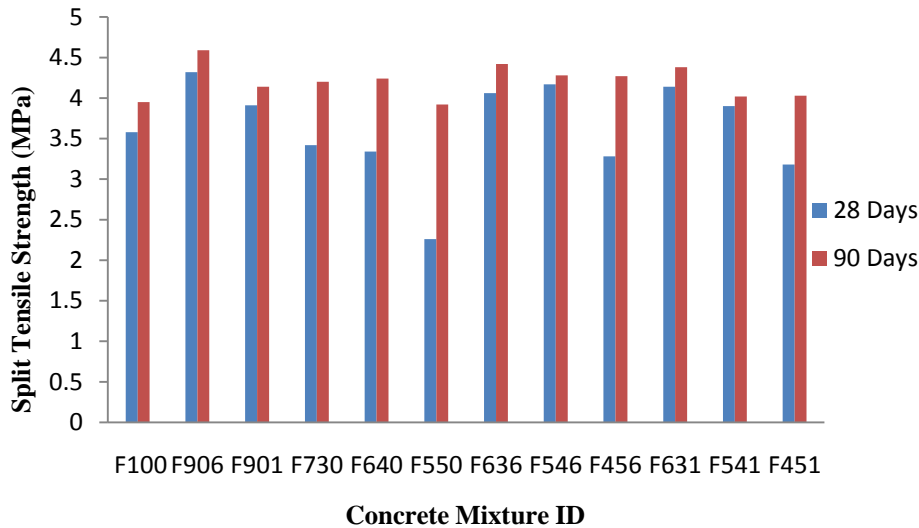


Fig. 4 Splitting tensile strength of binary and ternary blended concrete mixtures at 28 and 90 days

The binary mixtures incorporating FA was found to have lower tensile strength with increase in FA content at 28 days. The splitting strength was 4%, 8% and 35% lower for binary mixtures F730, F640 and F550 respectively, compared to the control mix. At 90 days, the gain in strength for FA binary mixtures was 4% for F730 and 8% for F640 compared to the control concrete. The FA binary mixture F550 still had a lower strength than the control mix. This strength gain is similar to the cube compressive strength of FA mixtures.

The test results for the various combinations of PC, FA and SF indicate that the split tensile strength of ternary mixtures with 30%- 40% FA and with 6%-10% SF at all ages is 8% to 15% higher than those of the control mixtures. The ternary mixture F636 showed a higher

strength gain of 15% compared to the control concrete and other ternary mixtures. However ternary mixtures with 50% FA exhibited an average of 10% lower split tensile strength compared to the control mix at 28 days, irrespective of the percentage of SF in it. Still, a strength gain of 3% was observed at 90 days for ternary mixtures with 50% FA compared to the control concrete.

3.4. Flexural Strength

The flexural strength test results are shown in Fig 5. Flexural strength of concrete mixtures increases with age, similar to the behaviour of compressive and splitting tensile strength.

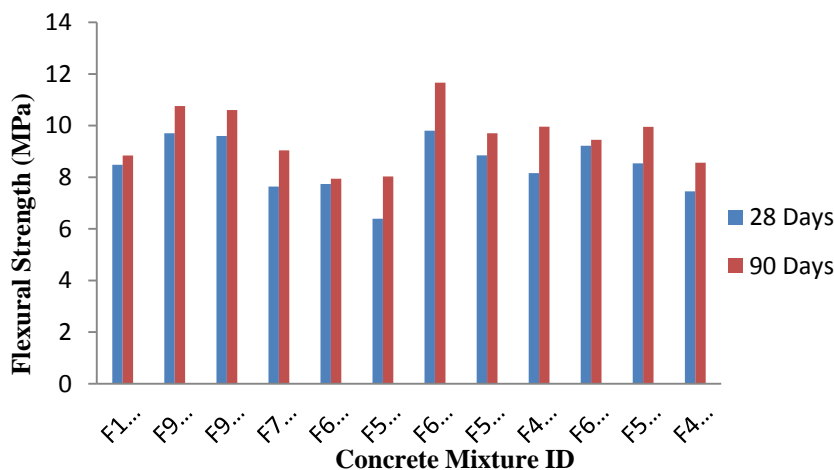


Fig. 5 Flexural strength of binary and ternary blended concrete mixtures at 28 and 90 days

The addition of SF as a binary mixture seems to have a more pronounced effect on flexural strength than for split tensile strength. The addition of 6% SF and 10% SF resulted in a higher strength of 18% and 13% on average compared to the control mix at both 28 and 90 days. However, the addition of 6% SF yielded a higher strength compared to 10% SF, similar to the case for split tensile strength and compressive strength.

The flexural strength of the binary FA concrete mixtures was affected by the percentage of FA in the total cementitious content, as was the case for compressive strength and split tensile strength. Compared to the control mix, the 28 day flexural strength of F730 was lesser by 9%, followed by F640 at 13% and F550 at 24%. The binary mixture F730 showed strength increase of 4% at 90 days, whereas F640 and F550 still showed 11% and 9% less strength compared to the control concrete.

The flexural strength of the ternary mixtures showed a similar behaviour of gain in strength, as was observed for the split tensile strength of concrete. It can be seen from

Fig. 5 that the ternary concrete mixture with 6% SF had greater strength than for 10% SF, and also the flexural strength was developed based on the FA content present in the mixtures.

4. Statistical Modeling

A statistical model between compressive strength-split tensile strength and compressive strength-flexural strength for predicting the split tensile strength, flexural strength values at 28 and 90 days is proposed. The regression analysis was used to establish a model from experimentally obtained values of compressive strength. Graphs are plotted between the experimentally obtained values of compressive strength versus the split tensile strength at 28 and 90 days as shown in Fig. 6 for different binary and ternary mixtures of FA and SF. In similar pattern, graphs are plotted between compressive strength and flexural strength as shown in Fig 7.

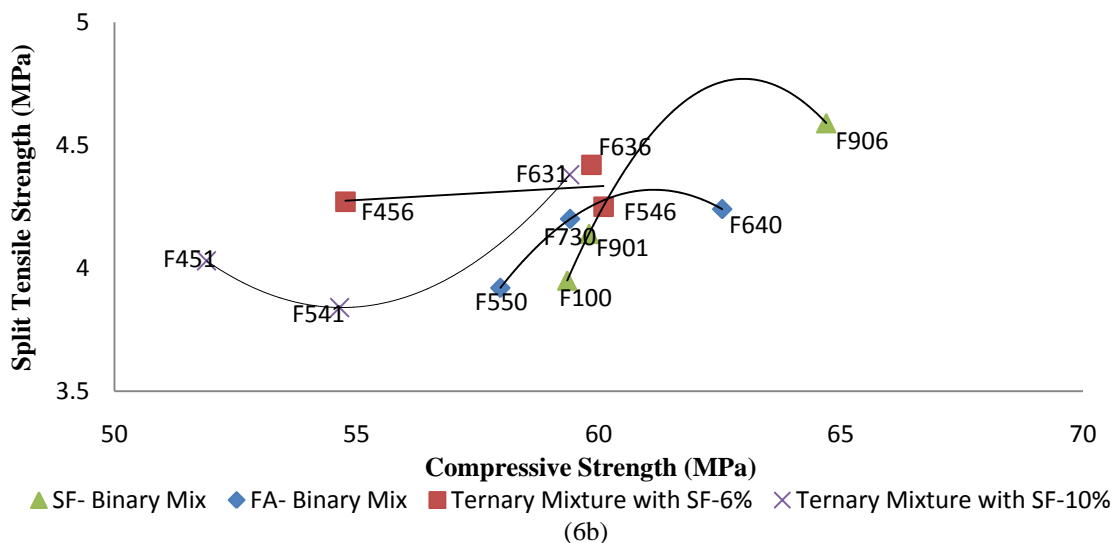
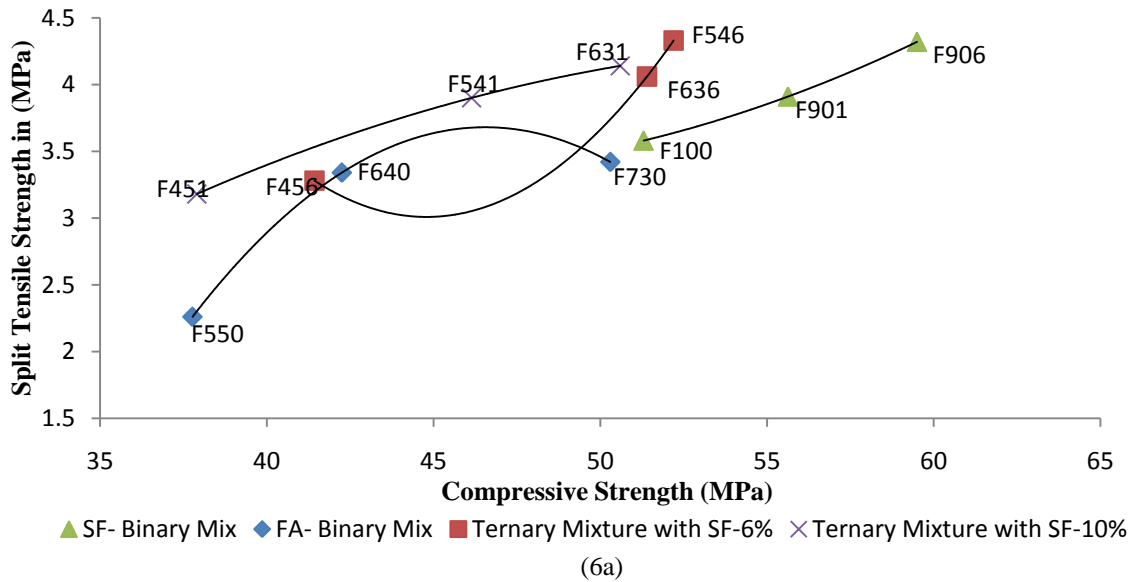


Fig. 6 Regression plot of compressive strength values in relation to the split tensile strength at 28 days and 90 days

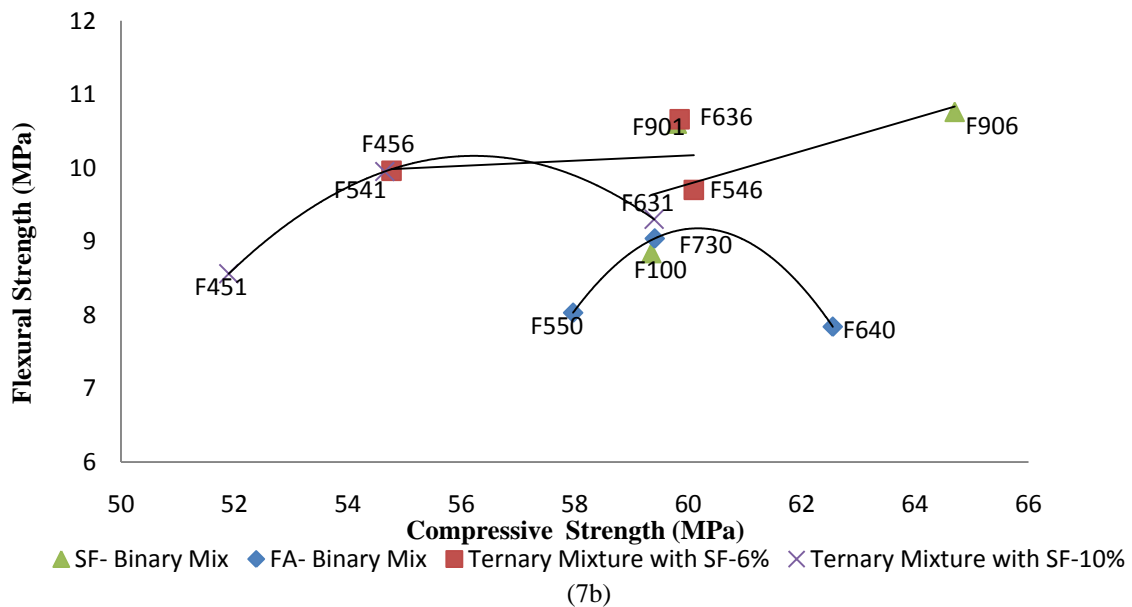
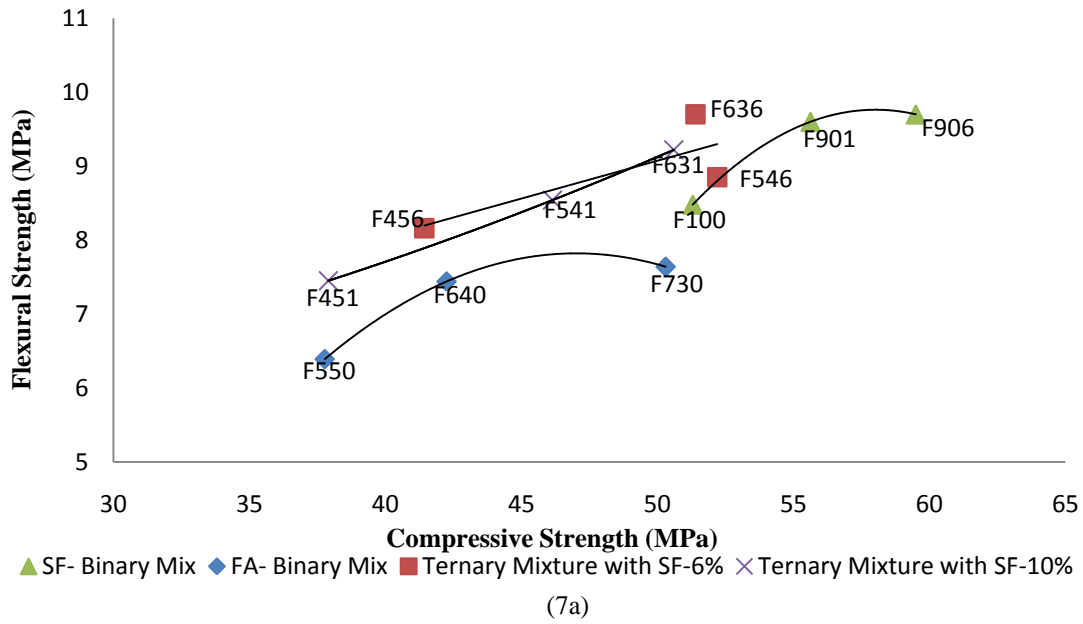


Fig. 7 Regression plot of compressive strength values in relation to the flexural strength at 28 days and 90 days.

Polynomial trend lines were adopted for all mixtures except ternary mixtures with 6% SF at 90 days. To obtain a best fit line for predicting the split tensile strength with minimum percentage error linear trend line was considered (Fig. 6); further for flexural strength also similar trend was followed for 3 mixtures (binary SF mixtures – 90 days and ternary mixtures with 6% SF – 28 days & 90 days) which is shown in Fig. 7.

Based on the regression analysis, the statistical model for flexural strength is expressed as

$$f_{fl} = k_1 f_c^2 + k_2 f_c + k_3 \quad (1)$$

Based on the regression analysis, the statistical model for split tensile strength is expressed as

$$f_t = k_1 f_c^2 + k_2 f_c + k_3 \quad (2)$$

where, f_c – compressive strength of concrete at 28 or 90 days, f_{fl} – flexural strength of concrete at predicted age (28 and 90 days), f_t – split tensile strength of concrete at predicted age (28 and 90 days) and k_1 , k_2 and k_3 are the regression constants.

The compressive strength of concrete at 28 and 90 days obtained from experiments are substituted in equations (1) and (2), and the corresponding split tensile strength and flexural strength are predicted. The input range of the proposed model is for the concrete mixtures with total binder content of 400 kg/m³ to 450 kg/m³ with 30% to 50% replacement of cement with fly ash and or silica fume. The regression coefficients for binary-ternary combinations of FA and SF are given in Table 5 & 6 at 28 and 90 days respectively.

Table 5 Regression coefficients for split tensile strength and flexural strength of binary-ternary concrete mixtures at 28 days

Description	Mixture ID	28 Days Strength							
		Split Tensile Strength				Flexural Strength			
		k_1	k_2	k_3	R^2	k_1	k_2	k_3	R^2
Control	F100	0.0036	-0.3115	10.017	1	-0.0618	7.7802	-240.28	1
Binary Mix PC+SF	F906	0.0036	-0.3115	10.017	1	-0.0618	7.7802	-240.28	1
	F901								
Binary Mix PC+FA	F730								
	F640	-0.0154	1.4596	-30.87	0.965	-0.0135	1.6943	-49.035	0.6379
Ternary Mix PC+SF+FA With 6% SF	F550								
	F636	0.0545	-5.0034	117.02	0.8662	0.0000	0.1023	3.9567	0.6335
Ternary Mix PC+SF+FA With 10% SF	F456								
	F631	-0.0098	0.9294	-17.965	0.8647	0.0191	-2.1053	61.898	0.4548
	F541								
	F451								

Table 6 Regression coefficients for split tensile strength and flexural strength of binary-ternary concrete mixtures at 90 days

Description	Mixture ID	90 Days Strength							
		Split Tensile Strength				Flexural Strength			
		k_1	k_2	k_3	R^2	k_1	k_2	k_3	R^2
Control	F100	-0.0284	3.2927	-85.817	1	-0.2173	26.152	-777.67	0.9846
Binary Mix PC+SF	F906								
	F901	-0.0284	3.2947	-85.817	1	0.0000	0.2242	-3.6758	0.3902
Binary Mix PC+FA	F730								
	F640	-0.0075	0.7873	-12.649	0.8403	-0.2173	26.152	-777.67	0.9846
Ternary Mix PC+SF+FA With 6% SF	F550								
	F636	0.0000	0.0113	3.6557	0.1336	0.0000	0.0356	8.0356	0.0464
Ternary Mix PC+SF+FA With 10% SF	F456								
	F631	-0.0114	1.1191	-18.662	0.8567	-0.0981	10.282	-277.84	0.891

From the observations (Table 5 & 6), it is clearly seen that the proposed equation 1 is suitable to predict the strength of control concrete and binary mixtures. Since the accuracy of the output for ternary mixture is less, the model is mainly recommended for binary mixtures. On the other hand, this can be used for the ternary mixture with some limitations. Therefore the dependency on the experimental results can be minimized.

The difference in the predicted values and actual experimental results obtained for the splitting strength and

flexural strength, along with the errors, are tabulated in Table 7 and 8. From these tables, it is observed that, the binary PC+SF mixtures have a maximum error of 8.19% for mixtures incorporating 6% SF for both split and flexural strength. On the other hand, the binary PC+FA mixtures showed a maximum of 5.36% error for mixtures incorporating 30% FA content. The percentage of error decreases with increase in FA content for both split tensile strength and flexural strength on all days.

Table 7 Predicted split tensile strength and flexural strength of binary-ternary concrete mixtures at 28 days and the corresponding error values

Mixture id	Curing age in days	Compressive Strength Vs Split Tensile Strength				Compressive Strength Vs Flexural Strength			
		f_c	$f_{t.act.}$	$f_{t.pre}$	% Error	f_c	$f_{\beta.act.}$	$f_{\beta.pre}$	% Error
F100	28	51.3	3.58	3.52	1.92	51.3	8.48	8.46	0.22
F906	28	59.5	4.32	4.22	2.13	59.5	9.7	9.67	0.26
F901	28	55.63	3.91	3.83	2.06	55.63	9.6	9.57	0.23

F730	28	50.3	3.42	3.58	4.8	50.3	7.64	7.97	4.4
F640	28	42.25	3.34	3.3	1.31	42.25	7.44	7.22	2.87
F550	28	37.77	2.26	2.29	1.30	37.77	6.39	6.38	0.03
F636	28	51.4	4.06	3.84	5.61	51.4	9.7	9.21	5.00
F546	28	52.2	4.33	4.35	0.38	52.2	8.85	9.29	5.04
F456	28	41.43	3.28	3.28	0.14	41.43	8.16	8.19	0.42
F631	28	50.6	4.14	3.98	4.07	50.6	9.22	8.77	4.81
F541	28	46.14	3.9	4.05	3.95	46.14	8.54	8.7	1.91
F451	28	37.9	3.18	3.18	0.07	37.9	7.45	7.37	0.98
Average Error					2.28	Average Error			2.18

Table 8 Predicted split tensile strength and flexural strength of binary-ternary concrete mixtures at 90 days and the corresponding error values

Mixture id	Curing age in days	Compressive Strength Vs Split Tensile Strength				Compressive Strength Vs Flexural Strength			
		fc	ft.act.	ft.pre	% Error	fc	f _l .act.	f _l .pre	% Error
F100	90	59.35	3.95	3.96	0.48	59.35	8.84	9.02	2.13
F906	90	64.7	4.59	4.38	4.45	64.7	10.76	10.82	0.65
F901	90	59.8	4.14	3.96	4.23	59.8	10.6	9.73	8.19
F730	90	59.41	4.2	3.97	5.36	59.41	9.04	9.04	0.1
F640	90	62.55	4.24	4.12	2.71	62.55	7.84	7.95	1.41
F550	90	57.97	3.92	3.81	2.63	57.97	8.03	8.12	1.12
F636	90	59.85	4.42	4.33	1.99	59.85	10.66	10.16	4.63
F546	90	60.1	4.25	4.34	1.96	60.1	9.7	10.17	4.89
F456	90	54.77	4.27	4.27	0.10	54.77	9.96	9.98	0.25
F631	90	59.4	4.38	4.23	3.31	59.4	9.3	9	3.14
F541	90	54.65	3.84	3.88	1.24	54.65	9.95	9.89	0.5
F451	90	51.9	4.03	4.08	1.26	51.9	8.56	8.52	0.43
Average Error					2.48	Average Error			2.29

For further analysis to gain a better understanding, the ternary mixtures are split into two categories; (1) (PC+SF+FA) incorporating 6% SF, and (2) (PC+SF+FA) incorporating 10% SF. The ternary mixtures with 6% SF showed a maximum error of 5.61% for mixtures with a lesser percentage of FA, i.e. F636, followed by F546 and F456. This behaviour can be directly attributed to the binary (PC+FA) mixture behaviour. Similarly the ternary mixture with 10% SF showed exactly the same behaviour as the ternary mixture with 6% SF; i.e. F631 showed a higher percentage of error followed by F541 and F451. The average percentage error values are 2.28 and 2.18 for split tensile strength and flexural strength respectively at

28 days, and the values are shown in Table 7. Similarly, the average error for split tensile strength and flexural strength at 90 days are 2.48 and 2.29 respectively as shown in Table 8. Therefore the average error is less than 5%, which is negligible.

For validation and to check the efficiency of the proposed model (equation 2) in predicting the output, the test data from L. Lam [18] was used. In that the binder content of 400 kg/m³ and 410 kg/m³ with replacement of 30 to 50% were only considered as it falls within the input range of the proposed model. The predicted values and error percentage are furnished in Table 9.

Table 9 Predicted split tensile strength of binary-ternary concrete mixtures at 28 days using L.Lam[18] experimental results

Mixture id	Compressive Strength Vs Split Tensile Strength			
	f _{c,Lam}	ft _{.act.Lam}	ft _{.pre.Equ-2}	% Error
S2-0-0	60.7	3.93	4.37	11.3
S2-15-0	56	3.75	3.57	-4.7
S2-25-0	49.3	3.91	3.66	-6.4
S2-45-0	43.9	2.91	3.53	21.2
S2-0-5	69.4	4.57	5.60	22.5
S2-40-5	44.6	3.36	2.42	-28.0

where, f_{c,Lam} – L. Lam [18] experimental compressive strength of concrete at 28, ft_{.act.Lam} – L.Lam[18] experimental split tensile strength of concrete at 28 and ft_{.pre.Equ-2} – Predicted split tensile strength of concrete using equation-2 at 28.

5. Conclusions

The following main conclusions can be drawn based on the experimental investigation:

1. The addition of 6% and 10% silica fume in concrete results in significant increase in the compressive strength, split tensile strength and flexural strength at all testing periods compared to control concrete.
2. The binary (PC+SF) silica fume mixtures performed well with 6% SF as replacement and they showed 7% higher cube compressive strength, 9% higher split tensile strength and 3% higher flexural strength compared to the binary mixtures with 10% SF on all days.
3. Incorporating FA by 30% - 40% as a cement replacement had beneficial effects on both the compressive strength and tensile strength of concrete only at 90 days
4. Ternary mixtures with 6% SF and 30% FA (F636) exhibited high compressive strength on all days compared to the control concrete, whereas F546 showed a strength gain at day 28 and had the highest strength at 90 days compared to other ternary and control mixtures. The other ternary mixtures made with 10% S For 50% FA showed slightly lesser or equivalent compressive strength compared to control concrete.
5. The ternary mixtures incorporating 30-40% FA with 6-10% SF developed a higher split tensile strength and flexural strength at both 28 and 90 days compared to control concrete. The ternary mixtures F456 and F451 demonstrated lower split tensile strength and flexural strength at 28 days compared to the control mix, but there was a strength gain observed at 90 days.
6. From the insight obtained from the - present study, it can be concluded that it is possible to reduce cement content by 36-46% by incorporating FA and SF together as a ternary mixture. This reduction in cement by using the available mineral admixture would offer ecological benefits, helping to cut down the use of cement, and save energy and reduce the costs of concrete construction in countries with an abundant supply of fly ash.
7. Equation (1) & (2) are proposed to describe two relationships between split tensile strength and flexural strength, based on compressive strength. The percentage error for the proposed equations is minimal. Hence these equations are useful for predicting the split tensile strength and flexural strength of binary and ternary mixtures of concrete incorporating fly ash and silica fume at 28 and 90 days.

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