A Model for Estimating the Aggregate Content for Self Compacting Fiber Reinforced Concrete (SCFRC)

P. Ghoddousi[1,*], R. Ahmadi[2] and M.Sharifi[3]

Abstract: *Superior performances of Self-Compacting Concrete (SCC) in fresh state to achieve a more uniform distribution encourage the addition of fibers in concrete which is a motivation for structural application of fiber-reinforced concrete. Fiber addition reduces the workability of Self-Compacting Fiber Reinforced Concrete (SCFRC). To provide required workability of the SCFRC, more paste is needed in the mixture. Therefore, the coarse aggregate content shall be adjusted to maintain its workability. The purpose of this study is to drive a model for estimating the aggregate contents for SCFRC. This model is based on constant covering mortar thickness theory. In this paper, all parameters which are participated in coarse aggregate content are discussed and presented in a relation. Then another relation is developed for predicting the void volume in the fibrous concrete. These relations are combined and a mathematical relation is deduced for predicting the coarse volume content in the function of the fiber factors. Proposed model is validated by conducting a rheological test. The result shows that the proposed model is simple, applicable and can be used as starting point in practical project.*
Finally in order to complete the proposed model, another relation has been derived that can show the interaction of parameters involved in SCFRC rheology behavior.

Keywords: *Self-Compacting Fiber Reinforced Concrete, Coarse Aggregate, Rheology, Covering Mortar Thickness*

1. Introduction

Self-compacting concrete (SCC) is a new category of high performance concrete characterized by its ability to spread into complicated shapes and congested reinforcements by its weight without any segregation and blocking. The superior performances of self-compacting concrete in fresh state to achieve a more uniform distribution encourage the addition of fibers in concrete and structural application of fiber-reinforced concrete. Discontinuous fibers added to concrete cause to increase the energy absorption capacity of concrete and post cracking behavior [1&2]. Synergy incorporation of the self-compacting concrete and fiber-reinforced technology allow the eliminating of vibration and reduction or complete substitution of transverse reinforcement [3 -7].

Although fiber addition reduces the workability of Self-Compacting concrete, it was found that using fiber in self-compacting concrete is feasible and Self-compacting properties can still be maintain at significant fibers with some required adjustment [8]. Type, diameter, aspect ratio and volume fraction of fiber come in addition of nominal size of aggregate and its contents are the major parameters that play an important role in flowability of self-compacting fiber reinforced concrete. ACI544 suggested to provide better workability of concrete, more paste is needed in the mixture of normal concrete [9]. Therefore the ratio of fine to coarse aggregate is adjusted accordingly.

Addition of fibers in self-compacting concrete has been studied by a number of researchers. Khayyat and Roussel [8] compared flowability and rheology of self-compacting concrete with different types of fibers. They illustrated that is possible to produce cohesive fiber reinforced self-compacting concrete. Groth and Nemegeer [10] studied the effect of steel fiber on flowability, segregation and toughness of self-compacting fiber reinforced concrete. Gustafsson [11] reported the data base on full scale production of self-compacting fiber reinforced concrete. Also some recommendations were

---

*   *Corresponding Author: ghoddousi@iust.ac.ir*

1   *Assistance Professor of IUST (Iran University of Science Technology)*
2   *Assistance Professor of BHRC (Building and House Research Centre)*
3   *PHD Candidate of IUST (Iran University of Science Technology)*

made by Grunewald [12] defined the maximum fiber factor related to the content and particle size distribution of aggregate and their risk of blockage through an equivalent bar spacing. Johnston [13] recommended to reduce the volume of coarse aggregate at least 10% compared with plain concrete.

The main of this study is to drive a model for estimating the aggregate contents for Self-Compacting Fiber Reinforced Concrete (SCFRC). This model is based on constant covering mortar thickness theory. In this paper, all parameters which are participated in coarse aggregate content are discussed and presented in a relation. Then another relation is developed for predicting the void volume in the fibrous concrete. These relations are combined and a mathematical relation is deduced for predicting the coarse volume content in the function of the fiber factors. Proposed model is validated by conducting a rheological test. The result shows that the proposed model is simple, applicable and can be used as starting point in practical project.

Finally in order to complete the proposed model, another relation has been derived that can show the interaction of parameters involved in SCFRC rheology behavior.

2. Model Concept

As mentioned earlier it is suggested that the volume of coarse aggregate should be reduced in fiber matrix to meet required workability. The purpose of this study is to develop a rheological model that suggests gravel content in SCSFR considering constant thickness of covering mortar (t_{cm}) in function of fiber factor ($v_f l_f / d_f$), where V_f is fiber fraction in concrete and l_f / d_f is fiber aspect ratio. This method is based on the fact that with a given total surface area of fibers, the total surface area of aggregates should be reduced to maintain tcm as reference mix (without fibers). In order to do that the multi aspect ratio concept used by Voigt [14] to relate the thickness of mortar layer cover fiber and gravel (t_{cm}) and maximum crack width to shrinkage has been used. According to Voigt [14] fiber reinforced concrete consists of two phases: The first phase includes gravel and fiber and the second phase is matrix (mortar) consisting of sand

and cementitous paste. The cementitous paste comprises cement, water, air, mineral and chemical admixture. Under the assumption of maximum compaction, the mixture of fibers and gravel contains a certain volume of air voids that solely depends on the volume, the size distribution and shape of fiber and gravel. If a certain volume of matrix is added to this conglomerate, the matrix has to fill up exactly that volume of air voids. The volume of matrix that exceeds this voids volume is equally used to cover the surface of fibers and gravel. The exact equation for calculating the covering mortar thickness, t_{cm}, proposed by Voigt et. all. [14] is given by Eq. (1):

$$t_{cm} = \frac{V_c - V_g - V_f - V_v}{A_g + A_f} \qquad (1)$$

Where t_{cm} is the average thickness of mortar layer covering fibers and gravel, V_c is the total volume of concrete, V_g is the total volume of gravel, V_f is the total volume of fiber, V_v is the total volume of voids (determined according to ASTM C29[15]), Ag is the total surface area of gravel and Af is the total surface area of fibers.

The volume of concrete, coarse aggregate are calculated from the mixture proportions. The total surface area of fiber calculated according to Eq. 2:

$$A_f = \frac{\%fiber \times density \times surface\ area\ of\ a\ fiber}{volume\ of\ a\ fiber \times density} \qquad (2)$$

If we assume a fiber with straight and circular section the Eq. 2 converts to Eq. 3:

$$A_f = \frac{\%fiber \times density \times \pi d_f \times l_f}{l_f \times \pi d_f^2 / 4 \times density} = \frac{4 \times V_f}{d_f} \qquad (3)$$

Also by applying an approximation A_g can be calculated as Eq. 4:

$$A_g = \frac{6Vg}{d_{ave}} \qquad (4)$$

Where d_{ave} is the average diameter of coarse aggregate particles.

Substituting Eq.3 and Eq.4 in Eq.2 the equation can be rewritten as Eq.5.

$$t_{cm} = (Vc - Vg - Vf - Vv) / (\frac{6Vg}{d_{ave}} + \frac{4Vf}{d_f}) \qquad (5)$$

This study focuses on how the coarse volume shown in Eq.6 can be estimated by a simple

mathematical relation.

$$V_g = (1 - Vf - Vv - \frac{4Vf\, t_{cm}}{d_f}) / (\frac{6t_{cm}}{d_{ave}} + 1) \qquad (6)$$

The Eq. 6 shows that V_g is the function of fibers fraction (V_f), voids volumes (V_V), fiber diameter (d_f), aggregate grading represented here by average diameter of coarse aggregate (d_{ave}) and covering mortar thickness (t_{cm}). So this relation can be written as Eq. 7:

$$V_g = f(t_{cm}, V_f, V_V, d_f, d_{ave}) \qquad (7)$$

Average diameter of aggregate is calculated with the Eq.8 where d_i is the average diameter of aggregate fraction i (defined as the average opening size of two consecutive sieves) and m_i is the mass of that fraction, i.e. the mass retained at the lower opening sieve.

$$d_i = \frac{\sum_i d_i m_i}{\sum_i m_i} \qquad (8)$$

The voids volume (V_v) in the Eq. 6 is a dependent parameter and is a function of fibers and aggregate properties as Eq. 9:

$$V_v = f(V_f, l_f/d_f, d_{ave}) \qquad (9)$$

Substituting the Eq.9 in the Eq.7, it can be inferred that the coarse volume contents relation (V_g) is the function of five independent parameters as presented in the Eq. 10:

$$V_g = f(t_{cm}, V_f, d_f, d_{ave}, l_f/d_f) \qquad (10)$$

Considering the above explanations, a mathematical relation has been developed for voids volume (V_v) for a specific coarse aggregate type. Then this relation substitutes in the Eq. 6 and a new relation has been prepared for the coarse volume content.

3. Experimental Program

Based on above theory, experimental investigation was carried out in two phases. In the phase 1, some tests were carried out to develop analytical model for voids volume. These tests have been done on one type of aggregate and three types of fibers according to ASTM C29 [15]. In the phase 2, the obtained relation from phase 1 has been used to develop model to estimate aggregate contents. Finally this model has been verified in some fiber reinforced concrete mixes.

3.1. Material Properties

- **Binders**
  The Portland cement type I with specific gravity of 3.15 and the Silica Fume (SF) with specific gravity of 2.2 has been used.

- **Aggregate**
  A natural river sand and crushed limestone with a maximum size of 12.5 mm was used as fine and coarse aggregates, respectively. The specific gravity and water absorption property of river sand is 2.53 and 4.1% and for crushed limestone is 2.5 and 1.4% . Average diameter (d_{ave}) of coarse aggregate is 5.17 mm . Particle size distribution of aggregate is presented in Fig.1.

- **Filler**
  A non-reactive limestone powder was used as filler material. The specific gravity and water absorption property of limestone powder is 2.63 and 10.1 % respectively.

- **Chemical Admixture**
  A Super Plasticizer (SP) with polycarboxylate-based has been used as an SP.

- **Fibers**
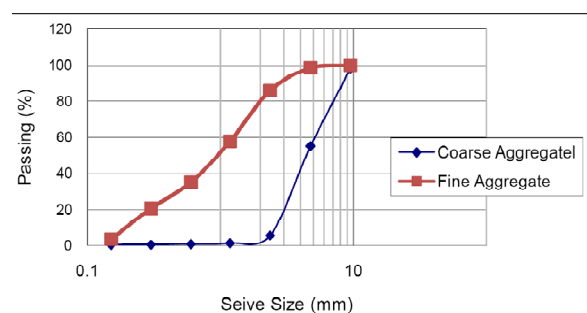 Three types of fiber have been used in this study as following:

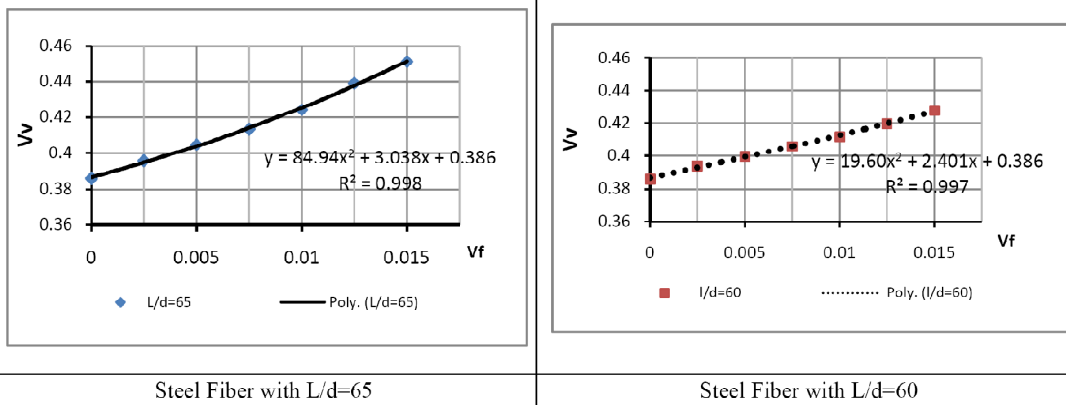**Fig. 1.** Particle Size Distribution of Aggregate

Fig. 2. Voids Volume Relation versus Steel Fiber Fraction

- Dramix hook ended steel fiber type ZP 30/0.5 with length of 30 mm, diameter of o.5 mm and aspect ratio (l_f/d_f)60
- Dramix hook ended steel fiber type 65/35 with length of 35 mm, diameter of o.53 mm and aspect ratio (l_f/d_f) 65
- A mono filament Poly Propylene (PP) fiber with 50 mm length and aspect ratio of 74.

3.2. Phase 1-Void Volume Relation

Voids volume of aggregate with various fiber fractions is determined with the method according to ASTM C29 [15]. The results and their mathematical relations for steel fibers and PP fiber are presented in Fig.2 and Fig.3 respectively.

As seen in prior figures, voids volume has a polynomial relation versus fiber fraction in both steel and PP fiber as following:

Steel Fiber, l_f/d_f=60;

$$Vv = 19.6\,V_f^2 + 2.401\,V_f + 0.386 \qquad (11)$$

Steel Fiber, l_f/d_f=65

$$Vv = 84.94\,V_f^2 + 3.038\,V_f + 0.386 \qquad (12)$$

PP Fiber, l_f/d_f=74;

$$Vv = -720.6\,V_f^2 + 13.92\,V_f + 0.387 \qquad (13)$$

Also voids volume relation in function of fiber factor $(V_f\,l_f/d_f)$ for steel fiber is shown in Fig. 4. This Figure implies that there is a similar relation with polynomial equation as:

$$Vv = 0.022(V_f\,l_f/d_f)^2 + 0.035\ (V_f\,l_f/d_f) + 0.0387 \qquad (14)$$

It should be noted that this figures are related to specified aggregate, i.e. with determined dave
.

3.3. Proposing the Mathematical Model

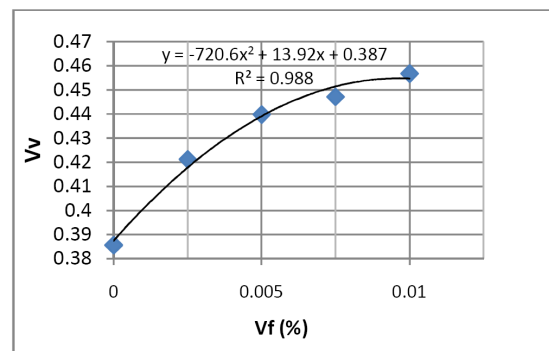If there is a mixture of self-compacting concrete that achieves the rheological properties

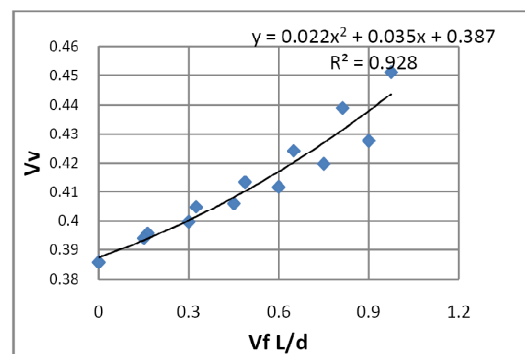**Fig. 3.** Voids Volume Relation versus PP Fiber Fraction

Fig. 4. Voids Volume Relation versus Fiber Factor
$(V_f\,l_f/\,d_f)$ - Steel Fiber

300

in fresh state, mortar covering thickness (t_{cm}) of this mix can be calculated with the use of Eq. 5. By substituting the calculated mortar covering thickness (t_{cm}) and voids volume relation (such as presented in clause 3.2) in the Eq. 6, a model will be obtained to estimate the coarse aggregate content in any self-compacting fiber reinforced concrete with desirable fiber fraction. For example an obtained model for self-compacting steel fibers reinforced is presented as following:

$$V_{g2} = \frac{1 - V_f - A - (\frac{4V_f t_{cm}}{d_f})}{(\frac{6t_{cm}}{d_{ave}} + 1)} \qquad (15)$$

Where:

$$A = \left[0.022 \left(\frac{V_f l_d}{d_f} \right)^2 + 0.035 \left(\frac{V_f l_d}{d_f} \right) + 0.387 \right] \qquad (16)$$

3.4. Model Validation

To verify the proposed model, the workability of a series of self-compacting steel fiber reinforced concrete has been tested. The test includes four mixtures with 0.25%, 0.5%, 0.75% and 1.0% of fiber fraction by using the hook ended steel fiber with aspect ratio of 60. Mixture proportions of the reference concrete and its rheological properties are given in Table 2 and Table 3 respectively. Average diameter of used coarse aggregate in this mixture is d_{ave}= 5.177 mm and the covering mortar is calculated as t_{cm}=1.259 mm.

The Eq. 11 has been used to develop rheological model. In this case, the aggregate content model is obtained as:

Table 1. Mixture Proportion for Reference Concrete (without fiber)

Component	Weight (kg/m^3)	Volume (m^3)
Cement	477	0.151
Silica Fume	53	0.024
Powder	240	0.091
Gravel	544.63	0.218
Sand	826.75	0.327
Water	150	0.150
SP	2.7	0.0027
w/c	0.3	

$$Vg_2 = 0.2179 - 6.956\,Vf^2 - 5.608\,Vf \qquad (17)$$

Coarse aggregate content of mixture has been calculated by using the Eq. 17. Calculated coarse aggregate volume is less than the coarse aggregate content in the reference mixture, so the difference volume of coarse aggregate shall be added to past volume to achieve a unit of volume concrete. The mixture proportions of fiber reinforced concrete and its obtained rheological properties are presented in Table 3 and Table 4 respectively. As presented in Table 4 the rheological properties of fiber reinforced concrete are near to rheological properties of the reference concrete.

4. Discussion on the Proposed Model

The major concern of the proposed model is that the presented relations are valid for specified aggregate (i.e. specified d_{av}) and cannot be used for other types and particle size effect [16]. In this clause the effect of aggregate type on proposed model has been discussed. In order to complete the proposed model, the used basic relation (i.e. covering mortar thickness) has been combined with another existing eminent relation.

Bui [17] proposed another formula to calculate the average spacing (d_{ss}) between the particle

Table 2. Rheology Properties of Reference Concrete

	Slump Flow (cm)	V Funnel (Sec)	J Ring (mm)
Allowable [1]	65 to 80	6 to 12	<15 mm
Measurement Values for Ref. Mixture	66	3.12	7

Table 3. Mixture Proportion of Fiber Reinforced Concrete

Mix No.		Mix1	Mix2	Mix3	Mix4
Fiber Fraction	%	0.25%	0.50%	0.75%	1.00%
Cement	kg/m^3	477	477	477	477
Sillica Fume	kg/m^3	53	53	53	53
Powder	kg/m^3	248.09	260.25	264.92	272.56
Gravel	kg/m^3	509.3	472.5	435.7	402.25
Sand	kg/m^3	854.69	880.25	912.828	939.12
Water	kg/m^3	150	150	150	150
SP	lit/m^3	2.7	2.7	2.7	2.7
w/c	-	0.3	0.3	0.3	0.3

Table 4. Rheology Properties of Fiber Reinforced Concrete

	Slump Flow (cm)	V Funnel (Sec)	J Ring (mm)
Allowable [**Error! Bookmark not defined.**]	65 to 80	6 to 12	<15 mm
0.25%	66	3.25	7
0.5%	66	3.28	12
0.75%	67	3.50	15
1.0%	64	4.43	20

surface in concrete in the prior study as:

$$d_{ss} = d_{ave}\left\{\sqrt[3]{1 + \frac{V_p - V_v}{V_c - V_p}} - 1\right\} \qquad (18)$$

Where d_{ss} is the average spacing between particle surface; V_p is past volume; V_v is volume of voids in densely compact aggregate; V_c is total concrete volume and dave is the average particle diameter (Eq. 8).

There are some common parameters in the Eq. 18 and the Eq. 1 as following relations:

$$\mathbf{t_{cm}} = \frac{1}{2}\mathbf{d_{ss}} \qquad (19)$$

$$V_{EP} = V_p - V_v = V_c - V_g - V_f - V_v \qquad (20)$$

Where; V_{EP} is the excess paste which needs to create a layer enveloping the particle.

The Eq.1 can be combined with Eq.18 considering the Eq.19 and 20; one obtains:

$$d_{ss} = d_{ave}\left\{\sqrt[3]{1 + \frac{t_{cm}(A_g + A_f)}{V_c - V_p}} - 1\right\} \qquad (21)$$

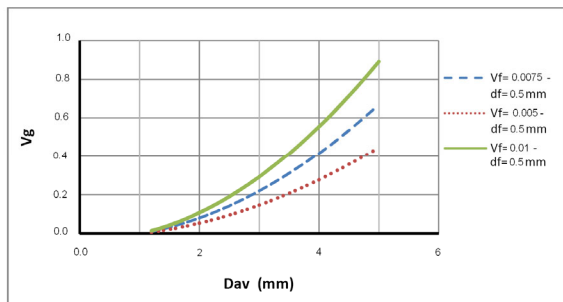Substituting the Eq. 3, Eq. 4 and Eq.22 in above equation, the Eq.23 can be derived.

Fig. 5. Variation of Vg versus dave

$$V_c - V_p = V_g + V_f \qquad (22)$$

$$d_{ss} = d_{ave}\left\{\sqrt[3]{1 + \frac{t_{cm}\left(\frac{6Vg}{d_{ave}} + \frac{4Vf}{d_f}\right)}{V_g + V_f}} - 1\right\} \qquad (23)$$

The Eq. 23 can be put in order and resulted in Eq24.

$$\left(\frac{d_{ss}}{d_{ave}}\right)^3 + 3\left(\frac{d_{ss}}{d_{ave}}\right)^2 + 3\left(\frac{d_{ss}}{d_{ave}}\right)\left(1 - \frac{V_g}{V_g + V_f}\right) = \frac{2\left(\frac{d_{ss}}{d_f}\right)V_f}{V_g + V_f} \qquad (24)$$

The Eq.24 presents the interaction between parameters involved in SCFRC rheology behavior. This relation can be used beside the proposed model to cover its deficiency. In this study the Eq.24 is put as following;

$$V_g = V_f\left(2\frac{d_{ss}}{d_f}d_{ave}^3 - d_{ss}^3 - 3d_{ss}^2 d_{ave} - 3d_{ss}d_{ave}^2\right)$$

$$/\left(d_{ss}^2 + 3d_{ss}^2 d_{ave}\right) \qquad (25)$$

The Eq.25 is plotted versus dave in Fig. 5 for reference mix in clause 3.3. As presented in this figure, the variation of coarse aggregate volume in function of coarse aggregate type (which is introduced with dav) can be estimated.

5. Conclusion

A model for estimating the aggregate contents for SCFRC has been presented and assessed by rheological test. Proposed model is based on constant covering mortar thickness theory. In order to derive the model, first all parameters which are participated in coarse aggregate content are discussed and presented in a relation. Then another relation is developed for predicting the void volume in the fibrous concrete. This study shows that a polynomial relation versus fiber fraction can be developed for voids volume. These relations are combined and a mathematical relation is deduced for predicting the coarse volume content in the function of the fiber factors. Proposed model is validated by conducting a rheological test. The result shows that the proposed model is simple, applicable and can be used as starting point in practical project.

Finally in order to complete the proposed model, another relation has been derived that can show the interaction of parameters involved in SCFRC rheology behavior.

6. Acknowledgment

References

[1]. N. Banthia, M. Sappakittipakorn, "Toughness enhancement in Steel Fibre Reinforced Concrete Through Fibre Hybridization", Cement and Concrete Research, 2007.

[2]. H.Oucief, M.F.Habita, B.Redjel, "Hybrid fiber reinforced self-compacting concrete: hardened properties", International Journal of Civil Engineerng. Vol.4 , No. 2, June 2006.

[3]. Shin SW, Oh JG, Ghosh SK. Shear behavior of laboratory-sized high strength concrete beams reinforced with bars and steel fibers, SP-142-10, American Concrete Institute; 1994.

[4]. Shah SP. Fiber reinforced concrete. Concr Int, Am Concr Inst 1990.

[5]. Frosch RJ. Behavior of large-scale reinforced concrete beams with minimum shear reinforcement. ACI Struct J 2000.

[6]. A. Foroughi-Asl, S. Dilmaghani, H. Famili,"Behavior Bond strength of reinforcement steel in self-compacting concrete. International Journal of Civil Engineerng. Vol. 6, No. 1, March 2008.

[7]. Mirsayah AA, Banthia N. Shear strength of steel fiber-reinforced concrete. ACI Struct Journals 2002;99(5).

[8]. Khayat H. K and Roussel Y., "Testing and Performance of Fiber-Reinforced Self Consolidation Concrete", First Int. Symposium on SCC, Stockholm, Edited by Skarendahl and Peterson, RILEM Publication, 1999.

[9]. ACI Committee 544, "Guide for Specifying proportioning mixing, placing and finishing Steel Fiber Reinforced Concrete", American Concrete Institute, Famington Hill, 1993.

[10]. Groth P and Nemegeer D, " The use of Steel Fiber in Self-Compacting Concrete", First Int. Symposium on SCC, Stockholm, Edited by Skarendahl and Peterson, RILEM Publication, 1999.

[11]. Gustafsson J. "Experience From Full Scale Production of Steel Fiber Self-Compaction Concrete", First Int. Symposium on SCC, Stockholm, Edited by Skarendahl and Peterson, RILEM Publication, 1999.

[12]. Grunewald S. "Performance-Based Design of Self-Compaction Fiber Reinforced Concrete", PhD Thesis, 2004.

[13]. Johnston C. D. ," Fiber-Reinforced Cement and Concrete", Gordon and Breach Science Publication, Amsterdam, 2001.

[14]. Voigt T., Bui V.K. and Shah, "Drying Shrinkage of Concrete Reinforced with Fiber and Welded-Wire Fabric", ACI Mat. J., Vol. 101, No. 3, 2004.

[15]. ASTM C 29/C 29M, " Standard Test Method for Bulk Density ("Unit Weight") and Voids in Aggregate, Annual Book of ASTM Standards, Vol. 04.02. 1997.

[16]. I. Rasoolan , S. A. Sadrnejad, A. R. Bagheri, "A Geometrical Inclusion-Matrix Model For Concrete", International Journal of Civil Engineering. Vol. 7, No. 2, June 2009.

[17]. Bui K. V., "A Method for the optimum proportioning of the Aggregate Phase of High Durable Vibration-Force Concrete", Master of Engineering Thesis, AIT Bangkk, Thailand,1994.