Technical Note:

New Approach for Estimating of Energy Dissipation over Stepped Spillways

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Abstract: *Hydraulics of stepped spillway is a very complicated phenomenon, as it consists of a two phase flow passing through a set of designed steps. The steps increase the rate of energy dissipation taking place on the spillway face. Turbulence, flow aeration and energy dissipation are the main tasks in the design of such structures. This study consists of the experimental investigation to determine the energy dissipation over stepped spillways. Experiments conducted at Water Research Institute on two physical models of the Siyah Bisheh stepped spillways in Iran. To develop a more generalized expression, the results of previous investigations were also considered in our study. Therefore, a wide range of variables were taken into account to estimate the energy dissipation along the non-uniform flow regime. Assuming the energy dissipation along the uniform flow regime to be equal to the vertical displacement of the jet, the total energy lost was calculated. A comparison of the results with those of measurements showed a regression of 0.92 for the total energy dissipation, which is one of the features of the present method for estimating of the energy dissipation, compared with the previous investigation.*

Keywords: *Hydraulic structure, Stepped spillway, Energy dissipation, Air-water flow*

1. Introduction

In the first half of the twentieth century, the use of concrete became more popular. The use of masonry at the downstream face of dams became obsolete and was quickly replaced with smooth chutes followed by hydraulic jump stilling basins [1]. Using flip buckets and ski jumps to throw overflowing waters into the air far downstream became popular also for high head dams. Moreover, chute blocks, baffle piers and end sills were added at the end of stilling basins to enhance energy dissipation. The application of Roller Compacted Concrete (RCC) for dam construction developed in the 1980s. This proved to be a more rapid and economic dam building technique. The use of stepped chutes increased as dam spillways became popular, mainly because of the increasing rate of the construction of RCC gravity dams.

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Flow over a stepped spillway can be divided into three separate flow regimes of nappe, skimming and transition flow depending upon the flow rate for a given stepped chute geometry. For smaller discharges, water flow down the chute as a succession of freefalling nappe impacting from one step to another, sometimes followed by a hydraulic jump [2]. For intermediate discharges, transition flows can be observed. Strong hydrodynamic fluctuations, splashing and spray near the free surface are the main features of this flow regime. Transition flows do not exhibit the quasi-smooth free-surface appearance of skimming flow or the succession of nappe falling down the spillway. For large discharges, the flow skims over the pseudo-bottom formed by the step edges as a coherent stream. Intense recirculation cavities can be observed between the step edges beneath the pseudo-bottom [3,4]. The vortices are maintained by the transmission of shear stress from the mainstream and contribute significantly to the skimming stream energy dissipation.

Stepped spillways have been used as hydraulic structures to dissipate energy, to enhance aeration rate in the flow, and also to comply with visual functions. They can be found acting as spillways in dams and weirs, as energy dissipaters in artificial channels, gutters and rivers, and as aeration enhancers in water treatment plants. As the flow passes over the spillway, a non-uniform flow develops along it. The thickness of boundary layer along the spillway is also increases so that it meets the free surface. Therefore, the turbulence initiates natural free-surface aeration, caused by turbulent velocities acting normal to the air-water boundary. Downstream of this inception point, a layer of air-water mixture forms, increasing in thickness with distance. Eventually, the flow becomes uniform. From this point, a quasi-Uniform flow forms downstream (Figure 1). Air concentration remains nearly constant between the steps. From this point, it can be assumed that the flow condition such as the velocity and the depth of water remain constant and it is reasonable to say that the energy dissipation is equal to the vertical displacement of the jet.

Based on this explanation, in this study, the flow was divided into two regimes of uniform and non-uniform flow. Then, the energy dissipation was calculated separately for every regime and the sum of the two energy dissipations was used to present the total energy lost. Experiments were conducted with two stepped spillways to determine the energy dissipations for the two regimes. The Results of the previous investigations were also included in this study to cover a wide range of geometry and flow condition. An expression was developed to determine the rate of energy dissipation along the non-uniform flow regime as E_1 (Figure 2). The rate of energy dissipation in the zone of uniform flow, E_2 was also assumed equal to the vertical displacement of the jet. Therefore, the total energy dissipation was calculated as the sum of E_1 and E_2.

2.Literature Review

Christodoulou [5] studied the flow over ungated stepped spillways. The slope of the spillway α varied between 25° to 55°. He found that the length of gradually varied flow formed from the crest L_u, was mainly a function of the flow discharge and relative height of the steps. He also showed that the slope of the spillway has little influence on L_u. Based on experimental information, he suggested the following expression;

$$L_u = \frac{8.60 q_w^{\ 0.713}}{k_s^{0.0695} (\sin\alpha)^{0.277}} \tag{1}$$

Boes and Minor [6] found that L_u is a function of the critical depth h_c and the slope of the spillway in the form of;

$$\frac{L_u}{h_c} = \frac{15}{\sin\alpha} \qquad \text{for} \qquad \alpha = 30°$$
$$\frac{L_u}{h_c} = \frac{35}{\sin\alpha} \qquad \text{for} \qquad \alpha = 50° \tag{2}$$

Boes and Hager [7] suggested an expression for estimating the location of uniform flow inception as a function of the critical depth h_c and the slope of the spillway in the form of;

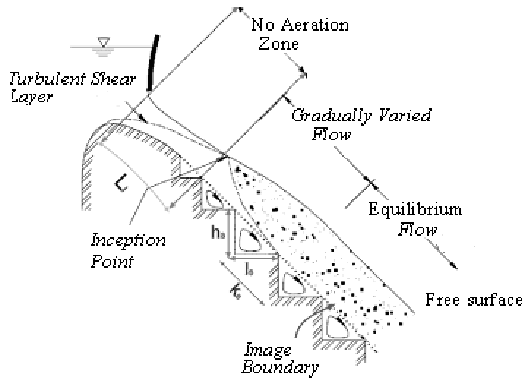

Fig.1 Schematic view of stepped spillway and air entrainment.

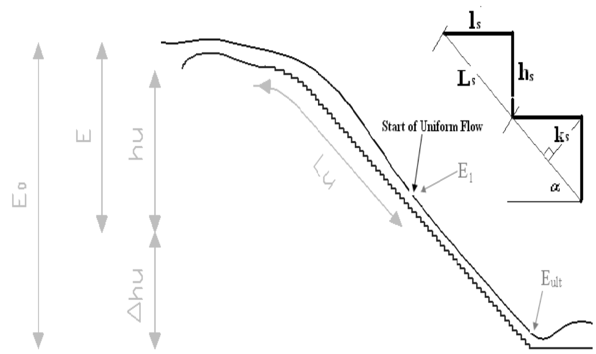

Fig. 2 Two models of stepped spillways used in this research.

$$\frac{L_u}{h_c}=25.52\times\left[1-0.055\times(\sin\alpha)^{-1/3}\right]\times(\sin\alpha)^{-1/3} \qquad (3)$$

Based on the previous investigations, it is expected that the rate of energy dissipation is mainly a function of the number of steps N_s and their geometry (i.e. the slope, height and length of the steps), the slope of the spillway α, flow regime, surface roughness, critical depth h_c, designed discharge, and flow velocity. In most of the studies it is mentioned that the energy dissipation mainly depends on the flow regime. In skimming flow regime, the energy dissipation is mainly caused by flow aeration and the destructive effect of the steps. For nappe flow regimes, the sum of shear caused by recirculation flow and hydraulic jump form on every step dominant the energy dissipation. In the transition regime, the energy dissipation consists of both the effects of the steps and the shear caused by recirculation zone. Therefore, based on the different regimes exists over the stepped spillways, it is necessary to study the energy dissipation for each of the flow regimes. The present study focuses on the regime of skimming flow.

Yasuda and Ohtsu [8] studied the formation of hydraulic jumps downstream of the stepped spillway. They found that the height of the steps has slight effect on the length of the jump. Comparison of the stepped spillways with similar slopes but different number of steps showed that with reducing the number of steps, the energy dissipation increases. Experimental results of Peruginelli and Pagliara [9] showed that in skimming flow regimes, the rate of energy dissipation increases as the slope of the spillway increases. The results are different from those of Fratino and Piccinni [10], who reported higher rate of energy dissipation for transition and skimming flow regimes as the slope of the spillway reduces. Studies also showed that the surface roughness of the stepped spillways has less effect on energy dissipation compared to the chute spillways.

Various expressions have been suggested for determining the rate of energy dissipation in skimming flow. Chanson [11] and Boes and Hager [7] introduced a relationship for the rate of energy dissipation $\Delta E_t/E_0$ as a function of the slope of the spillway α, the friction factor f, the height of spillway H_D, and the initial energy E_0 at the crest in the form of;

$$\frac{\Delta E_t}{E_\circ}=1-\frac{\left(\frac{f}{8\sin\alpha}\right)^{1/3}\cos\alpha+\frac{\phi}{2}\left(\frac{f}{8\sin\alpha}\right)^{-2/3}}{\frac{H_D}{h_c}+\frac{3}{2}} \qquad (4)$$

In some studies, a drag coefficient C_D for step was introduced in the following forms [12];

$$C_D=\frac{2.02}{\left(\frac{D_h}{k_s}\right)^{0.067}\times(Fr)^{1.96}} \qquad \text{(Nouri, 1984)}$$

$$C_D=\frac{3.285\times\left(\frac{D_h}{k_s}\right)^{0.015}\times(\tan\alpha)^{0.547}}{(\mathrm{Re})^{0.013}\times(Fr)^{2.021}} \qquad \text{(Yazdani,1994)}$$

They assumed uniform flow all over the spillway and the rate of energy dissipation was suggested as a function of this drag coefficient as follow;

$$\frac{\Delta E_t}{E_0}=104.33\times\left(\frac{H_D}{k_s}\right)^{-0.105}\times C_D^{\ 0.054} \qquad (5)$$

In which $k_s=h_s\cos\alpha$ is the step height vertical to the pseudo-bottom and D_h is the hydraulic depth.

3. Experimental Setup

In this study a set of experiments were performed at Water Research Center of Iran on two scaled physical models of stepped spillway (Figure 3). The models were made of plexy glass to visualize the flow. The scales of the models were large enough to reduce the scale effects [13].

Model 1: The width of the spillway is 133cm, the total height (crest to stilling basin) is 385.3cm and the slope of the spillway is 18.43º. The length of the stepped spillway is 874.7cm, which consists of 59 steps. The height of the steps is 4.67cm, $k_s=h_s\cos\alpha$ is 4.43cm and the length of the steps $l_s=h_s\cot\alpha$ is 14cm (see Figure 2).

Model 2: The width of the spillway is 150cm, the total height (crest to stilling basin) is 373.1cm

Model 1 **Model 2**

Fig. 3 A schematic view of stepped spillways used in this research.

and the slope of the spillway is 39.08º. The flow downstream of the crest enters a 150cm length of mild slope channel (θ <1º) with six variable steps to reach the main stepped spillway. The length of the stepped spillway is 438cm, which consists of 62 steps. The height of the steps is 6cm, $k_s=h_s Cos\alpha$ is 4.66cm and the length of the steps $l_s=h_s Cot\alpha$ is 7.2cm (see Figure 2).

In this study care was taken to improve and develop a more generalized expression for determining the rate of energy dissipation with different geometries and flow conditions. Therefore, additional information from previous investigations (three other stepped spillways) was also collected. Table 1 provides the range of the geometry condition of the steps and spillways used for this study. Table 2 also shows the range

Table 1 Present and previous geometry condition of stepped spillways used for this study.

h_s (cm)	L(cm)	H_D (cm)	α	Source	Exp. Series
61	244	146.4	52.04º	Sorenson [14]	A
25, 30	30	84.1	40º, 45º, 55º	Yazdani [12]	B
20, 24, 30, 40	30	84.1	52.04º	Ahmadyar [15]	C
46.7	133	285.3	18.43º	Model 1	D
60	150	438	39.08º	Model 2	E
60	150	438	39.08º	Model 2	F

Table 2 Present and previous flow condition of stepped spillways used for this study.

Fr	Re ($\times 10^4$)	h_e (m)	q (m^2/s)	Source	Exp. Series
1.4 - 2	1.9 - 18	0.0116 - 0.0969	0.0062 - 0.1115	Sorenson [14]	A
1.9 - 2.5	2 - 8	0.0109 - 0.0536	0.002 - 0.0186	Yazdani [12]	B
1.8 - 2.9	1.4 - 6.8	0.0096 - 0.071	0.0014 - 0.0226	Ahmadyar [15]	C
1.9 - 2.6	2.2 - 35.3	0.007 - 0.065	0.0095 - 0.1681	Model 1	D
1.6 - 2.8	2.2 - 41.5	0.01 - 0.07	0.0107 - 0.2246	Model 2	E
2.4 - 3	16.2 - 87.7	0.026 - 0.125	0.0839 - 0.464	Model 2	F

of the flow condition including the Froude and Reynolds numbers used for this study.

According to these tables, for every series of experiments, some of the parameters such as the slope of the spillway (α), the height of the spillway (H_D), the height of the steps (h_s), and the length of the spillways (L) kept constant. Also some of the parameters such as the discharge per unit width (q), critical depth and the entrance depth of flow at the crest (h_e), which affect the Reynolds (Re) and Froude (Fr) numbers of the flow may vary during each experiment. Using these set of experiments, a wide range of flow and geometry conditions were available to ensure a more generalized equation for determining the rate of energy dissipation over stepped spillways.

4. Results and Discussion

In this study, care was taken to select an expression for determining the length of the non-uniform flow or the boundary zone between the uniform and non-uniform flow over the spillway (L_u). This would be a complicated matter, because a gradually varied flow occurs over the spillway and thus, it is very difficult to exactly distinguish this boundary. Comparison of the results showed that the equation of Boes and Hager (equation 3) is more conservative as it provides a longer length of non-uniform flow over stepped spillways. Therefore, their equation was used to determine the location of starting uniform flow and the rate of energy dissipation for non-uniform flow regime over stepped spillways.

A dominant characteristic of stepped spillways is the strong flow aeration clearly seen in prototype and physical model. Theoretical analysis and numerical study is limited because of the large number of relevant equations (i.e. three basic equations per phase plus a phase transfer equation). Experimental investigations are also difficult but recent advances in air-water flow instrumentation brought new measuring systems enabling successful experiments. The relevant parameters needed for dimensional analysis must include fluid properties and the step and spillway geometries to develop an expression for the rate of energy dissipation in the form of;

$$\frac{\Delta E}{E} = f_1\left(\tan\alpha, \frac{L_u}{L_s}, \mathrm{Re}, fr, \frac{k_s}{D_h}\right) \tag{6}$$

where, ΔE is the energy dissipation, E is the entrance energy, k_s is the height of the step, L_u is the length of non-uniform flow made dimensionless with respect to the distance between the steps $L_s=h_s/Sin\alpha$, and is the slope of spillway (Figure 2). The parameter $k_s=h_sCos\alpha$ is a symbol of the height of the step which made dimensionless with respect the hydraulic depth (D_h).

To determine $\Delta E/E$ relative energy dissipation in non-uniform flow region, L_u the length of the non-uniform flow regime and h the total head on the crest should be verified to calculate $h_u=L_u.Sin\alpha$ and thus, $E=h_u+h$ (see Figure 2). If H_D is the height of the spillway, $\Delta h_u=H_D-h_u$ and then the energy at the beginning of the uniform flow $E_1=E_{ult}+\Delta h_u$ will be determined. It should be mentioned that E_{ult} is the energy at the entrance to the stilling basin, which can be calculated using the Bernoulli equation and the depth of water h_{90} (depth with 90% air concentration). The Reynolds and Froude numbers is also calculated based on the depth of water (h_{90}), the mean velocity and the hydraulic radius of the uniform flow. Finally, the energy dissipation along the non-uniform flow regime $\Delta E=E_0-E_1$ can be calculated.

Based on all data collected from five different stepped spillways (77 sets of data) for this study, an expression was derived to determine the rate of energy dissipation for non-uniform flow regime with R^2=0.53 as follow;

$$\frac{\Delta E}{E} = 0.2047\times\left(\frac{k_s}{D_h}\right)^{0.4708}\times(\mathrm{Re})^{0.2115}\times(Fr)^{-0.4970}\times(\tan\alpha)^{0.1615}\times\left(\frac{l_u}{l_s}\right)^{-0.0834} \tag{7}$$

An expression was also derived to cover only the present data (series D to F) for non-uniform flow regime, which provide a better regression of R^2=0.89 as follows;

$$\frac{\Delta E}{E}=4.51\times\left(\frac{k_s}{D_h}\right)^{-2.418}\times(\mathrm{Re})^{0.234}\times(Fr)^{1.214}\times(\tan\alpha)^{1.122}\times\left(\frac{l_u}{l_s}\right)^{-2.751} \quad (8)$$

In these equation the hydraulics radius and the Reynolds and Froude numbers of flow are based on the depth of water (h_{90}), which can be determined by the following relationship (Boes and Hager, 2003):

$$\frac{h_{90}}{h_s}=0.50F_s^{(0.1\tan\alpha+0.5)} \quad (9)$$

where, F_s is the step Froude number of flow in the form of;

$$F_s = q_w \Big/ \sqrt{g\sin\alpha . h_s^{\,3}}$$

For the uniform flow regime, the energy dissipation is regarded equal to the vertical displacement of the jet. Therefore, the total energy lost which is equal to the sum of the energy dissipations for uniform and non-uniform flow regimes were calculated. Supposing H_D as the height of the spillway, $(\Delta E)_1=(\Delta E/E)\times E$ and $(\Delta E)_2=H_D\text{-}h_u$, which are respectively the non-uniform and uniform parts of energy dissipations are calculated. Therefore, the total energy lost $(\Delta E)_t=(\Delta E)_1+(\Delta E)_2$ and thus the percentage rate of energy dissipation $(\Delta E/E)\times 100$ was determined.

Comparison of the results showed a reasonable regression of R^2=0.92. Figure 4 shows the comparison of the results of present expression for energy dissipation with those of previous relationships. In this figure the percentage rate of energy dissipations from the experiments on the horizontal axis were compared with those of calculated on the vertical axis. The figure also consists of a line of 45° and the lines of ±10% errors.

According to the figure, significant difference between the results of energy dissipation for gated and ungated spillways is not recognized. The figure also shows that the results of present expression are interestingly confined within the range of ±10% errors. However, it can be observed that the results of previous expressions provide a wider range of about ±60% errors.

5. Conclusion Remarks

A stepped chute spillway consists of an open channel with a series of drops so that energy dissipation is usually achieved along the steps and also within a standard stilling basin at the downstream end of the spillway. Water flowing over a stepped spillway can dissipate a major proportion of its energy. The steps increase significantly the rate of energy dissipation taking place along the spillway face and eliminate or

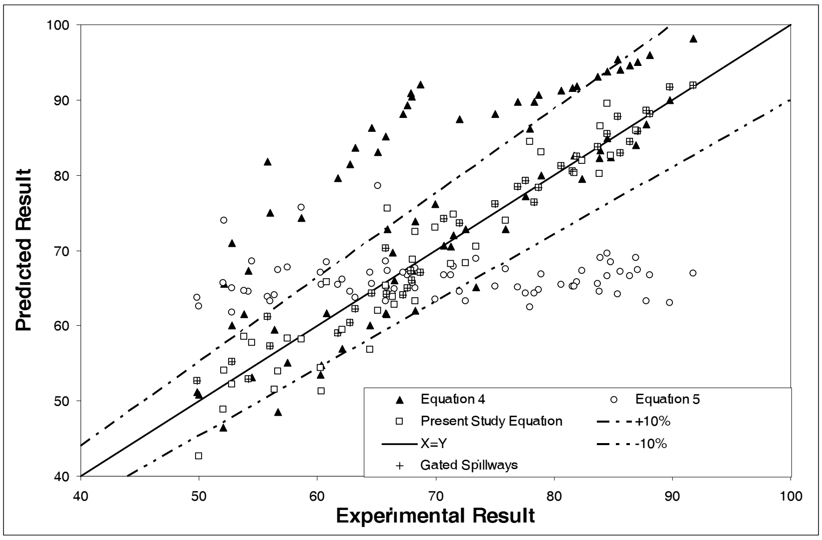

Fig. 4 Comparison of the experimental and those of present and previous expressions for total energy dissipation

reduce greatly the need for a large energy dissipater at the toe of the spillway. Various expressions have been introduced by previous investigators to predict the rate of energy dissipation, but due to the fact that these expressions are based on special hydraulic and geometry conditions, they can not be used for every situation.

This paper reviewed the characteristics of energy dissipation for the case of skimming flow along the zones of uniform and gradually varied flow regimes over stepped spillways. A set of experiments were performed with two physical models of the top and bottom stepped spillways of Siyah Bisheh dam in central Iran to determine the energy dissipation ΔE_t along the stepped spillways. The results of previous investigations have also been collected to develop a more generalized relationship for the energy lost along the non-uniform flow regime. Based on this information a general form for $\Delta E/E$ based on Froude and Reynolds numbers, angle of spillway, relative height of the step, and length of the gradually varied flow over stepped spillway was introduced. Energy dissipation along the uniform flow regime assumed equal to the vertical displacement. Also, the total energy lost, $(\Delta E)_t$ which is the sum of energy dissipation over the uniform and non-uniform regimes $(\Delta E)_t=(\Delta E)_1+(\Delta E)_2$ were calculated and the rate of total energy lost $(\Delta E_t/E_0)$ was determined. A comparison of the results showed a regression of 0.92 for the total energy lost over stepped spillways. Also, the present investigation showed that, a 10% error in measuring of the water depth causes about 5% error in estimating of the total energy lost over stepped spillway.

6. References

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