1. Introduction

The microclimatic conditions in an area are the main factors causing a soil deposit to be unsaturated (Thamer et al., 2006). Soils in arid and semi-arid regions of the world are dry and desiccated near the ground surface. Even under humid climate conditions the ground water can be well below the ground surface and the soils used in construction are unsaturated. Therefore, unsaturated soils or soils with negative pore water pressures can occur in essentially any geological deposit, such as residual soil and soils in arid and semi-arid areas with deep ground water table which is very common in Iran. When the ground water table is low it causes the soils to be mostly unsaturated except immediately after a rainfall.

The stability of cuts or trenches for laying pipelines involves unsaturated soils and is difficult to assess. The costs associated with temporary bracing are high, and each year lives are lost because of the instability of excavations. Stability analysis of temporary excavations needs the strength parameters of the unsaturated soils. The assessment of the stability of temporary excavations around construction sites is a difficult problem to solve and it is often left up to the contractor to handle in an appropriate manner (Fredlund and Rahardjo, 1992). To be more detailed, most of engineering problems involving heave, consolidation, collapse and dramatic changes in shear strength are directly related to the behavior of unsaturated soils (Fredlund and Rahardjo, 1985). The steep man-made cutting slopes made in residual soils to accommodate structures, require rational shear strength parameters to be established in partly saturated soils (Cornforth, 2005).

In this study, two vertical loose silty sand slopes are constructed and accelerated in a geotechnical centrifuge to fail. In the dry condition, the sand is cohesionless. However a vertical slope of this sand with limited height can be stand when it has some water. This is due to suction induced cohesion. In order to determine the cohesion in unsaturated condition in sand, a slope stability analysis will be performed according to the
geometry of the failure wedge and the failure acceleration in centrifuge models. Calculated cohesion will also be used in a Finite Element Method based software and a Finite Difference Method based one to compare the results with those of the physical modeling.

2. Centrifuge Modeling

Model tests using geotechnical centrifuge is a powerful method in physical modeling of soil samples. Centrifuge can artificially reproduce a behavior of soil samples, in which the distribution and magnitude of stress are the same as those of prototype. This feature is the main advantage of centrifuge modeling. Therefore, geotechnical centrifuge tests are capable of simulating the stress dependency of soil materials (Garg, 1992). Another advantage of the centrifuge tests is that it is not necessary to apply an external load on the top face of a soil sample (e.g. trenches), this is an advantage because the shape, size and location of the failure wedge is controlled by the loading conditions (Askarinejad, 2006, 2007). Figure 1 illustrates the concept of centrifuge modeling.

3. Shear strength parameters of unsaturated soils

When water seeps into soils, it can be absorbed and stored by the soil. The applied energy per unit volume of water is called soil suction or total suction (Lee and Wray 1995, Park 2000). Matric suction is the negative pore water pressure or capillary stress across the air-water interface and is associated with the capillary phenomenon from the surface tension of water.

Principally, theoretical development of unsaturated soil mechanics can be grouped into two tracks:

a) Effective stress approach of Bishop (1959) and Bishop and Blight (1963),

b) Independent variable approach of Fredlund and Morgenstern (1977) and Fredlund et al. (1978).

a) Effective Stress Approach: Bishop (1959) and Bishop et al. (1960) suggested that the effective stress ($\sigma'$) in unsaturated soils can be expressed as:

$$\sigma' = (\sigma_n - u_a) + \chi (u_a - u_w)$$

(1)

In which $\chi$ is the fractional cross sectional area of the soil occupied by water. Bishop et al. (1960) pointed out that although $\chi$ value primarily depends on the degree of saturation, it is also influenced by soil structure and stress pathway (wetting and drying) leading to a given degree of saturation. Bishop and Blight (1963) derived the deviation of $\chi$ value from the degree of saturation for two materials. Similar relationships have been presented in the literature. However all of them point out to the difficulty of using Eq. (1) because of the difficulty of predicting the $\chi$ value.
b) Independent State Variable Approach: The independent state variable approach was proposed by Fredlund and associates in a series of papers (Fredlund and Morgenstern, 1977; and Fredlund et al., 1978; Fredlund and Rahardjo, 1993; Fredlund, 1996). They suggested the following relationship for describing shear strength of unsaturated soils.

\[ \tau = c' + (\sigma_n - u_n) \tan \phi' + (u_a - u_w) \tan \phi^b \]  
(2)

Where \( \tan \phi^b \) is the slope of the shear strength vs. matric suction relationship. They also observed that although the Mohr-Coulomb failure criterion for saturated soil is plotted in two dimensions, the corresponding plot for unsaturated soil must be a 3-dimensional diagram (Fig. 2). Fredlund and Rahardjo (1993) further showed that since the intercept of the failure envelop intersects the shear stress vs. matric suction plane (Fig. 2), the relationship between the shear stress vs. matric suction can be described as:

\[ c = c' + (\mu_a - \mu_w) \tan \phi^b \]  
(3)

In which \( c \) is the intercept of the Mohr-Coulomb failure envelope at a specific matric suction and zero net normal stress.

In the case of saturation where all voids in the soil are filled by water, \( u_a = u_w = u \), the Eq. 3 is identical to Mohr-Coulomb criterion for saturated soils. However, in the most practical situations, the pore-air pressure in unsaturated soils \( u_a \) can be assumed zero (i.e. atmosphere).

Over the years, many experimental efforts have been made to verify the relationship between shear strength and soil suction (Eq. 2). One of the difficulties has been keeping soil suction constant as the specimen is being sheared.

A soil-water characteristic curve (SWCC) that relates the water content of a soil to matric suction is another important relationship for the unsaturated soil mechanics. The SWCC essentially shows the ability of an unsaturated soil to retain water under various matric suctions. While it is relatively easy to measure the soil-water characteristic curve in the laboratory, it is still quite costly and the test has not found its way into most conventional soils laboratories. Also computing the magnitude of \( \phi^b \) is a challenging in-situ and laboratory process. In this study a direct method of estimating the suction induced cohesion in unsaturated soils is explained. This direct method includes physical modeling of a sand trench by geotechnical centrifuge and performing a stability analysis on the failed wedge.

4. Physical modeling

In order to determine the suction induced cohesion in the semi saturated soil, a series of geotechnical centrifuge tests are organized. The apparatus specifications, the tested soil properties and the test procedure are explained as follows.

- Apparatus

The centrifuge tests of this research program were performed in the Geotechnical Centrifuge...
Centre of Iran University of Science and Technology. The centrifuge machine has a radius of R=1.0 m to the platform and a maximum capacity of 70 kg at 200 gravities (Salehzade, et. al., 2005). Table 1 gives the main specifications of this centrifuge machine.

The strong box used is 60 cm long, 18.5 cm wide and 14 cm high. The main structure of the box is a steel frame, and at the two longitudinal side walls, two sheets of anti-reflex Plexiglas are used. Two thin sheets of glass are also installed inside the longitudinal walls to reduce the friction between sand and the wall (Bransby, and Smith, 1975, Tie, 1993).

- The soil:
The sand used in these tests is Mahan Sand which is fine silty sand. Properties of this soil are summarized in Table 2.

According to the grading and hydrometry tests, the soil is composed of 85.5% sand, 12% silt, and 2.5% clay having a fairly uniform particle distribution ($C_c=3.3$, $C_s=0.7$).

As it can be seen in Table 2, the dry sand is cohesionless. In these series of tests the soil is mixed with water to have 5% water content. The unit weight of the sample is $\gamma = 15.6 \text{kN/m}^3$.

The saturation ratio of the sample can be estimated by using equation 5.

\[
\begin{align*}
\gamma &= (1 + \omega)\times \gamma_s \times G_s \\
\omega \times G_s &= S \times e
\end{align*}
\]

(5)

It should be mentioned that prior to the compaction stage of acceleration, the height of samples was 13 cm and they had a relative density of $D_r=10\%$. After compaction stage the samples settle to 11.5 cm leading to a relative density of $D_r=35\%$.

- Model Preparation and test procedure
To prepare a model the soil is poured in the box within the pre-specified height. During the sand pouring at different levels, target points are progressively installed. The sand is poured from a constant height to the surface of the soil. There is no compaction in this stage. To prepare models with the same density, a 20g centrifugal acceleration was applied to the models. This
acceleration level exerts a load equal to 20 times of the weight of particles on the underlying layers. During this stage, the face of cut is supported and no lateral displacement is allowed. Following this stage, lateral supports are removed. This process is performed at 1g and then the model is accelerated to fail.

During the in-flight condition, the positions of the target points are transferred by a wireless camera system to a computer and are saved.

5- The observed results and analysis

In figures 3 and 4 the condition of sample before and after the failure are shown.

As mentioned before, after applying the compaction acceleration (20g) and occurrence of a 1.5cm settlement at the top of the trench, the lateral supports of the trench are removed and the 11.5cm sand trench is accelerated up to failure. Model No.1 failed catastrophically at the acceleration of 4.25g. Failure of this model is shown in figure 4.

In order to investigate the virtual cohesion of used sandy materials, the force equilibrium can be written at the failure acceleration for the unstable wedge.

Calculating the failure wedge weight:

\[ W = 0.5 \times (H \times w \times d) \times \rho \times N_f \times g \rightarrow W = 159.26N \]  

(6)

Where

H: the height of the failure wedge
w: the width of the failure wedge
d: the depth of the failure wedge which is
assumed to be unit
\( \rho \): the density of the sample
\( N_f \): the acceleration at failure
\( g \): the gravity acceleration of the earth

Writing the force equilibrium,
\[
\begin{align*}
F_x \times \tan \phi + F_c &= W \times \sin i = 150.43 \, N \\
F_x &= W \times \cos i = 50.54 \, N
\end{align*}
\Rightarrow F_c = 115.044 \, N
\]
\[
\Rightarrow c = \frac{900 \, N}{\text{m}^2}
\]

In order to check the repeatability of tests, another model was tested with the same method. The model No.2 was 13 cm in height before the compaction stage of acceleration and after that stage it was 11.5 cm.

In the Figures 6 and 7 this model is shown before and after the failure. This model failed at the acceleration of 4.83g and the width of unstable wedge at the crest was 3 cm.

By using equations 6 and 7, the calculated virtual cohesion for model no. 2 is:
\[
\Rightarrow c = \frac{849 \, N}{\text{m}^2}
\]

This result is well comparable with the results of model No.1.

6. Numerical verification

6.1. Finite Element Method using PLAXIS software

The soil failure criterion is defined as Mohr-Coulomb and the cohesion is \( c = 900 \, \text{Pa}, \phi = 35^\circ \) and the height of the model is:
\[
H_p = H_m \times N_f \rightarrow H_p = 11.5 \times 4.58 \approx 52 \, \text{cm}
\] (8)

The mesh generation and boundary conditions are shown in the figure 8.

As the dilation angle is also required in analysis and has not been measured in the laboratory a parametric study is performed using different dilation angles. Results of this numerical modeling are listed in table 3. In this table the calculated factor of safety is compared with that of the physical model at failure.

6.2. Finite Difference Method using FLAC-SLOPE software

The soil failure criterion is defined as Mohr-Coulomb and the cohesion is \( c = 900 \, \text{Pa}, \phi = 35^\circ \) and the height of the model is:
\[
H_p = H_m \times N_f \rightarrow H_p = 11.5 \times 4.58 \approx 52 \, \text{cm}
\] (9)

A parametric study is performed on the effect of dilation angle and results are listed in table 4. In this table the factor of safety is compared with the F.S of physical model at the moment of failure.

7. Conclusion

Negative pore pressure and capillary forces develop a virtual cohesion between the grains of semi-saturated soils. The magnitude of this kind of cohesion is dependent on different factors such as grain size, water content, soil-water characteristic curve and relative density of the
Fig. 8 Mesh and boundary conditions

Fig. 9 Deformed shape of the model

Table 3 Factor of safety vs. dilation angle using PLAXIS

<table>
<thead>
<tr>
<th>H(cm)</th>
<th>C(pa)</th>
<th>Dilation Angle (deg)</th>
<th>F.S.</th>
<th>Difference with the F.S of physical model (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>900</td>
<td>0</td>
<td>0.723</td>
<td>27.7</td>
</tr>
<tr>
<td>52</td>
<td>900</td>
<td>10</td>
<td>0.777</td>
<td>22.3</td>
</tr>
<tr>
<td>52</td>
<td>900</td>
<td>20</td>
<td>0.843</td>
<td>15.7</td>
</tr>
<tr>
<td>52</td>
<td>900</td>
<td>30</td>
<td>0.86</td>
<td>14</td>
</tr>
<tr>
<td>52</td>
<td>900</td>
<td>35</td>
<td>0.877</td>
<td>12.3</td>
</tr>
</tbody>
</table>

Fig 10 Minimum factor of safety

Table 4 Factor of safety vs. dilation angle using FLAC-SLOPE

<table>
<thead>
<tr>
<th>H(cm)</th>
<th>C(pa)</th>
<th>Dilation Angle (deg)</th>
<th>F.S.</th>
<th>Difference with the F.S of physical model (%)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>16</td>
</tr>
<tr>
<td>52</td>
<td>900</td>
<td>10</td>
<td>0.85</td>
<td>15</td>
</tr>
<tr>
<td>52</td>
<td>900</td>
<td>20</td>
<td>0.86</td>
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<tr>
<td>52</td>
<td>900</td>
<td>35</td>
<td>0.87</td>
<td>13</td>
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</tbody>
</table>
soil. In this study, two vertical loose silty and slopes were constructed and accelerated in a geotechnical centrifuge to fail. To determine the cohesion, a slope stability analysis was performed according to the geometry of the failure wedge and the failure acceleration. To compare the results of physical and numerical, two numerical simulations were also done by FEM and FDM.

Results of this research are:

- The calculated cohesion in the two centrifuge tests were approximately the same.

- The observed geometry of failure wedge and Nf were also the same in this two models.

- The calculated factor of safety for the prototype by the FEM has a difference of fairly less than 30% compared to the centrifuge model.

- The calculated factor of safety of the prototype by the FDM has a difference of 16% compared to the centrifuge model.

- Results of this research show the validity of physical modeling for calculating the virtual cohesion in unsaturated silty sand.

8. Acknowledgments

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9. References


