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A super-element based on finite element method for latticed columns

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Abstract

Latticed columns are frequently used in industrial steel structures. In some countries these built-up columns might be even used in other types of steel structures such as residential and commercial buildings. Besides, latticed columns are parts of skeletons of many historic buildings all around the world. To analyze a steel structure with latticed columns a more accurate numerical model for such a column seems to be essential. The lay-out and connectivity of constructing main profiles of a latticed column leads to formation of many shear zones along the length of a column. Therefore, considering shear effects on the behavior of a lattice column is inevitable. This paper proposed a new super-element with twelve degrees of freedom to be used in finite element modeling of latticed columns. The cross sectional area, moments of inertia, shear coefficient and torsional rigidity of the developed new element are derived. To compute these parameters with less complexity a model using only beam elements is also introduced. A general purpose finite element program named LaCE is developed. This FE program is capable of performing linear and nonlinear analysis of 3D-frames with latticed columns, considering shear deformation. To show the accuracy of the proposed element, several cases are studied. The outcome of these investigations revealed that the current-in-practice model for latticed columns suffers from some major shortcomings which to some extends are resolved by the proposed super-element. The developed element showed the capability of modeling a lattice column with good accuracy and less computational cost.

Keywords: *Finite element, Super-element, Latticed column, Shear deformation, Torsional rigidity, Nonlinear analysis, Stiffness reduction.*

1. Introduction

Latticed columns used to be the most dominant type of columns during nineteen and early twentieth century [1]. These columns have been serving as the pillars of many historic buildings and monuments all around the world for many years. Due to historical values, these buildings must be preserved. For maintaining, health monitoring and retrofitting of a historic building, an extensive structural analysis is inevitable. To the best knowledge of the authors, hardly an appropriate and convenient numerical model for analyzing a structure with some latticed columns is at our disposal. In such columns, the effect of the stiffness reduction due to shear deformations is significant and must be included in any analytical or numerical solution. Ignoring this effect could be very serious and might lead to collapse of buildings which are designed without considering this vital effect, especially during an earthquake.

Nowadays latticed columns are basically used in industrial structures. Besides, in some countries mostly in Asia, it is a common practice to use this type of columns in commercial and residential buildings with steel structures. Despite of its abundant uses, there is no extensive literature on this subject. The current research on the latticed columns has mainly focused on the derivation of critical loads for these columns [2-12]. Here we briefly address some of these methods.

In 1970 Fung et al [2] calculated the critical load of some latticed columns by introducing, a dimensionless parameter which was considered to include the shear effect. This parameter contained the effect of axial force, eccentricity of local diagonal members and extra stiffness due to the connections of the batten plates to the main profiles. The effects of end plates on the critical load as well as on the coefficients of slope-deflection equations were studied and graphically presented. Bruce in 1971 [3] studied the effects of end plates on critical loads. He effectively showed that end plates played an important role on the buckling strength of the latticed columns. Elastic and plastic analysis of latticed columns was the subject of a research conducted by Zhaomin and Zhikang in 1984 [4]. In 1990, a fundamental theoretical method was presented by Gjelsvik [5]. This method was based on simulating the open web of a latticed column by a continuous web with only shear stiffness. The effects of end plates on the magnitudes of the critical load were also studied. He constructed his theory on the following kinematic assumptions. (*i*) Each main profile behaves as a

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simple beam; (*ii*) the shear deformation within each main profile is negligible; (*iii*) the transverse displacements of both main profiles are the same (the web is transversely rigid); (*iv*) the web behaves as a shear panel which is continuously connected to the main profiles. By using this model the order of the governing differential equations changed from four to six. Gjelsvic obtained the critical load by solving the new governing equation. It is worthy of mention that the Euler theory, the Engesser theory and the spaced column theory are all special cases of his theory. In 1995, Pual [6] arranged a theoretical and experimental research to calculate critical loads of the latticed columns. The basic method and assumptions in his research were the same as those had been described by Gjelsvik [5] except that he assumed the replaced web would behave as a Timoshenko shear beam. In another research that was conducted by Pual in the same year, he assumed that the replaced web could carry bending, shear and axial forces. Besides, the effects of end plates on the critical load were considered. The obtained results were compared with the results from the Euler, Engesser and spaced column theories [7]. Li and Li in 2004 [8] proposed a generalized finite element for buckling analysis of tapered columns. They used Chebyshev polynomial approach and studied the effect of shear deformation on the buckling capacity of lattice columns. In 2010 Mijailovic [9] tried to optimize of lattice columns by considering buckling and deformation criteria as the constraints functions. He computed the optimization parameters by Lagrange's multipliers method. Razdolosky [10, 11] investigated the flexural buckling and slenderness ratio of laced columns. Finally, Jiang et al in 2011 [12] used both numerical methods and experimental results for studying ultimate load capacity of a special type of latticed columns.

1.1. The Proposed super-element

By reviewing the literature on the subject of latticed columns, one might notice that existing analytical and numerical models for studying the behavior of this type of columns specially subjected to seismic loads are not be very appropriate. To narrow the gap between the analytical results from the customary modeling of latticed columns and the actual behavior of these columns, developing a new numerical element could be constructive.

In order to be able to calibrate the results of our proposed element with the experimental data a vast investigation was performed. But no experimental results regarding the general behavior of a latticed column was available. Therefore, in this paper the actual behavior of a lattice column will be simulated by the behavior of a reference model. The reference model is a 3-D latticed column constructed by using 3-D solid elements introduced in *SAP2000*. To examine the performance of the proposed model, the results obtained from the developed model must be compared with the results from the reference model. The behavior of the reference model will also been compared with the behavior of the current customary model. The current model is a column which is defined by an individual steel section with the following equivalent parameters. (*i*) the cross-sectional area is equal to the total cross-sectional areas of both main profiles; (*ii*) the torsional rigidity is equal to the total torsional rigidities of both main profiles; (*iii*) the moment of inertia for bending in the plane perpendicular to the batten plates is equal to total moments of inertia of both main profiles; (*iv*) the moment of inertia for bending in the plane of batten plates is equal to the moments of inertia of both main profiles about the central axis perpendicular to the batten plates. One must notice that the current model neglects the effects of the panel shear deformations of the latticed column. By comparing the results from the reference model with the results from the current model, some equivalent parameters for the current model such as the equivalent cross-sectional area and moment of inertia in the plane perpendicular to batten plates could be derived. Then these parameters are employed to develop the stiffness matrix of the super element. Other proposed equivalent parameters of the super element have been modified to consider the panel shear deformations of the latticed column. Some formulas have been proposed to calculate the equivalent cross-sectional area, the equivalent moment of inertia and the shear factor for bending in the plane perpendicular to the batten plates and the equivalent torsional rigidity. To calculate these parameters, a model has been constructed by using only beam elements. To consider the effects of batten plates and end plates, a procedure is proposed.

By using this developed model, it is expected to obtain a good approximation of the actual behavior of a latticed column with a considerable reduction in the number of degrees of freedom which used in the reference model. At the end, the stiffness matrix of the super element, a 12 by 12 matrix in 3-D, has been developed by using the proposed equivalent parameters.

2. Modeling and Examining the Latticed Columns Behavior

To examine the behavior of a latticed column, a few models of this type of columns are constructed in 3-D space by using a large number of solid elements. The main profiles which have been chosen ranged from IPE140 to IPE270. In order to calculate the elements of the stiffness matrix of the proposed super-element, one end of the latticed column has been considered to be fixed and a rigid plate has been connected to the other end. Then, each time, only one of the end degrees of freedom has been released. By applying a force in the direction of the released degree of freedom, the corresponding deflection has been calculated. By dividing the applied force to the computed deflection, the stiffness component of the equivalent column of the latticed column, corresponds to the considered degree of freedom, has been calculated. Following this procedure, a large number of 3-D latticed columns have been analyzed using *SAP2000* and the equivalent stiffness components have been computed. In all studied cases, the width of the end plates was *200 mm*, batten plates width was *100 mm* and the length of all latticed columns was assumed to be *2100 mm*, Fig. (1). Table (1) shows some results of the analysis. In this table, t_e and t_b are respectively end and batten plates thicknesses, b is the distance between the center lines of both main profiles, k_X^s, k_Y^s and k_Z^s are displacement stiffness elements in the X ,Y and Z directions, respectively and k_θ^s is the rotational stiffness element. From the stiffness relations, one can approximately derive the equivalent cross-sectional area. The moment of inertia in the plane perpendicular to the batten plates is as follows

$$\frac{AE}{l} = k_Y^s \Rightarrow A_{eq} = \frac{lk_Y^s}{E} \tag{1}$$

$$\frac{12EI}{l^3} = k_Z^s \Rightarrow I_{eqZ} = \frac{l^3 k_Z^s}{12E} \tag{2}$$

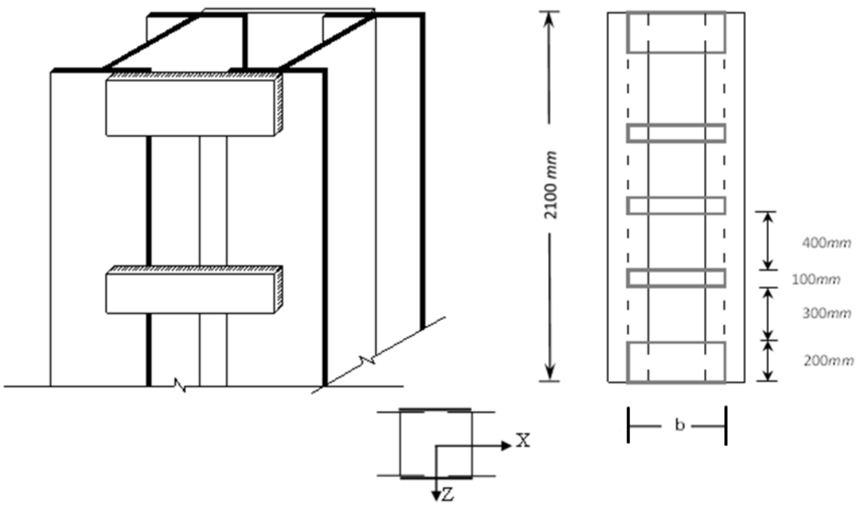


Fig. 1 The latticed column with the end and batten plates

Table 1 Linear analysis of the reference model (without shear deformations)

Main profile	b (mm)	$t_e = t_b$ (mm)	k_X^s (kg/mm)	k_Y^s (kg/mm)	k_Z^s (kg/mm)	$k_\theta^s \times 2.6e-4$ (kg/mm)
IPE140	150	6	326.5	34286.7	333.1	36.2
IPE140	150	10	346.6	34975.6	354.2	41.4
IPE200	200	8	863.2	57777.7	1078.9	155.9
IPE240	260	10	1547.3	79283.9	2015.9	344.0
IPE240	260	14	1671.3	80053.1	2075.8	387.1

Table (2) shows the results obtained by applying these proposed relations. In this table A_{eq} is the equivalent cross-sectional area, A_I is the cross-section of each main profile, I_{eqZ} is the equivalent moment of inertia in the plane perpendicular to the batten plates and I_{CZ} is the moment of inertia of each main profile about its strong axis.

The equivalent stiffness elements of a latticed column, in the linear state but with shear deformation, have been defined by using solid elements and the *SAP2000* program. To compute the equivalent stiffness elements with fewer calculations, we constructed a model which exclusively uses the beam elements.

Table 2 A comparison between the results of Table (1) and the proposed A_{eq} and I_{eqZ}

Main profile	b (mm)	$t_e = t_b$ (mm)	A_{eq} (mm^2)	$I_{eqZ} \times e-4$ (mm^4)	$2A_I$ (mm^2)	$2I_{CZ} \times e-4$ (mm^4)	*Error* *A%*	*Error* I_Z*%*
IPE140	150	6	3428	1224	3280	1082	4.33	11.6
IPE140	150	10	3497	1301	3280	1082	6.21	16.6
IPE200	200	8	5777	3965	5700	3880	1.33	2.3
IPE240	260	10	7928	7409	7820	7780	1.36	4.7
IPE240	260	14	8005	7629	7820	7780	2.31	1.9

2.1. Mathematical modeling of the lattice columns

Using the results of Table (2), it can be concluded, with a good approximation, that

$$A_{eq} \cong 2A_I \tag{3}$$

Although the equivalent moment of inertia in the plane perpendicular to the batten plates increases in some cases and decreases in some others with respect to I_{CZ}, but still it is fair to say that

$$I_{eqZ} \cong 2I_{CZ} \tag{4}$$

For computation of displacements in the *X* direction and bending moments about the *Z* axis, based on the properties of the end and batten plates in the actual model (Fig. (2-a)), a model which is constructed by using only the beam elements has been proposed (Fig. (2-b)). In the regions of end and batten plates, the latticed column behaves almost like a solid section. Since the proposed model is constructed by only beam elements (Fig. (2-b)), in order to consider these solid areas, the properties of end and batten plates have been divided between these areas as follows. In Fig. (2-b) the cross-sectional area and the moment of inertia for the element number *1* are equal to those of the end plate. The cross-sectional area and the moment of inertia for the element number *2* are equal to half of those quantities computed for the end plate and finally, the cross-sectional area and the moment of inertia for the element number *3* are equal to half of those quantities computed for a batten plate.

Elements number *4*, *5* and *6* are also considered because in these areas, end and batten plates are welded to main profile webs (Fig. (2-a)). Therefore, in bending and lateral displacement, these areas work with the main profile webs and make them stiffer. The properties of these areas are given in Eqns. (5) through (7). The assigned additional cross-sectional area and the moment of inertia for the element number *4* are equal to half those values assigned for the main profile webs in the direction of the end plates width (Fig. (2-b)). Therefore,

$$A = 0.50 h_e t_w \text{ and } I = 0.50\left(\frac{h_e^3 t_w}{12}\right) \tag{5}$$

where, h_e is the end plates width, t_w is the web thickness of main profiles, A and I are the additional cross-sectional area and the moment of inertia added to corresponding quantities of element number *5*. Thus

$$A = 0.50\left(\frac{1}{2} h_e t_w\right) \text{ and } I = 0.50\left(\frac{1}{2}\frac{h_e^3 t_w}{12}\right) \tag{6}$$

For element number *6* these parameters are as follows

$$A = 0.50(h_b t_w) \text{ and } I = 0.50\left(\frac{h_b^3 t_w}{12}\right) \tag{7}$$

where h_b is the batten plate width. The displacement stiffness, k_X^s, obtained for the latticed column by analyzing this column using both the reference model (Table (1)) and the stiffness, k_X^b, obtained from the proposed model (Fig. (2-b)) by the *SAP2000* program have been compared in Table (3).

For controlling the behavior of the proposed model in bending about the *Z* axis, through an intensive investigation many columns have been analyzed. Some of the results are summarized in Table (4). In all these cases, one end of the latticed column is fixed and the other end is released for displacement in the *X* direction and rotation about the *Z* axis.

Parameters used in Table (4) are defined as follows. Δ_X^s is the end horizontal displacement of the latticed column analyzed by the reference model in the *X* direction, θ^s is the end rotation of the latticed column analyzed by the reference model about the *Z* axis. Δ_X^b and θ^b have the same definitions as Δ_X^s and θ^s, respectively but for the latticed column analyzed by the proposed model.

Based on the results summarized in the Tables (3) and (4), one can conclude that the proposed model is capable, with a good accuracy, to present the behavior of the latticed column for displacement in the *X* direction and rotation about the *Z* axis.

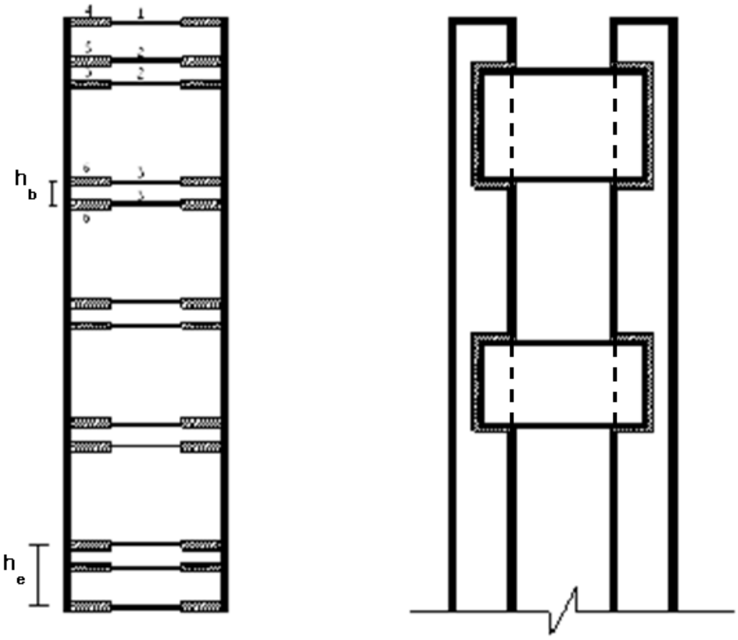
Table 3 A comparison between k_X^s and k_X^b

Main profile	b (*mm*)	$t_e = t_b$ (*mm*)	k_X^s (*kg/mm*)	k_X^b (*kg/mm*)	Error %
IPE140	150	6	326.59	338.11	3.4
IPE140	150	10	346.69	347.83	0.3
IPE200	200	8	863.27	887.61	2.7
IPE240	260	10	1547.32	1492.26	-3.5

IPE240	260	14	1671.38	1640.31	-1.8

Table 4 A comparison between the SAP2000 analysis results of the latticed columns using the reference model and the proposed model

Main profile	b (mm)	$t_e = t_b$ (mm)	M (T-m)	Δ_X^s (mm)	θ^s (rad)	Δ_X^b (mm)	θ^b (rad)	error $\Delta\%$	Error $\theta\%$
IPE140	150	6	6.0	32.1	0.030	32.5	0.031	1.3	1.6
IPE140	150	10	6.0	31.7	0.030	32.5	0.031	2.4	2.8
IPE200	200	8	8.0	14.1	0.013	14.0	0.013	-0.7	0.8
IPE240	260	10	8.0	8.0	0.007	7.9	0.007	-1.1	0.1
IPE240	260	14	10.4	7.9	0.007	7.9	0.007	-0.4	1.6



(**a**) The real model in the X-Y plane (**b**) The proposed model in the X-Y plane

Fig. 2 Different models of the latticed column

To calculate the equivalent torsional rigidity of a latticed column, the following features must be considered. In the zones of end and batten plates, the latticed column behaves like a thin-walled section, so the torsional rigidity in these areas is obtained based on the equations derived for thin-walled sections. Elsewhere, the latticed column behaves similar to an open section. In the regions where end and batten plates exist, both main profiles cannot be twisted separately. Fig. (3) shows the approximate twisting angle and the approximate internal forces in the cross section. Therefore,

$$T = \frac{GJ}{l}\theta \qquad T = f_1 b + f_2 d \tag{8}$$

where θ is the twisting angle, T is the applied torque, J is the torsional rigidity, and f_1 and f_2 are internal forces in the webs and flanges of each main profile, respectively. From Fig. (3), the compatibility equation can be written is as follows

$$\theta = \frac{2\Delta_1}{b} = \frac{2\Delta_2}{d} \tag{9}$$

Using Eqns. (8) and (9) one can obtain

$$\frac{f_1}{\Delta_1} b\Delta_1 + \frac{f_2}{\Delta_2} d\Delta_2 = \frac{GJ}{l}\theta \tag{10}$$

where $\frac{f_1}{\Delta_1}$ and $\frac{f_2}{\Delta_2}$ are the stiffnesses in the Z and X directions in the global coordinate system, respectively. Thus

$$k_1 = \frac{f_1}{\Delta_1} = \frac{12EI_{CZ}}{l^3} \quad \text{and} \quad k_2 = \frac{f_2}{\Delta_2} = \frac{12EI_{fl}}{l^3} \tag{11}$$

where I_{fl} is defined in the following equation.

$$I_{fl} = 2\left[b_f t_f \left(\frac{b}{2}\right)^2 + \frac{b_f^3 t_f}{12} \right] \tag{12}$$

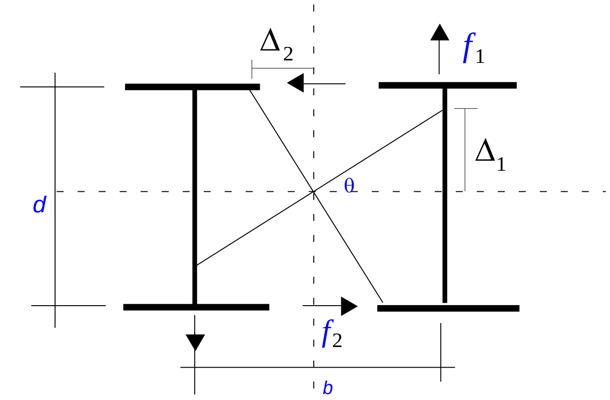


Fig. 3 The approximate twisting shape of the latticed columns in the zones without the end and batten plates

Equations (11) and (12) have been derived by neglecting the effects of batten plates and assuming that in twisting action due to presents of end plates, the webs and flanges of main profiles somehow could work together and behave as an integrated solid member. Therefore, Eqn. (10) could be written as shown here.

$$J_1 = \frac{1}{2G}\left(k_1 b^2 + k_2 d^2\right) \tag{13}$$

The torsional rigidity in the zones with batten plates, Fig. (4), is calculated based on the relations derived for thin-walled sections as follows

$$J = \frac{Q^2}{\oint \frac{ds}{t}} \tag{14}$$

where in this equation, the numerator and denominator are defined as

$$Q = bd\ , \quad \oint \frac{ds}{t} = 2\left[\frac{b - b_f}{t_b} + \frac{b_f}{t_f + t_b} + \frac{d}{t_w} \right] \tag{15}$$

Derivation of Eqn. (14) can be found in reference [13]. Thus,

$$J_2 = \frac{2b^2 d^2}{\dfrac{b - b_f}{t_b} + \dfrac{b_f}{t_f + t_b} + \dfrac{d}{t_w}} \tag{16}$$

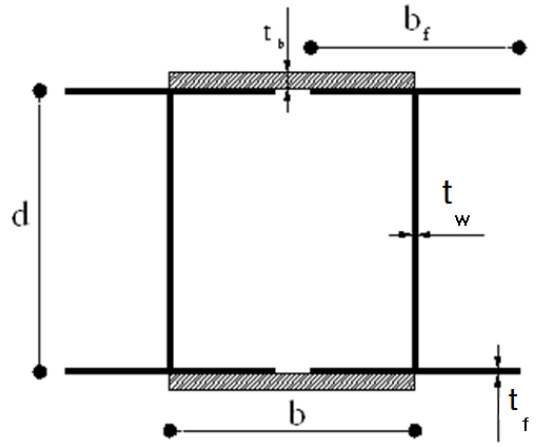


Fig. 4 The section of the latticed column in the zones with batten plates

Based on the assumptions used for computation of the torsional rigidity, a good approximation is that the torsional rigidities of the zones with batten plates are uniformly distributed through the entire length of the latticed column. Thus,

$$J_{eq} = J_1 + \frac{nh_b}{(1 - 2h_e)} J_2 \tag{17}$$

where, n is the number of batten plates. Table (5) shows a comparison between the results of Eqn. (17) with the analysis results of the latticed column computed by using the reference model. In this table, T is the applied torque at the free end of the latticed column and J_{eq}^s is the torsional rigidity calculated from the reference model.

Table 5 A comparison between the J_{eq}^s and J_{eq}

Main profile	b (mm)	$t_e = t_b$ (mm)	T (T-m)	$J_{eq}^s \times e-4$ (mm^4)	$J_{eq} \times e-4$ (mm^4)	Error %
IPE140	150	6	5.8	362	348	-3.8
IPE140	150	10	5.8	414	388	-6.3
IPE200	200	8	8.0	1559	1377	-11.6
IPE240	260	10	10.0	3440	3690	6.7
IPE240	260	14	10.0	3871	3867	0.1

2.2. The equivalent parameters of the latticed columns

The equivalent moment of inertia for bending about the Z axis based on the proposed model and the concept of condensation can be computed using Eqn. (18) [14],

$$I_{eqX} = \frac{k_X^h l\left(l^2 + 12 g_{eqX}\right)}{12E} \tag{18}$$

Since the panel-shear deformations of a latticed column play a crucial role on the behavior of this type of columns, the equivalent shear factor is obtained as follows (the procedure for finding this relation can be found in the reference [15]).

$$\alpha_{eqX} = \frac{EI_{eqX}}{GA_{eq}\left[\frac{k_Y^b}{k_X^b} - \frac{l^2}{3}\right]} \tag{19}$$

By using these equivalent parameters, the stiffness matrix of the proposed super element is obtained as follows.

$$\mathrm{K} = \begin{bmatrix} \mathrm{k}_1 & \mathrm{k}_2 \\ \mathrm{k}_2^T & \mathrm{k}_4 \end{bmatrix} \tag{20}$$

The elements of the stiffness matrix without considering the shear effects are introduced in Eqns. (21) through (23).

$$\mathrm{k}_1 = \begin{bmatrix} A_{eq}E/l & 0 & 0 & 0 & 0 & 0 \\ 0 & 12EI_{eqZ}/l^3 & 0 & 0 & 0 & 6EI_{eqZ}/l^2 \\ 0 & 0 & 12EI_{eqX}/l^3 & 0 & -6EI_{eqX}/l^2 & 0 \\ 0 & 0 & 0 & GJ_{eq}/l & 0 & 0 \\ 0 & 0 & -6EI_{eqX}/l^2 & 0 & 4EI_{eqX}/l & 0 \\ 0 & 6EI_{eqZ}/l^2 & 0 & 0 & 0 & 4EI_{eqX}/l \end{bmatrix} \tag{20}$$

$$\mathrm{k}_2 = \begin{bmatrix} -A_{eq}E/l & 0 & 0 & 0 & 0 & 0 \\ 0 & -12EI_{eqZ}/l^3 & 0 & 0 & 0 & 6EI_{eqX}/l^2 \\ 0 & 0 & -12EI_{eqX}/l^3 & 0 & -6EI_{eqX}/l^2 & 0 \\ 0 & 0 & 0 & -GJ_{eq}/l & 0 & 0 \\ 0 & 0 & 6EI_{eqX}/l^2 & 0 & 2EI_{eqX}/l & 0 \\ 0 & -6EI_{eqZ}/l^2 & 0 & 0 & 0 & 2EI_{eqZ}/l \end{bmatrix} \tag{22}$$

$$\mathrm{k}_3 = \begin{bmatrix} A_{eq}E/l & 0 & 0 & 0 & 0 & 0 \\ 0 & 12EI_{eqZ}/l^3 & 0 & 0 & 0 & -6EI_{eqZ}/l^2 \\ 0 & 0 & 12EI_{eqX}/l^3 & 0 & 6EI_{eqX}/l^2 & 0 \\ 0 & 0 & 0 & GJ_{eq}/l & 0 & 0 \\ 0 & 0 & 6EI_{eqX}/l^2 & 0 & 4EI_{eqX}/l & 0 \\ 0 & -6EI_{eqZ}/l^2 & 0 & 0 & 0 & 4EI_{eqZ}/l \end{bmatrix} \tag{23}$$

2.3. Stiffness matrix of a 3-D frame element

To develop a general purpose3-D finite element program for analyzing 3-D frames with latticed columns subjected to gravity and lateral loads, the stiffness matrix of a frame element, with shear deformation and axial force effects, can be written as follows

$$K = K_e + K_g \tag{24}$$

where K_e and K_g are the linear and geometric stiffness matrixes. The elements of these matrices are given in appendix (A).

By employing the developed stiffness matrix for the super element and stiffness matrix for the frame element, a general purpose finite element program, named *LaCE*, has been written in *MATLAB* environment.

3. Numerical Study

To show the capabilities of the proposed model, a single bay frame is considered, Fig.(5). The right column of the considered frame is latticed. This frame is analyzed using both the reference model and the proposed model, separately. Table (6) shows displacements of the frame computed at the point of load application using these two models by *SAP2000*. One can observe that the behavior of the proposed model follows with the behavior of the reference model with a reasonable approximation. The second column in Table (6) expresses the efficiency of the proposed model in reducing the computational cost with respect to the computational cost required for the reference model.

Beam: IPE 300
Main column: IPE 200
Battened Plates: 100×8 mm
End plates: 200×8 mm

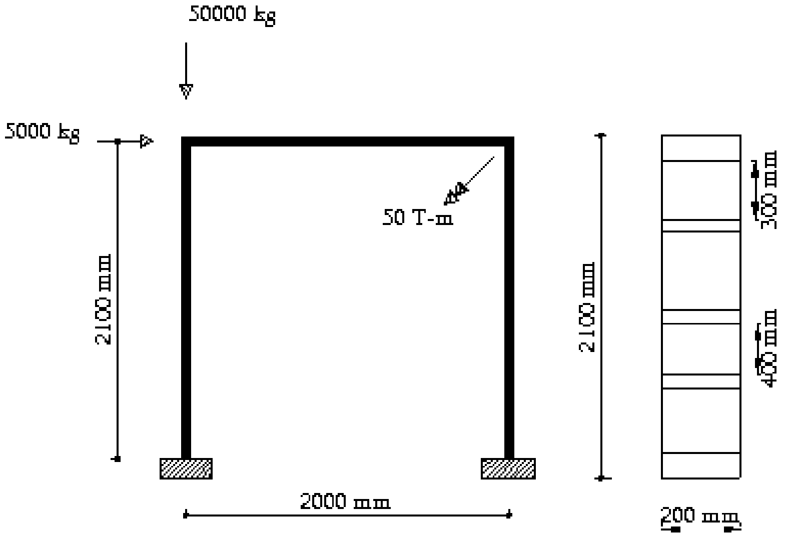


Fig. 5 A simple frame with a latticed column.

Table 6 Analysis results of case (1) (using the proposed and the reference models)

Model	Degrees of freedom	Δ_x Disp. (*mm*)	Δ_y Disp. (*mm*)	θ Rotation(*rad*)
Reference model	3438	1.68	-0.447	-0.00053
Proposed model	122	1.71	-0.447	-0.00054
Error %	-	1.66	-0.070	2.04

In order to quantify the discrepancy between the current model and the proposed model, quite a few problems have been analyzed using the *LaCE* program. Here, as an example, the results of a five story building frame are presented, Fig. (6). In this frame two kinds of latticed column have been used in the first and second stories, Table (7). The beams are the same and made of *IPE300* and all columns which are not latticed are the same and made of *IPE240*. First, the frame has been linearly analyzed while considering the shear deformations. Then, the latticed columns were replaced by their current customary equivalent columns and the frame has been linearly reanalyzed without considering the effect of shear deformations. Table (8) shows the comparison between the story drifts for these two different cases. The second and third columns in Table (8) show the story drifts computed from the frame analysis which employed the proposed model and the current customary model (S.D.L, S.D.C). One can observe from Table (8) that using the current model instead of the latticed column causes a significant error in the story drifts of the building frames without lateral bracing system.

Table 7 Geometrical properties of the latticed columns used in case (2)

Element location	b (*mm*)	Main Profile	Batten plates (*mm*x*mm*)	End plates (*mm*x*mm*)
Lateral columns	200	**IPE**160	100×10	200×10
Middle columns	200	**IPE**180	100×10	200×10

Table 8 The analysis results of case (2) (using the current and the proposed models)

Story	*S.D.L.*$^{+}$ (*mm*)	*S.D.C.*$^{++}$ (*mm*)	Error %
1	102.5	73.3	28.44
2	214.9	158.2	28.49
3	251.1	190.9	23.97
4	278.7	216.1	22.46
5	297.7	233.7	21.51

+ Story drift from the proposed model
++ Story drift from the current model

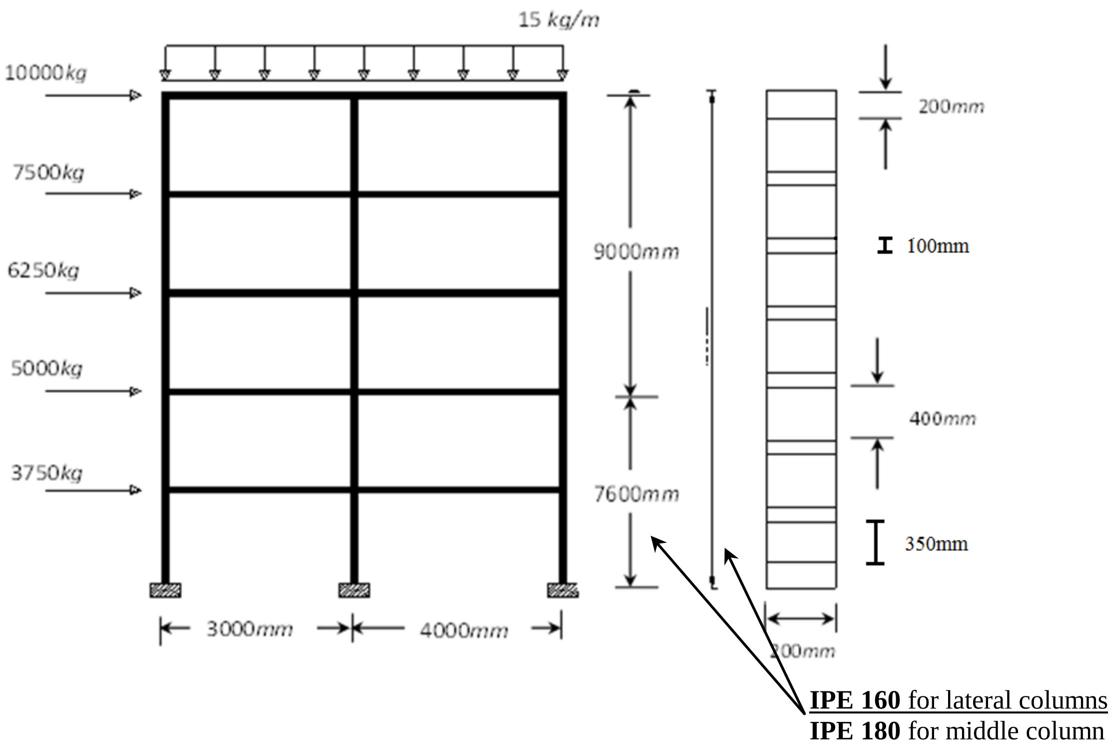


Fig. 6 A five story building frame.

4. Conclusion

Based on the relations obtained for calculation of the equivalent parameters of the proposed super-element and also the results shown in the Tables (1) through (5), one can conclude that the stiffness matrix of the proposed super element, (Eqn. 20), is capable of providing a suitable model of a latticed column stiffness matrix. From the case studies and the results summarized in Tables (6) and (8), it is shown that the current and customary models of latticed columns cannot suitably represent the behavior of these columns. Finally, it is demonstrated that the proposed model can achieve this goal with a good approximation and a considerable reduction in computational cost.

Symbols

A	cross-sectional area (Eqn.(1), L^2)
A_I	cross-sectional area of main profiles (Table (2), L^2)
A_{eq}	equivalent cross-sectional area of the latticed column (Eqn. (1), L^2)
b	distance between the center lines of main profiles (Table (1), L)
b_f	flang width of the main profile (Eqn. (15), L)
d	depth of the main profile (Eqn. (15), L)
E	modulus of elasticity (Eqn.(1), ML^{-2})
f_1	approximate force due to twist in the web of main profiles (Eqn. (8), MLT^{-2})
f_2	approximate force due to twist in the web of main profiles (Eqn. (8), MLT^{-2})
G	shear modulus (Eqn. (8), ML^{-2})
h_b	batten plate width (Fig. (2-b), L)
h_e	end plate width (Fig. (2-b), L)
I	moment of inertia (Eqn. (2), L^4)
I_{CZ}	moment of inertias of the main profiles about their strong axes (Eqn. (2), L^4)
I_{eqX}	equivalent moment of inertia of the latticed column in the X-Y plane (Eqn. (18), L^4)
I_{eqZ}	equivalent moment of inertia of the latticed column in the Y-Z plane (Eqn. (2), L^4)
J	torsional rigidity (Eqn.(8), MLT^{-2})
J_1	equivalent torsional rigidity of the latticed column in regions with no batten plates (Eqn.(13), MLT^{-2})
J_2	equivalent torsional rigidity of the latticed column in regions with batten plates (Eqn. (16), MLT^{-2})
J_{eq}	equivalent torsional rigidity of the latticed column (Eqn. (17), MLT^{-2})
K	stiffness matrix of the super element (Eqn. (20), ML^{-1})
K_e	linear stiffness matrix (Eqn. (24), ML^{-1})
K_g	geometric stiffness matrix (Eqn. (24), ML^{-1})
K_X^s	displacement stiffness of the reference model in the X-direction (Table (1), ML^{-1})
K_Y^s	displacement stiffness of the reference model in the Y-direction(Table (1), ML^{-1})
K_Z^s	displacement stiffness of the reference model in the Z-direction (Table (1), ML^{-1})
K_θ^s	torsional rigidity of the reference model (Table (1),

	ML^{-1})	
l	length of the latticed column (Eqn. (1), L)	
M	bending moment (Table (4), ML)	
n	number of batten plates (Eqn. (17))	
T	twisting torque (Eqn. (8), ML)	
t_b	thickness of the batten plates (Table (1), L)	
t_e	thickness of the end plates (Table (1), L)	
t_f	flang thickness of the main profiles (Eqn. (15), L)	
t_w	web thickness of the main profiles (Eqn. (15), L)	
α_{eqX}	equivalent shear factor of the latticed column in the X-direction (Eqn. (19))	
θ	twisting angle (Eqn. (8))	
θ^b	bending angle in the proposed model about the Z-axis (Table (4))	
θ^s	bending angle in the reference model about the Z-axis (Table (4))	
Δ_1	lateral displacement in the latticed column in the Z-direction due to the torque about the Y-axis (Fig. (3), L)	
Δ_2	lateral displacement in the latticed column in the X-direction due to the torque about the Y-axis (Fig. (3), L)	
Δ_X^b	lateral displacement in the proposed model in the X-direction (Table (4), L)	
Δ_X^s	lateral displacement in the reference model in the X-direction (Table (4), L)	

Appendix (A)

The stiffness matrix of the frame element with shear and axial deformations is as follows

$$\mathrm{K} = \mathrm{K}_e + \mathrm{K}_g \tag{A-1}$$

where K_e is the linear part and K_g is the geometric part of the stiffness matrix defined in the following equations

$$\mathrm{K}_e = \begin{bmatrix} \mathrm{k}_{1e} & \mathrm{k}_{2e} \\ \mathrm{k}_{2e}^T & \mathrm{k}_{3e} \end{bmatrix}_{12\times12} \tag{A-2}$$

and $$\mathrm{K}_g = \begin{bmatrix} \mathrm{k}_{1g} & \mathrm{k}_{2g} \\ \mathrm{k}_{2g}^T & \mathrm{k}_{2g} \end{bmatrix}_{12\times12} \tag{A-3}$$

The elements of the matrix K_e are

$$\mathrm{k}_{1e} = \begin{bmatrix} e_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & e_{1z} & 0 & 0 & 0 & e_{2z} \\ 0 & 0 & e_{1y} & 0 & -e_{2y} & 0 \\ 0 & 0 & & e_5 & 0 & 0 \\ 0 & 0 & -e_{2y} & 0 & e_{3y} & 0 \\ 0 & e_{2z} & 0 & 0 & 0 & e_{3z} \end{bmatrix}$$

$$\mathrm{k}_{2e} = \begin{bmatrix} -e_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -e_{1z} & 0 & 0 & 0 & e_{2z} \\ 0 & 0 & -e_{1y} & 0 & -e_{2y} & 0 \\ 0 & 0 & 0 & -e_5 & 0 & 0 \\ 0 & 0 & e_{2y} & 0 & e_{4y} & 0 \\ 0 & -e_{2z} & 0 & 0 & 0 & e_{4z} \end{bmatrix} \tag{A-4}$$

$$\mathrm{k}_{3e} = \begin{bmatrix} e_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & e_{1z} & 0 & 0 & 0 & -e_{2z} \\ 0 & 0 & e_{1y} & 0 & e_{2y} & 0 \\ 0 & 0 & 0 & e_5 & 0 & 0 \\ 0 & 0 & e_{2y} & 0 & e_{3y} & 0 \\ 0 & -e_{2z} & 0 & 0 & 0 & e_{3z} \end{bmatrix}$$

The parameters used in Eqn. (A-4) are defined in Eqn. (A-5).

$$\begin{gathered} e_1 = \frac{AE}{a}\ ;\ e_{1z} = \frac{12EI_Z}{a(a^2+12g_Z)}\ ;\ e_{2z} = \frac{6EI_Z}{(a^2+12g_Z)}\ ;\ e_{3z} = \left(\frac{a^2}{3}+g_z\right)\frac{12EI_Z}{a(a^2+12g_z)}; \\ e_{4z} = \left(\frac{a^2}{6}-g_z\right)\frac{12EI_z}{a(a^2+12g_z)}\ ;\ e_5 = \frac{GJ}{a}\ ;\ e_{1y} = \frac{12EI_y}{a(a^2+12g_y)}\ ; \\ e_{2y} = \frac{6EI_y}{(a^2+12g_y)}\ ;\ e_{3y} = \left(\frac{a^2}{3}+g_y\right)\frac{12EI_y}{a(a^2+12g_y)}\ ;\ e_{4y} = \left(\frac{a^2}{6}-g_y\right)\frac{12EI_y}{a(a^2+12g_y)} \end{gathered} \tag{A-5}$$

where a is the length of the beam element, A is cross-sectional area, E is modulus of elasticity, G is shear modulus, I_y and I_z are moment of inertias about the y and z axes, respectively Fig. (5), and J is torsional moment of inertia. Parameters g_y and g_z represent the shear deformation effects and are defined as follows

$$\begin{gathered} g_z = \frac{EI_z}{\alpha_z GA} \\ g_y = \frac{EI_y}{\alpha_y GA} \end{gathered} \tag{A-6}$$

The elements of geometric stiffness matrix, K_g, are

$$
k_{1g} = P\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{1z} & 0 & 0 & 0 & c_{2z} \\ 0 & 0 & c_{1y} & 0 & -c_{2y} & 0 \\ 0 & 0 & 0 & c_5 & 0 & 0 \\ 0 & 0 & -c_{2y} & 0 & c_{3y} & 0 \\ 0 & c_{2z} & 0 & 0 & 0 & c_{3z} \end{bmatrix}
$$

$$
k_{2g} = P\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c_{1z} & 0 & 0 & 0 & c_{2z} \\ 0 & 0 & -c_{1y} & 0 & -c_{2y} & 0 \\ 0 & 0 & 0 & -c_5 & 0 & 0 \\ 0 & 0 & c_{2y} & 0 & c_{4y} & 0 \\ 0 & -c_{2z} & 0 & 0 & 0 & c_{4z} \end{bmatrix} \qquad \text{(A-7)}
$$

$$
k_{3g} = P\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{1z} & 0 & 0 & 0 & -c_{2z} \\ 0 & 0 & c_{1y} & 0 & c_{2y} & 0 \\ 0 & 0 & 0 & c_5 & 0 & 0 \\ 0 & 0 & c_{2y} & 0 & c_{3y} & 0 \\ 0 & -c_{2z} & 0 & 0 & 0 & c_{3z} \end{bmatrix}
$$

The parameters that used in these matrixes are defined in Eqn. (A-8).

$$
\begin{aligned}
c_{1z} &= \left[1.2a^3 + 144g_z^2/a + 24g_z a\right] / \left(a^2 + 12g_z\right)^2 \\
c_{2z} &= 0.1a^4 / \left(a^2 + 12g_z\right)^2 \\
c_{3z} &= \left[\frac{2}{15}a^5 + 12g_z^2 a + 2g_z a^3\right] / \left(a^2 + 12g_z\right)^2 \\
c_{4z} &= \left[\frac{-1}{30}a^5 - 2g_z a^3 - 12g_z^2\right] / \left(a^2 + 12g_z\right)^2 \\
c_5 &= J/(Aa) \qquad \text{(A-8)} \\
c_{1y} &= \left[1.2a^3 + 144g_y^2/a + 24g_y a\right] / \left(a^2 + 12g_y\right)^2 \\
c_{2y} &= 0.1a^2 / \left(a^2 + 12g_y\right)^2 \\
c_{3y} &= \left[\frac{2}{15}a^5 + 12g_y^2 a + 2g_y a^3\right] / \left(a^2 + 12g_y\right)^2 \\
c_{4y} &= \left[\frac{-1}{30}a^5 - 2g_y a^3 - 12g_y^2\right] / \left(a^2 + 12g_y\right)^2
\end{aligned}
$$

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