

High-Temperature Mechanical Properties of Concrete

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Abstract: Structural fire safety capacity of concrete is very complicated because concrete materials have considerable variations. In this paper, constitutive models and relationships for concrete subjected to fire are developed, which are intended to provide efficient modeling and to specific fire-performance criteria of the behavior of concrete structures exposed to fire. They are developed for unconfined concrete specimens that include residual compressive and tensile strengths, compressive elastic modulus, compressive and tensile stress-strain relationships at elevated temperatures. In this paper, the proposed relationships at elevated temperatures are compared with experimental result tests and previous existing models. It affords to find several advantages and drawbacks of present stress-strain relationships and using these results to establish more accurate and general compressive and tensile stress-strain relationships. Additional experimental test results are needed in tension and the other main parameters at elevated temperatures to establish well-founded models and to improve the proposed relationships. The developed models and relationships are general, rational, and have good agreement with experimental data.

Keywords: constitutive models, strength of concrete, fire, residual compressive and tensile strengths, stress-strain relationship, elastic modulus, elevated temperature

1. Introduction

The concrete behaves differently under different types and combinations of stress conditions due to the progressive microcracking at the interface between the mortar and the aggregates (transition zone) [1]. Structural fire safety is one of the primary considerations in the design of high-rise buildings and infrastructures, where concrete is often the material of choice for structural members. At present, the fire resistance (structural fire safety) of reinforced concrete (RC) members is generally established using prescriptive approaches that are based on either the standard fire resistance tests or empirical calculation methods. These approaches have major drawbacks and do not provide rational and realistic fire safety assessment. As the world is

moving toward performance-based fire codes, there is an increased focus on the use of numerical methods for evaluating fire performance of structural members. Because the fire performance of structural members depends on the properties of the constituent materials, knowledge of high-temperature properties of concrete is critical for fire resistance assessment under performance-based codes [2].

The parameters that control concrete behavior are: compressive strength, tensile strength, peak strain, modulus of elasticity, creep strain, thermal conductivity, thermal strain, and etc that are nonlinear functions of temperature. Also, aggregate types of concrete influence the concrete behavior exposed to fire [3]. The aggregates thermal expansion is partly opposed to the drying of cement paste. This phenomenon makes it possible to think that limestone aggregates whose thermal coefficient of expansion is lower than that of siliceous aggregates is more favorable to the behavior at high temperature of concrete [4]. Many compressive and tensile constitutive models for concrete at normal temperatures are developed. The constitutive laws of concrete materials under fire condition are complicated and knowledge of current thermal properties is based on the limited material properties. There are either limited test

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data for some high-temperature properties, or there are considerable variations and discrepancies in the high-temperature test data for other properties of concrete [5-7]. These variations and discrepancies are mainly due to the differences in test methods, condition of procedures, and the environmental parameters accompanying the tests [8-9]. Thus, at present, there are no reliable constitutive relationships in codes and standards for many of the high-temperature properties of concrete [6, 9]. There have been significant efforts in computational mechanics to describe the behavior of concrete using various proposed models [10]. Although the computational methods and techniques for estimating the fire performance of structural members of buildings are developed but researches that provide inputting data such as constitutive laws of concrete materials into these computational methods has not kept pace [11]. Much of information in ACI216R [12] is based on experimental test results undertaken during 1950s and 1960s that contains no comprehensive constitutive relationships [2]. The modeling of concrete considers cracking, crushing failure modes and nonlinear behavior [13].

There is an urgent need to establish constitutive relationships for modeling the fire

response of concrete members. Regression analyses are conducted on available experimental data in literature to propose compressive strength, tensile strength and compressive elastic modulus. Firstly, the proposed relationships for mechanical properties, i.e. compressive strength, tensile strength and modulus of elasticity, are compared with test results. Secondly, the influence of high temperatures is discussed in light of the available models of peak strain (strain at peak stress). Thirdly, the proposed compressive and tensile stress-strain relationships for concrete at elevated temperature are compared with test results.

2. Compressive Strength of Concrete at Elevated Temperatures

The residual compressive behavior of concrete has been under investigation since the early 1960s (see the contributions by Zoldners, Dougill, Harmathy, Crook, Kasami et al., Schneider and Diederichs, all quoted in RILEM, 1985 [14]). Attention has been focused mostly on the compressive strength (the strength at room temperature after a specimen has been heated to a test temperature and subsequently cooled) as such, on the residual strain and on strength

Table1. Compressive strength models of concrete at high temperatures

Ref.	Residual Compressive Strength at Elevated Temperatures
Lie and Lin [16]	$f'_{cT} = f'_c \left(2.011 - 2.353 \frac{T-20}{1000} \right) \leq f'_c$
Lie et al. [17]	$f'_{cT} = f'_c (1 - 0.001T)$ $T \leq 500^\circ\text{C}$; $f'_{cT} = f'_c (1.375 - 0.00175T)$ $500^\circ\text{C} \leq T \leq 700^\circ\text{C}$; $f'_{cT} = 0$ $T \geq 700^\circ\text{C}$
EN1992-1-2 [18]	$f'_{cT} = f'_c$ $T \leq 100^\circ\text{C}$; $f'_{cT} = f'_c (1.067 - 0.00067T)$ $100^\circ\text{C} \leq T \leq 400^\circ\text{C}$; $f'_{cT} = f'_c (1.44 - 0.0016T)$ $T \geq 400^\circ\text{C}$
ASCE manual [19]	$f'_{cT} = f'_c$ $20^\circ \leq T \leq 450^\circ\text{C}$; $f'_{cT} = f'_c \left[2.011 - 2.353 \left(\frac{T-20}{1000} \right) \right]$ $450^\circ \leq T \leq 874^\circ\text{C}$; $f'_{cT} = 0$ $T > 874^\circ\text{C}$
Lie and Irwin [20]	$f'_{cT} = f'_c$ $0^\circ\text{C} \leq T < 450^\circ\text{C}$; $f'_{cT} = f'_c (2.06 - (T/425))$ $T \geq 450^\circ\text{C}$
Jau [21]	$f'_{cT} = (1 - 0.001T)f'_c$ $0^\circ\text{C} \leq T \leq 500^\circ\text{C}$; $f'_{cT} = \left[1.6046 + (1.3T^2 - 2817T) \times 10^{-6} \right] f'_c$ $T > 500^\circ\text{C}$
Kodur et al. [22]	$f'_{cT} = \begin{cases} f'_c [1.0 - 0.003125(T-20)] & T < 100^\circ\text{C} \\ 0.75f'_c & 100^\circ\text{C} \leq T \leq 400^\circ\text{C} \\ f'_c [1.33 - 0.00145T] & 400^\circ\text{C} < T \end{cases}$
Li and Purkiss [23]	$f'_{cT} = f'_c \left(0.00165 \left(\frac{T}{100} \right)^3 - 0.03 \left(\frac{T}{100} \right)^2 + 0.025 \left(\frac{T}{100} \right) + 1.002 \right)$
Hertz [24]	$f'_{cT} = f'_c \left[1 / \left(1 + \frac{T}{T_1} + \left(\frac{T}{T_2} \right)^2 + \left(\frac{T}{T_8} \right)^8 + \left(\frac{T}{T_{64}} \right)^{64} \right) \right]$ Siliceous aggregate: $T_1 = 15000, T_2 = 800, T_8 = 570, T_{64} = 100000$ Lightweight aggregate: $T_1 = 100000, T_2 = 1100, T_8 = 800, T_{64} = 940$ Other aggregates: $T_1 = 100000, T_2 = 1080, T_8 = 690, T_{64} = 1000$
Chang et al. [25]	1. $f'_{cT} = f'_c \left(1.008 + \frac{T}{450 \ln(T/5800)} \right) \geq 0.0$ $20^\circ\text{C} < T \leq 800^\circ\text{C}$; 2. $f'_{cT} = f'_c \left[\begin{cases} 1.01 - 0.00055T & 20^\circ\text{C} < T \leq 200^\circ\text{C} \\ 1.15 - 0.00125T & 200^\circ\text{C} \leq T \leq 800^\circ\text{C} \end{cases} \right]$

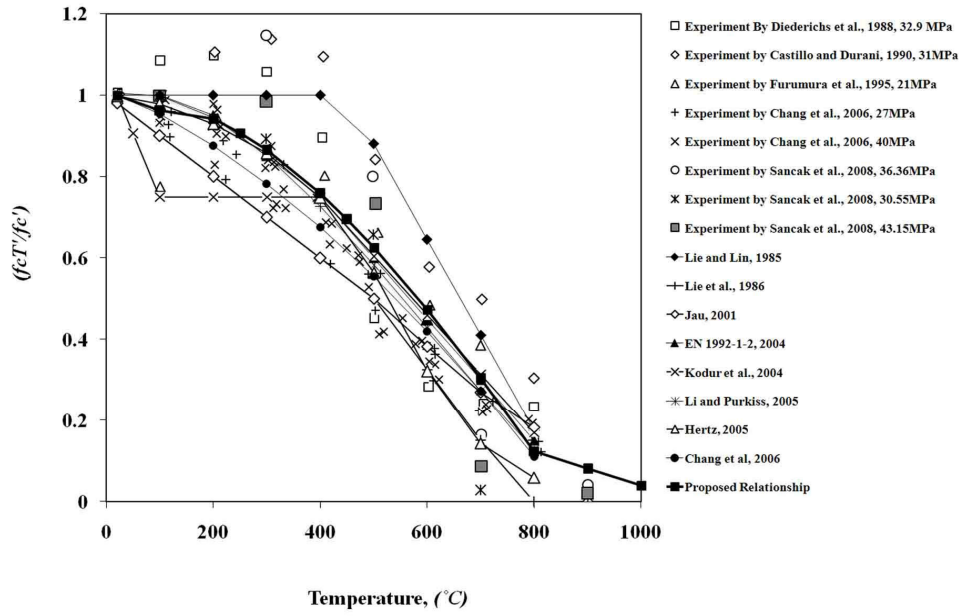


Fig.1. Comparison between compressive strength of siliceous aggregate concrete at elevated temperatures with experimental data

recovery with time [15]. The most important models of the compressive strength of concrete at high temperature in the literature are summarized in Table 1. In this study, the relationships proposed for the compressive strength of siliceous aggregate, calcareous and lightweight aggregate concrete at elevated temperature that regression analyses are conducted on existing experimental data to propose them are expressed as Eqs. (1-3). These proposed relationships are compared separately with test results and with the models in Table 1, as shown in Figures 1-3.

Siliceous aggregate concrete:

$$f'_{cT} = f'_c \begin{cases} 1.012 - 0.0005T \leq 1.0 & 20^\circ\text{C} \leq T \leq 100^\circ\text{C} \\ 0.985 + 0.0002T - 2.235 \times 10^{-6}T^2 + 8 \times 10^{-10}T^3 & 100^\circ\text{C} < T \leq 800^\circ\text{C} \\ 0.44 - 0.0004T & 900^\circ\text{C} \leq T \leq 1000^\circ\text{C} \\ 0 & T > 1000^\circ\text{C} \end{cases} \quad (1)$$

Carbonate aggregate concrete:

$$f'_{cT} = f'_c \begin{cases} 1.01 - 0.0006T \leq 1.0 & 20^\circ\text{C} \leq T \leq 200^\circ\text{C} \\ 1.0565 + 0.0017T + 5 \times 10^{-6}T^2 - 5 \times 10^{-9}T^3 & 200^\circ\text{C} < T \leq 900^\circ\text{C} \\ 0 & 900^\circ\text{C} < T \end{cases} \quad (2)$$

Lightweight aggregate concrete:

$$f'_{cT} = f'_c \begin{cases} 1.01 - 0.00037T \leq 1.0 & 20^\circ\text{C} \leq T \leq 300^\circ\text{C} \\ 1.0491 - 0.00036T + 10^{-6}T^2 - 2 \times 10^{-9}T^3 & 300^\circ\text{C} < T \leq 900^\circ\text{C} \\ 0 & T \geq 1000^\circ\text{C} \end{cases} \quad (3)$$

Figures 1-3 show the variation of compressive strength test results and the available models with temperature for concrete. Figure 1 makes comparison between the models in Table 1 and the proposed relationship for concrete at different temperatures against published unstressed experimental test results (unstressed tests: the specimen is heated, without preload, at a constant rate to the target temperature, which is maintained until a thermal steady state is achieved) (Diederichs et al. [26], Castillo and Durrani [27], Furumura et al. [28], Chang et al. [25], and Sancak et al. [29]). Concrete typically loses 10 - 20% of its original compressive strength when heated to 300 °C, and 60 - 75% at 600 °C. The models described by Lie and Lin [16] and Lie et al. [17] provide the upper and lower bounds for f'_{cT} . The proposed relationship fits the test results well. Figure 2 shows a comparison between the models in Table 1 and the proposed relationship for high-strength calcareous aggregate concrete against the unstressed experimental results reported by Abrams [30] and Savva et al. [31]. The proposed

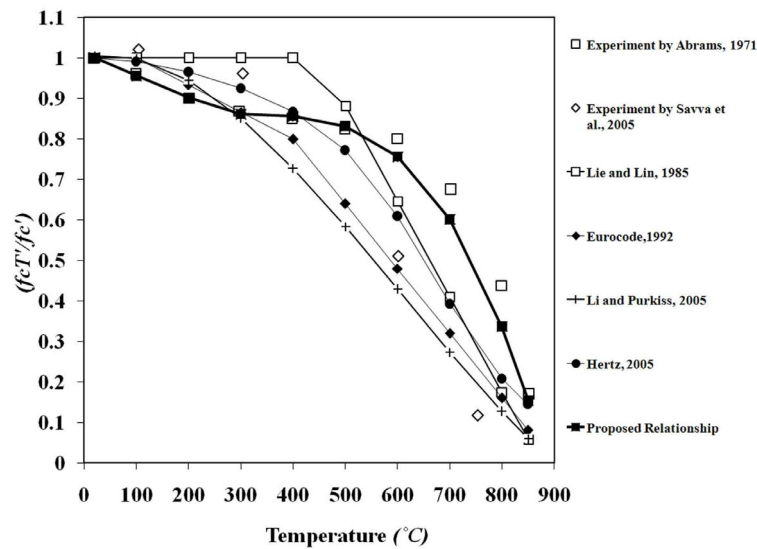


Fig.2. Comparison between compressive strength of carbonate aggregate concrete at elevated temperatures with experimental data

relationship agrees with the test results fairly well. Figure 3 shows a comparison between the previous models (Table 1) and the relationship proposed here for high-strength lightweight aggregate concrete and the unstressed experimental results reported by Abrams [30] and Sancak et al. [29]. The proposed relationship fits the experimental results well in comparison with others. Lightweight concrete has less strength loss at high temperature compared to ordinary aggregate concrete. The behavior of calcareous aggregate and lightweight aggregate concrete

was about the same over the entire temperature range and retained more than 75% of the original strength at temperatures up to 649 °C.

3. Tensile Strength for Concrete at Elevated Temperatures

Research studies on tensile strength of concrete at elevated temperatures are much more limited. As documented in the literature, four models are available to evaluate the residual tensile strength of concrete at elevated

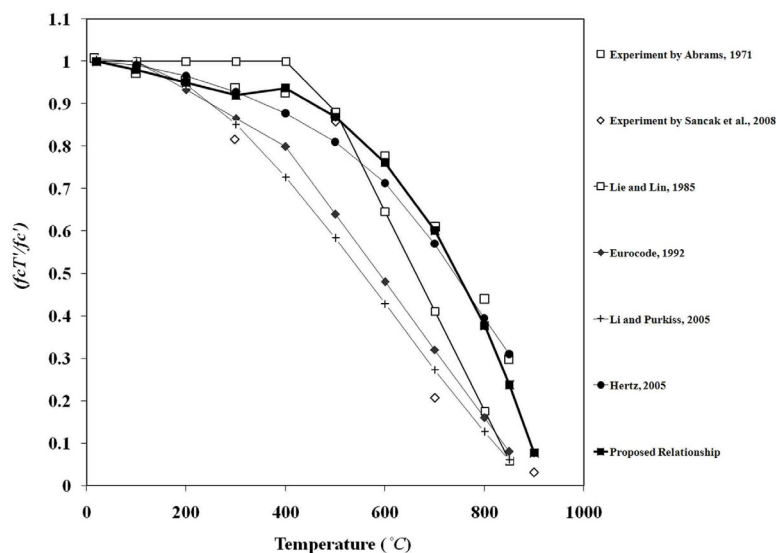


Fig. 3. Comparison between compressive strength of lightweight aggregate concrete at elevated temperatures with experimental data

Table 2. Tensile strength models of concrete at high temperatures

Ref.	Residual Tensile Strength at Elevated Temperatures
Bazant and Chern [32]	$f_{crT} = f_{cr} \begin{cases} -0.000526T + 1.01052 & 20^\circ C \leq T \leq 400^\circ C \\ -0.00252T + 1.8 & 400^\circ C \leq T \leq 600^\circ C \\ -0.0005T + 0.6 & 600^\circ C \leq T \leq 1000^\circ C \end{cases}$
EN1992-1-2 [18]	$f_{crT} = k_{ck,T} f_{cr}$; $k_{ck,T} = \begin{cases} 1 & 20^\circ C \leq T \leq 100^\circ C \\ 1 - (T - 100) / 500 & 100^\circ C < T \leq 600^\circ C \end{cases}$
Terro [33]	$f_{crT} = f_{cr} (f'_{crT} / f'_c)$; $f_{cr} = (f'_c / 10)$
Li and Guo, (reported in [34])	$f_{crT} = f_{cr} (1 - 0.001T)$ $20^\circ C \leq T \leq 1000^\circ C$
Chang et al. [25]	$f_{crT} = f_{cr} \begin{cases} 1.05 - 0.0025T & 20^\circ C \leq T \leq 100^\circ C \\ 0.80 & 100^\circ C < T \leq 200^\circ C \\ 1.02 - 0.0011T \geq 0.0 & 200^\circ C < T \leq 800^\circ C \end{cases}$

temperature, and these are summarized in Table 2. Here, a model is proposed to evaluate the tensile strength of concrete at elevated temperature that regression analyses are conducted on existing experimental data to propose it which is expressed as Eq. (4).

$$f_{crT} = f_{cr} \begin{cases} 1.02 - 0.00098T \leq 1.0 & 20 < T \leq 300^\circ C \\ 0.965 - 0.0001T - 9 \times 10^{-7}T^2 - 3 \times 10^{-9}T^3 + 3.2 \times 10^{-12}T^4 & 300^\circ C < T \leq 900^\circ C \\ 0 & T \geq 1000^\circ C \end{cases} \quad (4)$$

Figure 4 shows a comparison between the models in Table 2 and the proposed relationship for tensile strength of concrete against the experimental results reported by Lie [35], Andeberg and Thelandersson [36], Noumowe et al. [37] and Xu et al. [38] that indicates the accuracy of the proposed relationship. The residual tensile strengths of concrete decreased similarly and almost linearly with increase of temperature.

4. Elastic Modulus at Elevated Temperatures

The elastic modulus of concrete could be affected primarily by the same factors influencing its compressive strength concrete [39]. The most important available models for elastic modulus of concrete at high temperatures are summarized in Table 3. Here, a relationship for the elasticity modulus of concrete at elevated temperatures is proposed that regression analyses are conducted on existing experimental data to propose it and is expressed as Eq. (5).

$$E_{crT} = E_c \begin{cases} 1.0 & 20^\circ C \leq T < 100^\circ C \\ 1.015 - 0.00154T + 2 \times 10^{-7}T^2 + 3 \times 10^{-10}T^3 & 100^\circ C < T \leq 1000^\circ C \\ 0 & T > 1000^\circ C \end{cases} \quad (5)$$

Figures 5 provides a comparison between the Table 3 models and the developed model for elasticity modulus of normal and high strength concretes against experimental results of

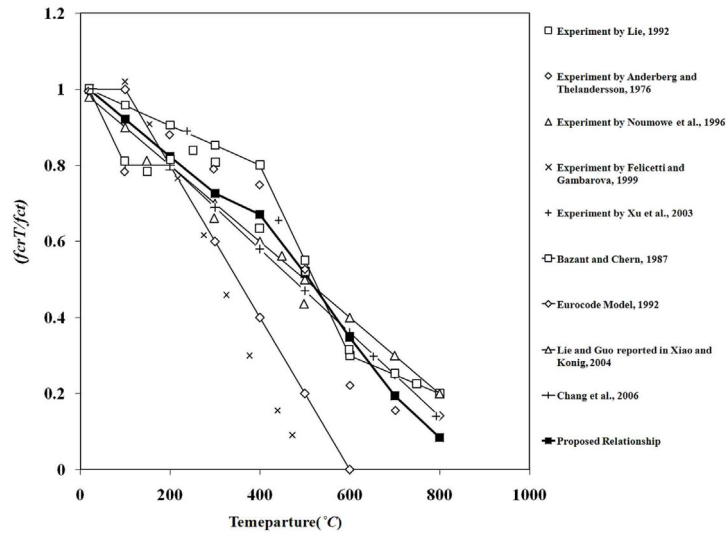


Fig. 4. Comparison between tensile strength for concrete at elevated temperatures with experimental data

Table 3. Compressive elastic modulus at elevated temperatures

Ref.	Compressive Elastic Modulus at Elevated Temperatures
Anderberg and Thelandersson [36]	$E_{crT} = 2f'_{cr} / \epsilon'_{crT}$
BSI [40]	$E_{crT} = (700 - T / 550) E_c \leq E_c$
Schneider [41]	Normal weight concrete: $E_{crT} = (-0.001552 T + 1.03104) g E_c \quad 20^\circ C \leq T \leq 600^\circ C$ $E_{crT} = (-0.00025 T + 0.25) g E_c \quad 600^\circ C \leq T \leq 1000^\circ C$ Lightweight concrete: $g = 1 + \frac{f_{ci}}{f'_c} \frac{T - 20}{100} ; \frac{f_{ci}}{f'_c} \leq 3.0 \quad E_{crT} = (-0.00102 T + 1.0204) g E_c \quad 20^\circ C \leq T \leq 1000^\circ C ;$
Khennane and Baker [42]	for preloaded concrete: $E_{crT} = (-0.000634 T + 1.012673) E_c$ if $20^\circ C \leq T \leq 525^\circ C$ $E_{crT} = (-0.002036 T + 1.749091) E_c$ if $525^\circ C \leq T \leq 800^\circ C$ for unloaded concrete: $E_{crT} = (-0.001282 T + 1.025641) E_c$ if $20^\circ C \leq T \leq 800^\circ C$
Lu, (reported in [34])	$E_{crT} = (1 - 0.0015 T) E_c \quad 20^\circ C \leq T \leq 200^\circ C ; E_{crT} = (0.87 - 0.00084 T) E_c \geq 0.28 E_c$
Li and Guo, (reported in [34])	$E_{crT} = (0.83 - 0.0011 T) E_c \quad 60^\circ C \leq T \leq 700^\circ C \quad E_{crT} = E_c \quad 20^\circ C \leq T \leq 60^\circ C ;$
Li and Purkiss [23]	$E_{crT} = (800 - T / 740) E_c \leq E_c$
Chang et al. [25]	1. $E_{crT} = \begin{cases} -0.00165 T + 1.033 & 20^\circ C < T \leq 125^\circ C \\ 1 / (1.2 + 18(0.0015 T)^{4.5}) & 125^\circ C < T \leq 800^\circ C \end{cases} E_c$ 2. $E_{crT} = (-0.00165 T + 1.033) E_c \quad 20^\circ C < T \leq 600^\circ C$

Table 4. Peak strain at elevated temperatures

Ref.	Peak Strain at Elevated Temperatures
Bazant and Chern [32]	$\epsilon_{\max} = 0.0000064 T + 0.00216 \quad 20^\circ C \leq T \leq 600^\circ C ; \epsilon_{\max} = 0.000015 T - 0.003 \quad 600^\circ C \leq T \leq 650^\circ C$
Lie [35]	$\epsilon_{\max} = 0.0025 + (6.0 T + 0.04 T^2) \times 10^{-6}$
Khennane and Baker [42]	$\epsilon_{\max} = 0.003 \quad 20^\circ C \leq T \leq 200^\circ C ; \epsilon_{\max} = 0.00001156 T + 0.000686 \leq 0.0082 \quad T \geq 200^\circ C$ for preloaded concrete: $\epsilon_{\max} = 0.00000167 T + 0.002666 \geq 0.003 \quad \text{if } T \leq 800^\circ C$
Terro [33]	$\epsilon_{\max} = (50\lambda_L^2 - 15\lambda_L + 1) \epsilon'_{c1} + 20(\lambda_L - 5\lambda_L^2) \epsilon'_{c2} + 5(10\lambda_L^2 - \lambda_L) \epsilon'_{c3}$ $\epsilon'_{c1} = 2.05 \times 10^{-3} + 3.08 \times 10^{-6} T + 6.17 \times 10^{-9} T^2 + 6.58 \times 10^{-12} T^3$ $\epsilon'_{c2} = 2.03 \times 10^{-3} + 1.27 \times 10^{-6} T + 2.17 \times 10^{-9} T^2 + 1.64 \times 10^{-12} T^3 ; \epsilon'_{c3} = 0.002$
Li and Purkiss [23]	$\epsilon_{\max} = 2f'_c / E_c + 0.21 \times 10^{-4} (T - 20) - 0.9 \times 10^{-8} (T - 20)^2$
Lu and Yao, (reported in [34])	$\epsilon_{\max} = \epsilon'_c (0.0019 T + 0.9615)$
Kodur et al. [22]	$\epsilon_{\max} = 0.018 + (6.7f'_c + 6.0T + 0.03T^2) \times 10^{-6}$
Chang et al. [25]	$\epsilon_{\max} = \begin{cases} 1.00 & 20^\circ C < T \leq 200^\circ C \\ (-0.1f'_c + 7.7) \left[\frac{\exp(-5.8 + 0.01T)}{1 + \exp(-5.8 + 0.01T)} - 0.0219 \right] + 1.0 & 200^\circ C < T \leq 800^\circ C \end{cases} \epsilon'_c$

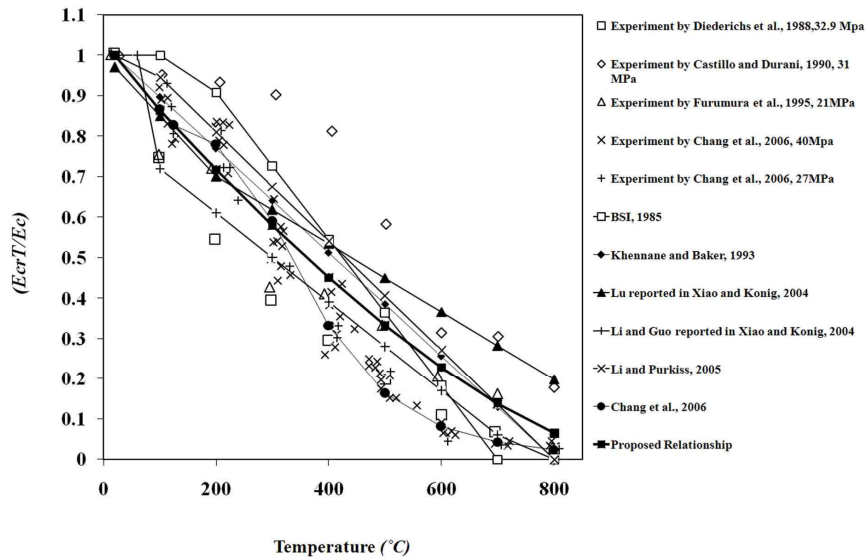


Fig.5. Comparison between elastic modulus of concrete at elevated temperatures with experimental data

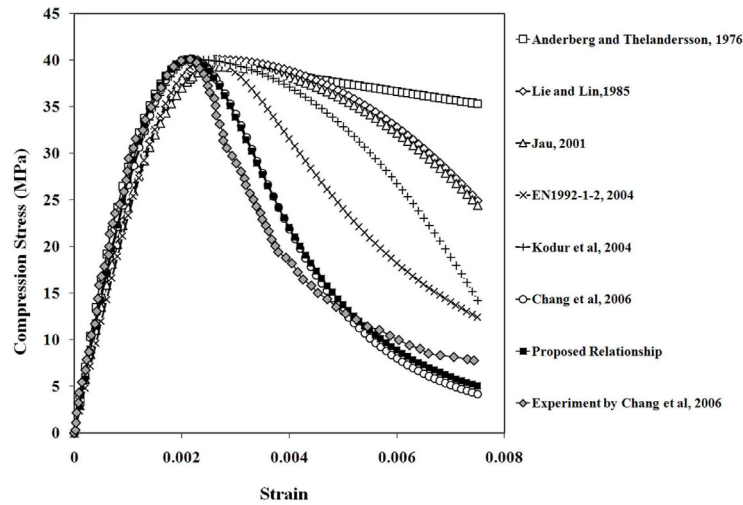


Fig. 6. Comparison between compressive stress-strain relationships for concrete at elevated temperatures with Chang et al. [25] experimental data at 20°C

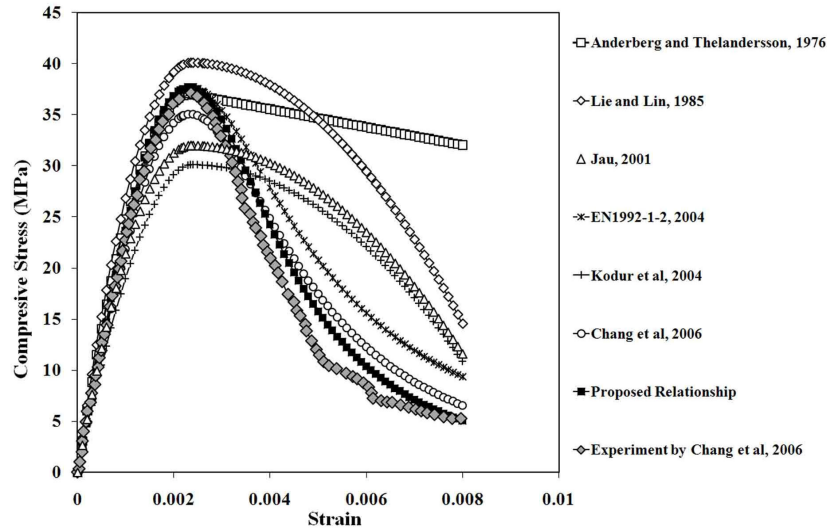


Fig.7. Comparison between compressive stress-strain relationships for concrete at elevated temperatures with Chang et al. [25] experimental data at 203°C

Diedreichs et al. [26], Castillo and Durani [27] and Furumura et al. [28]. The developed model is fitted well with mostly of the experimental results.

5. Peak Strain at Elevated Temperatures

The most important models for peak strain of concrete at high temperatures are summarized in Table 4. As reported by Youssef and Mofteh [43], the models from Lie [35] and Li and Purkiss [23] provide an upper bound for peak strain at elevated temperatures and Lu and Yao (reported in [34]) provides a lower bound. Among the

available models in the literature, Terro's model [33] has the advantage of accounting for different compressive stress levels and providing good accuracy.

6. Concrete Stress-Strain Relationship at Elevated Temperatures

6.1. Compressive Stress-Strain Relationships at Elevated Temperatures

The most important available compressive stress-strain relationships for concrete at high temperatures are summarized in Table 5. In this

Table 5. Compressive stress-strain relationships at elevated temperatures

Ref.	Compressive Stress-Strain Relationships at Elevated Temperatures
Anderberg and Thelandersson [36]	$\sigma_{cT} = f_1 - 880 (\epsilon_{cT} - \epsilon_1)$ $\epsilon_{cT} \geq \epsilon_1$ $\sigma_{cT} = E_{crT} \left[\epsilon_{cT} - (\epsilon_{cT}' / 2\epsilon_{cT}') \right]$ $\epsilon_{cT} \leq \epsilon_1$; $\epsilon_1 = \epsilon_{cT}' (1 - (880 / E_{crT}))$ $f_1 = E_{crT} (\epsilon_1 - \epsilon_1' / 2\epsilon_{cT}')$
Lie and Lin [16]	$\sigma_{cT} = f_{cT}' \left[1 - ((\epsilon_{cT} - \epsilon_{max}) / 3\epsilon_{max})^2 \right]$, $\epsilon_{cT} > \epsilon_{max}$ $\sigma_{cT} = f_{cT}' \left[1 - ((\epsilon_{max} - \epsilon_{cT}) / \epsilon_{max})^2 \right]$, $\epsilon_{cT} \leq \epsilon_{max}$
Schneider [41]	$\epsilon_{cT} = \left[1 + (1/1-n) (\epsilon_{cT}' / \epsilon_{cT}')^n \right] \frac{\sigma_{cT}}{E_{crT}}$; $n = 2.5$ for lightweight concrete $n = 3.0$ for normal-weight concrete
Terro [33]	$\epsilon_{cT} = \left[1 + (1/1-n) (\epsilon_{cT}' / \epsilon_{cT}')^n \right] \frac{\sigma_{cT}}{E_{crT}}$; $n = 2.0$
EN1992-1-2 [18]	$\sigma_{cT} = \left[3\epsilon_{cT}' f_{cT}' / \epsilon_{max} (2 + (\epsilon_{cT}' / \epsilon_{max})^3) \right]$ $\epsilon_{cT} \leq \epsilon_{cu}$
Kodur et al. [22]	$\sigma_{cT} = f_{cT}' \left[1 - \left(30(\epsilon_{cT} - \epsilon_{max}) / (130 - f_c') \right)^2 \right]$, $\epsilon_{cT} > \epsilon_{max}$ $\sigma_{cT} = f_{cT}' \left[1 - ((\epsilon_{max} - \epsilon_{cT}) / \epsilon_{max})^2 \right]$, $\epsilon_{cT} \leq \epsilon_{max}$;
Chang et al. [25]	$M = \frac{E_r}{E_{cr}} M_o = \frac{E_c}{E_c'} \frac{\sigma_{cT}}{f_{cT}'}$; $\frac{\sigma_{cT}}{f_{cT}'} = \frac{1}{1 + \left(M - \frac{n}{n-1} \left(\frac{\epsilon_{cT}'}{\epsilon_{cr}} \right) + \left(\frac{1}{n-1} \right) \left(\frac{\epsilon_{cT}'}{\epsilon_{cr}} \right)^n \right) M \left(\frac{\epsilon_{cT}'}{\epsilon_{cr}} \right)}$; $E_c = 5000 \sqrt{f_c'}$ $n_o = \left[f_c' / 12 \right] + 0.77 > 1.0$; $n = n_o (M / M_o)^{0.014 - 0.0007T}$;
Youssef and Mofteh [43]	1. $\epsilon_{cT} \leq \epsilon_{oTc} + \epsilon_{tr}$ $\sigma_{cT} = K_{hT} f_{cT}' \left[2.0 \left(\frac{\epsilon_{cT}}{\epsilon_{oTc} + \epsilon_{tr}} \right) - \left(\frac{\epsilon_{cT}}{\epsilon_{oTc} + \epsilon_{tr}} \right)^2 \right]$ $\epsilon_c \geq \epsilon_{oTc} + \epsilon_{tr}$ $\sigma_{cT} = K_{hT} f_{cT}' \left[1 - 2(\epsilon_{cT} - \epsilon_{oTc} - \epsilon_{tr}) \right] \geq 0.2 K_{hT} f_{cT}'$ $Z = 0.5 / (\epsilon_{50uT} + \epsilon_{50h} - \epsilon_{oTc} - \epsilon_{tr})$ $\epsilon_{oTc} = \epsilon_{oT} \times K_{hT}$; $K_{hT} = 1 + \frac{\rho_s f_{yT}}{f_{cT}'}$ $\epsilon_{50uT} = \frac{3 + 0.29 f_c'}{145 f_c' - 1000} \frac{\epsilon_{oTc}}{\epsilon_c} + \epsilon_{tr}$ 2. $\epsilon_{ocT} = \epsilon_{oT} \left[1 + 5 \left(\frac{f_{ccT}' - 1}{f_{cT}'} \right) \right]$ $f_c = 2 f_{ccT}' \epsilon_{cT}' / \left[(\epsilon_{ocT} + \epsilon_{tr}) \left(1 + \left(\frac{\epsilon_{cT}'}{\epsilon_{ocT} + \epsilon_{tr}} \right)^2 \right) \right]$; $f_{ccT}' = f_{cT}' \left[-1.254 + 2.254 \sqrt{1 + \left(7.94 f_{IT}' / f_{cT}' \right)} - \left(2 f_{IT}' / f_{cT}' \right) \right]$ $f_{IT}' = K_e (2 f_{yT} A_s / d_s S_h)$

study, a compressive stress–strain relationship for concrete at elevated temperatures that is based on Carreira and Chu [44]’s model with several modifications and is developed by using proposed compressive strength and elastic modulus relationships (i.e. Eq. (1-3 and 5)), which is expressed as Eq. (6).

$$\frac{\sigma_{cT}}{f_{cT}'} = \frac{\beta_{mT} \left(\frac{\epsilon_{cT}}{\epsilon_{max}} \right)}{\beta_{mT} - 1 + \left(\frac{\epsilon_{cT}}{\epsilon_{max}} \right)^{\beta_{mT}}};$$

$$\beta_{mT} = \beta_{mT,a}(\text{fitted}) = [1.02 - 1.17(E_p / E_c)]^{0.74} \quad \text{if } \epsilon_{cT} \leq \epsilon_{max} \quad (6)$$

$$\beta_{mT} = \beta_{mT,d}(\text{fitted}) = \beta_{mT,a}(\text{fitted}) + (a + bt) \quad \text{if } \epsilon_{cT} \geq \epsilon_{max}$$

$$a = 2.7 \times \left(12.4 - 1.66 \times 10^{-2} f_{cT}' \right)^{-0.46}; \quad b = 0.83 \exp(-911 / f_{cT}')$$

Figures 6-7 provide a comparison between the Table 5 relationships and the developed relationship for concrete against experimental results of Chang et al. [25] at 20 °C and 203 °C. The proposed model has good agreement with the experimental results. Figure 8 shows a comparison between the Table 5 relationships and the developed relationship for concrete against experimental results of EN 1992-2-1 [18] at 100 °C. Figures 9-10 show a comparison

between the relationships in Table 5 and the developed compressive stress-strain relationship for concrete against experimental results of Furumura et al. [28] at 500°C and 700°C temperatures.

6.2. Tensile Stress-Strain Relationships at Elevated Temperatures

Tensile stress–strain relationships for concrete at elevated temperatures are limited. A linear relationship is widely used to represent the pre-cracking behavior. After cracking, Terro [33] suggested a linear degrading branch that joins the point of cracking and a point on the horizontal axis with a strain of 0.004. Fracture toughness is often utilized to define the softening branch. Zhang and Bicanic [45] assessed the residual fracture toughness of cooled concrete after heating to 600°C. Similar research is needed to assess fracture toughness of concrete after heating to different temperatures and before cooling. Also, Youssef and Mofteh [43] proposed tensile stress-strain relationships for confined concrete. In this study, a tensile stress–strain relationship for concrete at elevated temperature

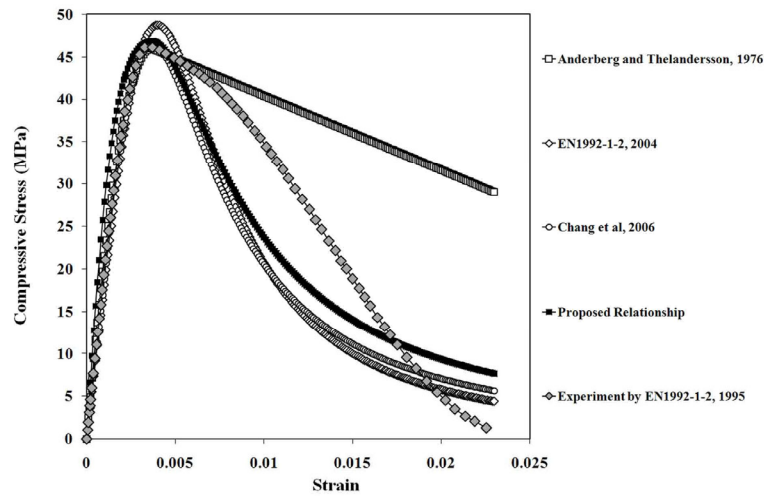


Fig. 8. Comparison between compressive stress-strain relationships for concrete at elevated temperatures with En 1992-1-2 [18] experimental data at 100°C

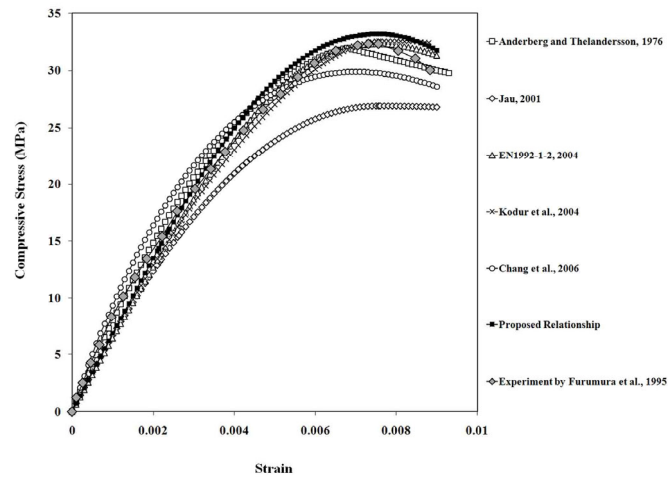


Fig. 9. Comparison between compressive stress-strain relationships for concrete at elevated temperatures with Furumura et al. [28] experimental data at 500°C

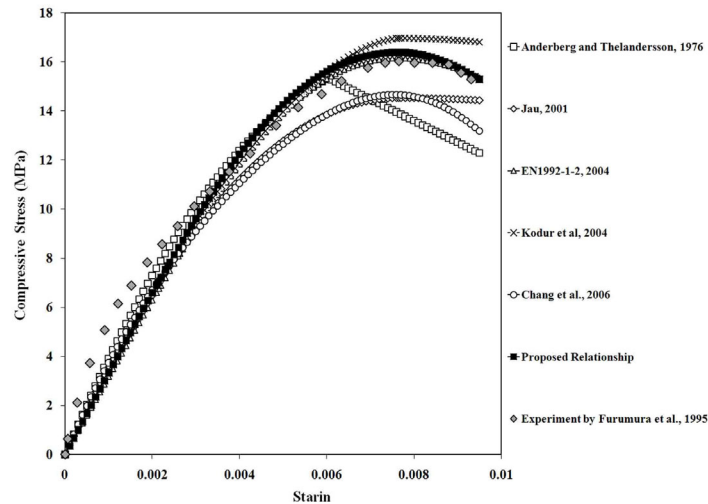


Fig. 10. Comparison between compressive stress-strain relationships for concrete at elevated temperatures with Furumura et al. [28] experimental data at 700°C

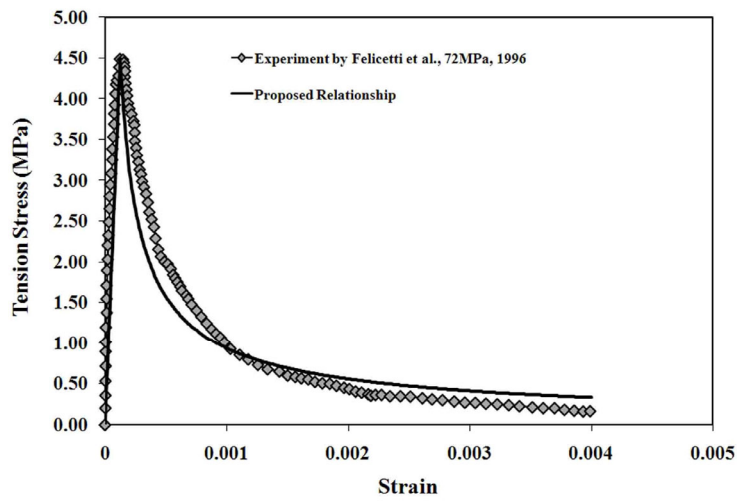


Fig. 11. Comparison between the developed tensile stress-strain relationship for concrete (72 MPa) at elevated temperatures with Felicetti et al. [46] experimental data at 20°C

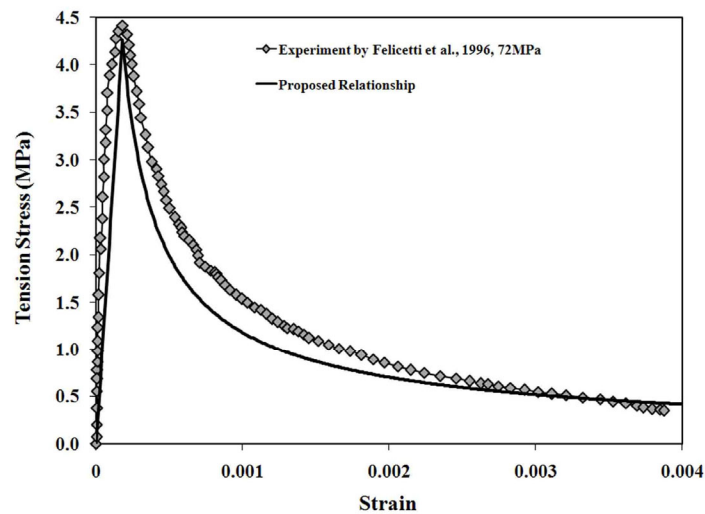


Fig. 12. Comparison between the developed tensile stress-strain relationship for concrete (72 MPa) at elevated temperatures with Felicetti et al. [46] experimental data at 105°C

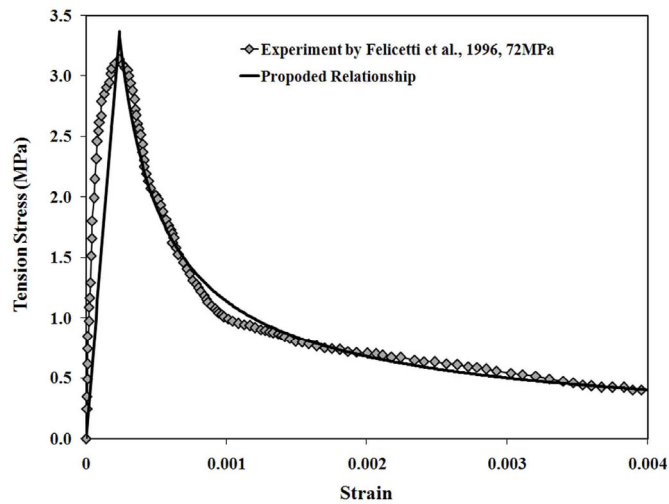


Fig. 13. Comparison between the developed tensile stress-strain relationship for concrete (72 MPa) at elevated temperatures with Felicetti et al. [46] experimental data at 250°C

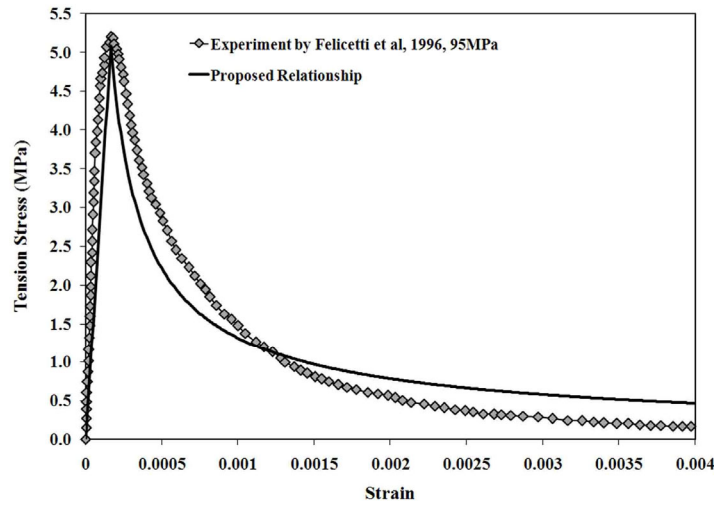


Fig.14. Comparison between the developed tensile stress-strain relationship for concrete (95 MPa) at elevated temperatures with Felicetti et al. [46] experimental data at 105°C

is developed by using proposed residual tension strength and elastic modulus relationships (i.e. Eqs. (4-5)), which is expressed as Eq. (7).

$$f_{ctT} = \begin{cases} f_{crT} E_c & \varepsilon_{cT} \leq \varepsilon_{ctT}' \\ f_{crT} \left(\varepsilon_{ctT}' / \varepsilon_{cT} \right)^{0.75} & \varepsilon_{cT} > \varepsilon_{ctT}' \end{cases} \quad (7)$$

Figures 11-14 compare the developed tensile relationship and experimental results of Felicetti et al. [46] for concrete at 20°C, 105°C and 250°C temperatures. The developed relationship is rational and has good agreement with the experimental results.

7. Conclusions

In this paper, constitutive models and relationships for concrete subjected to fire are developed, which are intended to provide efficient modeling and to specific fire-performance criteria of the behavior of concrete structures exposed to high temperatures. Attempts made towards achieving rational and well-founded constitutive models and relationships for concrete elevated temperatures. The major conclusions derived from the present work are:

1. The developed models for compressive strength of concrete at elevated temperatures for siliceous, carbonate and

lightweight aggregate concretes are verified well to the experimental results.

2. The developed model for elasticity modulus of concrete at elevated temperatures is rational and compatible with the experimental results.
3. The developed compressive stress-strain relationship of concrete at elevated temperatures is made based on the well-established relationships for concrete at ambient temperatures, which has a good conformity with the experimental test results of concrete at different temperatures.
4. The developed tensile stress-strain relationship for concrete at elevated temperatures has a linear branch until reaching the crack stress and after cracking, the developed relationship for tension at high temperature is modified by accounting for the decreasing tensile strength of concrete. The relationship is uncomplicated and compatible with the experimental test results.
5. The available models for compressive strength at elevated temperatures did not notice to the aggregate types.
6. Available researches and experimental test data on tensile strength of concrete at elevated temperatures are limited.
7. The additional experimental tests are needed to investigate the significance and

role of several parameters of thermal and mechanical properties of concrete.

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Notation

- σ_c : Concrete compressive stress at ambient temperature
- f'_c : Concrete compressive strength at ambient temperature
- f'_{28} : Compressive strength at 28 days
- f_{cr} : Tensile strength of concrete
- f_{ci} : Initial compressive stress before heating
- f_l : Stress at the point of intersection of the two equations defining the stress strain curve of concrete
- f_{yT} : Yield strength of reinforcing bars at elevated temperature
- σ_{cT} : Concrete compressive stress at elevated temperature
- f'_{cT} : Concrete compressive stress at elevated temperature
- f_{crT} : Tensile resistance of concrete at elevated temperature
- f'_{ccT} : Compressive strength of confined concrete at elevated temperature
- f'_{iT} : Effective lateral confining stress at elevated temperature
- ϵ_c : Concrete strain at ambient temperature
- ϵ'_c : Strain at maximum stress for concrete at ambient

temperature	l_{ch} : Characteristic length or crack bandwidth
ε_{cu} : Ultimate strain for concrete at ambient temperature	k_t : Initial tangent stiffness to the stress-displacement curve
ε_0 : Strain at the elastic limit in compression	c : Stiffening parameter
ε_{tu} : Cracking strain	g : Function to account for increase in modulus of elasticity due to external loads
ε_1 : Strain at point of intersection of the two equations defining the stress strain curve of concrete	t : Age (day)
ε_{max} : Strain at maximum stress of concrete at elevated temperature	Z : Slope of the decaying branch of the concrete stress-strain curve
ε_{oTc} : Strain at maximum stress of confined concrete at elevated temperature	K_{hT} : Confinement factor at elevated temperature
δ : Total displacement, measured over the specified gage length	K_e : Confinement effectiveness coefficient
δ_t : Displacement corresponding to the tensile strength	A_s : Cross sectional area of transverse reinforcement
E_C : Initial modulus of elasticity at ambient temperature	d_s : Diameter of the transverse reinforcing bars
E_{crT} : Initial modulus of elasticity at elevated temperature	S_h : Center-to-center spacing of the transverse reinforcement
E_p : Secant modulus at peak stress	λ_L : Factor accounting for the initial compressive stress level
G_f : Fracture energy of the concrete in tension	ρ_s : Ratio of the volume of transverse reinforcement to the volume of concrete core measured to outside of the transverse reinforcement