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A bi-level model for location-allocation problem of construction & demolition waste management under fuzzy random environment

Jiuping Xu[1,2,*], **Pei Wei**[2]

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Abstract

In this paper, a location allocation (LA) problem in construction and demolition (C&D) waste management (WM) is studied. A bi-level model for this problem under a fuzzy random environment is presented where the upper level is the governments who sets up the processing centers, and the lower level are the administrators of different construction projects who control C&D waste and the after treatment materials supply. This model using an improved particle swarm optimization program based on a fuzzy random simulation (IPSO-based FRS) is able to handle practical issues. A case study is presented to illustrate the effectiveness of the proposed approach. Conclusions and future research directions are discussed.

Keywords: Location-allocation optimization, Construction waste management, Fuzzy random, PSO, Bi-level models

1. Introduction

Location-allocation (LA) problems are often encountered in many practical systems, such as emergency service systems, telecommunication networks, and the public service [30, 2, 23, 35, 36, 39]. In this paper, the LA problem in construction and demolition (C&D) WM in large-scale engineering projects is considered.

The significant increase in construction activity in China associated with the rapid economic development has produced a large amount of C&D waste over the past three decades. Existing research suggests that construction activity is a major contributor to environmental pollution. Dong et al. [4] found that China produced approximately 30% of the world's municipal solid waste (MSW), and more recently Wang et al. [29] found that amongst China's MSW, construction activities were responsible for nearly 40%, after having consumed about 40% of total natural resources and around 40% of energy. Consequently, C&D waste has become a serious problem domestically with C&D waste recycling and processing being gradually focused on.

C&DWM research and practice has been guided by the 3Rs principles; reduce, reuse and recycle [15, 26]. One advantage of reuse and recycling is the reduction in landfill quantities and a partial displacement of the environmental impact of quarrying activities [21]. However, transporting the processed available material to the demand nodes increases the C&D waste recycling burden, so decisions on the location of the recycling and processing centers are especially important. Up to now, only a few studies have been conducted looking at the WMLA problem. Kao et al. [14] studied a model to reduce the drawbacks of recycling depot locations; Erkut et al. [6] presented a new multi-criteria mixed-integer linear programming model to solve an LA problem for regional municipal solid WM; Louwers et al. [12] gave an LA facility model for the collection, preprocessing and redistribution of carpet waste; Valeo et al. [28] improved an LA model within a geographic information systems (GIS) software package. In this paper, two hierarchies in the C&D WM LA problem are developed, to embody the game relationship, and BLP is adopted to deal with this problem.

In practice, many parameters, for example, customer demand and, transport costs are uncertain rather than deterministic, so they are usually very hard to determine and need to be estimated from historical data. Traditionally, customer demand in an LA problem is assumed to be a random variable. Wang et al. [32] presented LA decisions in a two-echelon supply chain with stochastic demand; Logendran and Terrell [16] considered stochastic demand in their incapacitated plant LA problem.

A new approach to the LA problem has been developed where customer demand and other parameters are characterized as fuzzy variables as sometimes these

* *Corresponding Author: xujiuping@scu.edu.cn*
1 State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu 610064, P. R. China
2 Uncertainty Decision-Making Laboratory, Sichuan University, Chengdu, 610064, P. R. China

parameters can be imprecise. Zhou and Liu [36] developed a model of a capacitated LA problem with fuzzy demand; Wen and Iwamura [30] studied an LA facility problem with fuzzy demand; El-Sayed et al. [7] assumed demand to be stochastic in their model for a forward - reverse logistics network design under risk; Shen and Liu [24] studied a new class of fuzzy LA problems and applied an approximation method; Liu [18] considered transportation costs as a fuzzy variable.

In practice, sometimes randomness and fuzziness coexist in an LA problem. For instance, if a company wishes to open new shops due to an increase in customer demand, they need to decide whether these shops should be located in a new region or in an established region. Because customers in the established regions have been supplied by the company for a long time, demand can be summarized by probability distributions. However, customers in the new regions have never been supplied by the company and thus demand can be described using fuzzy variables and the choice of potential locations is a combination of random factors. In the real world, many institutions face such problems when they want to expand their scale and customer reach. These problems include two kinds of data: random and fuzzy thus FRV can be introduced into the LA problem with a mixed uncertainty of randomness and fuzziness depending on the nature of the actual problem. This dual uncertainty in the LA problem and other problems has been the subject of focused research. Wen and Kang [33] studied an LA facility problem with random fuzzy demands; Wang et al. [31] gave a two-stage fuzzy-random facility-location model; Xu and Yang [34] studied travel time and demand as fuzzy random variables (FRVs); Xu and Yao [38] studied a logistics distribution center location problem with random fuzzy coefficients.

This paper contributes to the literature as follows: first, this paper proposes a BLP model which considers two hierarchies of C&DWM in large-scale projects and solves the LA problem. In addition, an FRV is adopted to describe the demand and transport costs which help decision makers make more effective and precise decisions. Secondly, an improved particle swarm optimization program based on a fuzzy random simulation (IPSO-based FRS) is used to solve this problem. Thirdly, this model and method are applied to a practical case to arrive at an optimum decision. This model has the ability to provide guidance to C&D LA waste recycling decision makers.

This paper is organized as follows: in Section 2, the location-allocation problem mathematical model is introduced; in Section 3, an IPSO-based FRS to solve this bi-level problem is given; in Section 4, a case study in a large-scale water conservancy and hydropower construction project is presented to demonstrate the efficiency of the proposed method; finally, in Section 5, concluding remarks are outlined.

2. Problem statement and model formulation

The problem in this paper focuses on C&D WM LA. The concrete issues are the location of processing centers and the optimal allocation of waste recycling and processed material supply. The proposed fuzzy random BLP model optimizes cost on two separate levels.

2.1. Problem description

In this paper, the construction project LA WM problem involves two participants i.e. the government who invests in waste collection depots and processing centers and the administrators of different construction WM systems who control where construction waste should go and the materials demand after treatment. Due to this two part hierarchy, the two participants who make decisions successively in the model on the upper and lower-level are first described.

The investor on the upper-level decides the type, quantity and position of the processing centers whereas the administrator on the lower-level decides which processing centers are the best in accordance with the upper-level decision. The investor on upper-level affects the decision of the administrator on the lower-level, but does not fully control it. The administrators make decisions autonomously based on the decision of the upper-level. Figure 1(A) describes this kind of system in C&D waste management, and Figure 1(B) shows a motivating example of this problem. Traditionally, reverse logistics is the process of recycling products. However, in the C&D WM process, reverse logistics includes waste collection, recycling and processing between the construction project demand nodes, recycling depots and processing centers, all of which are affected by the processing center location

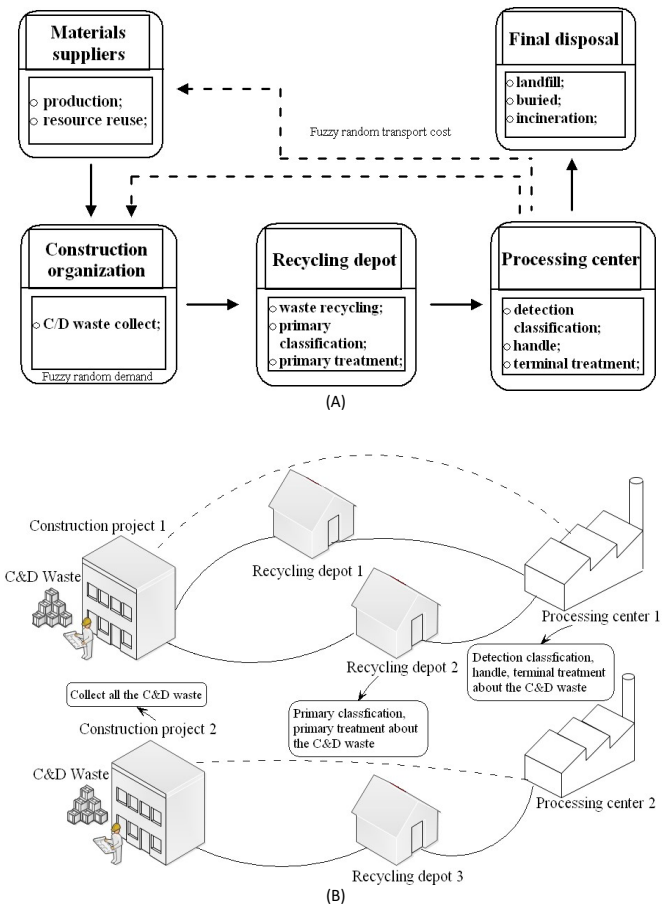


Fig. 1. The forward and reverse logistics system of construction waste management

2.2. Model assumptions

The model was built on the following assumptions:

(1) Only one kind of waste material is considered; (2) The upper level is the processing center location decision maker, and the lower level is the construction WM system administrator who decides the quantity to be conveyed from one point to another; (3) The maximum capacity of the recycling depots and processing centers are known in advance; (4) The transit between every demand node, recycling depot and processing center are assured and can be measured.

Based on the assumptions above, a BLP model of the C&D WM under a fuzzy random environment can be developed.

2.3. Model formulation

C&D WM is a complex problem that involves decisions both at the strategic and the operating level. The proposed model focuses on a bi-level decision process with the investor (i.e. government) and the administrators of different WM systems under a fuzzy random environment.

2.3.1. Lower-level programming

The lower-level programming is the choice-behavior of the WM system processing center administrators. The assignment of a certain customer's demand is influenced by the assignment of another customers' demand. The administrators on the lower-level decide which processing center is the best so that transport costs are minimized.

Objective function:

The lower-level administrators want to achieve minimum transport costs, so the objective is a reduction or minimization of total transportation costs from one point to another.

Thus, the minimum objective can be described as $\min_x H\left(\tilde{\bar{C}}, x\right)$.

x is the vector of x_{ij}, x_{jk}, x_{ki}. is $\tilde{\bar{C}}$ he vector $\tilde{\bar{C}}_{ij}$, $\tilde{\bar{C}}_{jk}$, $\tilde{\bar{C}}_{ki}$. For FRVs, Kwakernaak [9, 10] introduced a mathematical model which was later formalized more clearly by Kruse and Meyer [11]. In the Kwakernaak/Kruse and Meyer approaches, FRV is viewed as a fuzzy perception/observation/report of a classical real-valued random variable. Because the FRVs cannot be calculated, an expected operator is used to deal with the objective functions. Probabilistic and statistical studies for FRVs in Kwakernaak/Kruse and Meyer's approach usually concern either 'crisp' parameters of the 'original' random variable or fuzzy-valued parameters defined on the basis of Zadeh's extension principle. More precisely, Kruse and Meyer [11] have defined.

Definition 1 If θ(X) is a real-valued parameter of a random variable X:$\Omega \to R$ associated with the probability space (Ω, A, P), and $\xi(\Omega, A, P)$ denotes the class of all possible 'originals' of an FRV χ:$\Omega \to F_C$(R) associated with (Ω, A, P), then the induced fuzzy parameter of χ corresponds to: $\theta(\chi)$:R$\to$(0,1] such that for all $t \in R$,

$$\theta\left(\mathrm{X}(t)\right) = \sup_{\chi \in \xi(\Omega,\mathrm{A},\mathrm{P}) | \theta(\mathrm{X})=t} \inf_{\omega \in \Omega} \left\{ \chi(\omega)\left(\mathrm{X}(\omega)\right) \right\}$$

as an example of an induced fuzzy parameter, so that if θ(X)=E(X|P), then θ(X) corresponds to the so-called fuzzy expected value of χ which is the fuzzy set in F_C(R) such that for each $\alpha \in (0,1]$,

$$\left(\theta(\chi)\right)_\alpha = \left[E\left(\inf \chi_\alpha \mid \mathrm{P}\right), E\left(\sup \chi_\alpha \mid \mathrm{P}\right)\right]$$

and an FRV can be transformed to a fuzzy interval. Here an example is given to illustrate the method for the determination of a fuzzy random variable expectation.

Based on the definitions and description above, the FRVs can be transformed, to $\tilde{\bar{C}}_{ij}$ for example,

$$\inf\left(\bar{E}\left(\tilde{\bar{C}}_{ij}\right)\right)_\alpha = E\left(\inf \tilde{\bar{C}}_{ij\alpha} \mid \mathrm{P}\right); \qquad \sup\left(\bar{E}\left(\tilde{\bar{C}}_{ij}\right)\right)_\alpha = E\left(\sup \tilde{\bar{C}}_{ij\alpha} \mid \mathrm{P}\right);$$

Thus, the fuzzy expected value may be represented by a single fuzzy number. Simultaneously, the objective functions with FRVs is equal to

$$\min_x E\left(H\left(\tilde{\bar{C}}, x\right)\right) = \sum_{i=1}^{I} \sum_{j=1}^{J} t_{ij} E\left(\tilde{\bar{C}}_{ij}\right) x_{ij} + \sum_{j=1}^{J} \sum_{k=1}^{K} t_{jk} E\left(\tilde{\bar{C}}_{jk}\right) x_{jk} + \sum_{i=1}^{I} \sum_{k=1}^{K} t_{ki} E\left(\tilde{\bar{C}}_{ki}\right) x_{ki}$$

where $E(\tilde{\bar{C}}_{ij})$, $E(\tilde{\bar{C}}_{jk})$, $E(\tilde{\bar{C}}_{ki})$ are the expected values of the FRVs $\tilde{\bar{C}}_{ij}$, $\tilde{\bar{C}}_{jk}$ and $\tilde{\bar{C}}_{ki}$., respectively. Because it is still difficult to deal with an objective function involving fuzzy variables, the optimum cannot be directly determined. In this paper, without loss of generality, an expected value operator is used for the conversion of the uncertain model to a deterministic one based on the theory proposed by Heilpern [8]. From this, the objective functions of the upper level and the lower level can be transformed into their crisp equivalences as in Eq. (1):

$$\min_x E\left(H\left(\tilde{\bar{C}}, x\right)\right) = \sum_{i=1}^{I} \sum_{j=1}^{J} t_{ij} E\left(\tilde{\bar{C}}_{ij}\right) x_{ij} + \sum_{j=1}^{J} \sum_{k=1}^{K} t_{jk} E\left(\tilde{\bar{C}}_{jk}\right) x_{jk}$$
$$+ \sum_{j=1}^{J} \sum_{k=1}^{K} t_{jk} E\left(\tilde{\bar{C}}_{jk}\right) x_{jk} + \sum_{i=1}^{I} \sum_{k=1}^{K} t_{ki} E\left(\tilde{\bar{C}}_{ki}\right) x_{ki} \tag{1}$$

where $E_d(\tilde{\bar{C}}_{ij})$, $E_d(\tilde{\bar{C}}_{jk})$ and $E_d(\tilde{\bar{C}}_{ki})$ are the expected values of fuzzy variables $E(\tilde{\bar{C}}_{ij})$, $E(\tilde{\bar{C}}_{jk})$ and $E(\tilde{\bar{C}}_{ki})$, respectively. Note the E_d above shows the twice expected values: the first being the fuzzy random variables converted into fuzzy numbers based on the theory proposed by Kruse and Meyer in 1987, and the second is used to transform the fuzzy numbers into deterministic numbers based on the theory proposed by Heilpern in 1992. In order to avoid confusion and facilitate understanding, the note E_d is given a double E.

Constraints:

The volume of available processed materials supplied to all demand nodes which have been trans- ported from processing centers is considered more or less equal. Demand at each node $\tilde{\bar{Q}}_i$ is an FRV. A chance-constrained operator is used to deal with this constraint.

Chance-constrained programming which was introduced by Charnes and Cooper is one of the most useful approaches in tackling problem uncertainty. The probability of meeting the demand at a demand node from a processing center must be at least at a pre-determined confidence level $\eta_i(\eta_i \geqq 0.5)$ specified by the decision maker(s). The confidence level frequently used

is greater than the 0.5 of the literature [27, 1]. η_i Here, is assumed to be greater than 0.5.

Where $\tilde{\tilde{Q}}_i(\omega)$ is a realization of the fuzzy random variable $\tilde{\tilde{Q}}_i$ under the occurrence of each elementary event ω. Given the membership function of a fuzzy random variable $\tilde{\tilde{Q}}_i$, the degree of possibility is defined as

$$Pos\left\{\sum_{k=1}^{K} x_{ki} \geq \tilde{\tilde{Q}}_i\right\} \geq \theta_i \ .$$

So, this constraint can be written as a set of chance-constraints as follows (2) [37],

$$\Pr\left\{\omega \mid Pos\left\{\sum_{k=1}^{K} x_{ki} \geq \tilde{\tilde{Q}}_i\right\} \geq \theta_i\right\} \geq \eta_i \tag{2}$$

The recycling depots and processing centers have capacity limits so that the capacity of processing center k is more than or equal to the amount which is transported to it, so Eq.(3) is as follows:

$$\sum_{j=1}^{J} x_{jk} \leq W_k y_k \tag{3}$$

The amount of waste materials supplied to recycling depot j by demand node i should be less than or equal to meet the capacity restraints, so the constraint is determined as in Eq. (4):

$$\sum_{j=1}^{J} x_{jk} \leq W_k y_k \tag{4}$$

The inflow and the outflow needs to be balanced, so that the quantity of rough handling waste materials entering the processing centers is equal to both the discarded and handling part. The handling part is the already processed available materials supplied to the demand node. In the C&D waste handling process, some raw materials may be added, so the weight may change, with a similar situation occurring at recycling depots. With this in mind, the discard rate is σ_1 at the processing center, and σ_2 at the recycling depot. These constraints Eq. (5) and Eq. (6) are as below:

$$\sum_{j=1}^{J} x_{jk}\left(1+\sigma_1\right) = \sum_{i=1}^{I} x_{ki} \tag{5}$$

$$\sum_{i=1}^{I} x_{ij}\left(1-\sigma_2\right) = \sum_{k=1}^{K} x_{jk} \tag{6}$$

The constraints below can be used to simply represent the relationship between the x_{jk}, x_{ki} and the processing center location patterns y_i under an equilibrium condition. Since M is an arbitrarily large positive constant, if $y_i=0$, then x_{ij} cannot be positive; but if $y_i=1$, then x_{ij} can be as large as desired, so the constraint Eq. (7) is as below:

$$x_{jk} \leq My_k, \qquad x_{ki} \leq My_k \tag{7}$$

2.3.2. Upper-level programming

The investors (i.e. government) on the upper-level make decisions about how many processing centers should be built and whether each processing center is in the right location.

Objective function:

The investor aims to minimize costs so the objective at the upper-level is the minimization of total costs which includes the lower-level decision of total transportation costs denoted as $H(\tilde{\tilde{C}},x)$, which has been described in the lower-level objectives . After analysis, three parts should be included in this objective, as follows: The first part is the total transportation costs, marked as: $E_d(H(\tilde{\tilde{C}},x))$; The second part is the waste material total processing costs:

$$\sum_{i=1}^{I}\sum_{j=1}^{J} D_j x_{ij} + \sum_{j=1}^{J}\sum_{k=1}^{K} D_k x_{jk} \quad ;$$

The third part is the basic processing center construction costs:

$$\sum_{k=1}^{K} B_k y_k \ ;$$

Thus, the minimum objective can be described as:

$$\min_y E_d\left(f\right) = E_d\left(H\left(\tilde{\tilde{C}},x\right)\right) + \sum_{i=1}^{I}\sum_{j=1}^{J} D_j x_{ij} + \sum_{j=1}^{J}\sum_{k=1}^{K} D_k x_{jk} + \sum_{k=1}^{K} B_k y_k \tag{8}$$

Constraints:

There should be at least one processing center, and, because of capital restraints, the decision maker determines the maximum number of processing centers, thus the following constraints Eq. (9),

$$1 \leq \sum_{k=1}^{K} y_k \leq N^k \tag{9}$$

In order to distinguish the two cases, (0, 1) variables are introduced. y_k with 1 indicating recycling center j has been selected and 0 means it has not, thus the following Eq. (10) constraint,

$$y_k \in \{0,1\} \tag{10}$$

2.3.3. Bi-level programming model

The complete bi-level programming model under a fuzzy random environment (A1) is established based on the above discussion.

$$(A1)\begin{cases} \min_y E_d\left(f\right) = E_d\left(H\left(\tilde{\tilde{C}},x\right)\right) + \sum_{i=1}^{I}\sum_{j=1}^{J} D_j x_{ij} + \sum_{j=1}^{J}\sum_{k=1}^{K} D_k x_{jk} + \sum_{k=1}^{K} B_k y_k \\ s.t. \begin{cases} 1 \leq \sum_{k=1}^{K} y_k \leq N^k \\ y_k \in \{0,1\}, \quad \forall k \in K \\ \min_x E_d\left(H\left(\tilde{\tilde{C}},x\right)\right) = \sum_{i=1}^{I}\sum_{j=1}^{J} t_{ij} E_d\left(\tilde{\tilde{C}}_{ij}\right) x_{ij} + \sum_{j=1}^{J}\sum_{k=1}^{K} t_{jk} E_d\left(\tilde{\tilde{C}}_{jk}\right) x_{jk} + \sum_{i=1}^{I}\sum_{k=1}^{K} t_{ki} E_d\left(\tilde{\tilde{C}}_{ki}\right) x_{ki} \\ s.t. \begin{cases} \Pr\left\{\omega \mid Pos\left\{\sum_{k=1}^{K} x_{ki} \geq \tilde{\tilde{Q}}_i\right\} \geq \theta_i\right\} \geq \eta_i \\ \sum_{j=1}^{J} x_{jk} \leq W_k y_k \\ \sum_{i=1}^{I} x_{ij} \leq S_j \\ \sum_{j=1}^{J} x_{jk}\left(1+\sigma_1\right) = \sum_{i=1}^{I} x_{ki} \\ \sum_{i=1}^{I} x_{ij}\left(1-\sigma_2\right) = \sum_{k=1}^{K} x_{jk} \\ x_{jk} \leq My_k, x_{ki} \leq My_k \quad \forall i \in I, j \in J, k \in K \\ x_{ij}, x_{jk}, x_{ki} \geq 0, \quad \forall i \in I, j \in J, k \in K \\ i = 1,2,\cdots,I; j = 1,2,\cdots,J; k = 1,2,\cdots,K \end{cases} \end{cases} \end{cases}$$

3. A modified PSO program based on bi-level problem fuzzy random simulation

It is very difficult to solve the BLP model with the main reason being is that it is an NP-hard problem. Ben-Ayed and Blaint [3] studied this problem deeply, and pointed out that even very simple bi-level programming is still an NP-hard problem. Particle Swarm Optimization (PSO), developed by Eberhart and Kennedy [5], is a form of swarm intelligence in which the behavior of a biological social system like a flock of birds or a school of fish is simulated.

This technique uses collaboration among a population of simple search agents (called particles) to find the optimum in a search space, and has been shown to be effective in optimizing difficult multidimensional problems in a variety of fields [13, 22]. In this paper, an IPSO-based FRS algorithm is proposed to solve this BLP model including FRVs.

3.1 Fuzzy random simulation

A Fuzzy random simulation for the expected value model is proposed. FRV expected values are usually difficult to transform into their equivalent forms. Xu and Zhou [35] put forward a fuzzy random simulation by combining stochastic simulation and fuzzy simulation to solve these types of problems. In this paper this kind of simulation is used to determine the equivalent value of the objective functions dealt with by the expected operator. For example, fuzzy random transport cost $\tilde{\tilde{c}}$.

In a fuzzy random expected value model (1), one problem is to calculate the expected value $E_d(\tilde{\tilde{c}})$. Note that, for each $\omega \in \Omega$ the expected value $E_d(\tilde{\tilde{c}}(\omega))$ can be calculated using fuzzy simulation. Since $E_d(\tilde{\tilde{c}})$ is essentially the expected value of stochastic variable $E_d(\tilde{\tilde{c}}(\omega))$, stochastic simulation and fuzzy simulation can be combined to produce a fuzzy random simulation. The detailed program is in the Appendix B: Programs.

For the chance-constrained programming, the Fuzzy random simulation it is also used and a chance-constrained operator is used to deal with the FRV in the constraint above. After that, the constraint becomes similar to Eq. (4). Xu and Zhou proposed a fuzzy random simulation aimed at fuzzy random objective functions and fuzzy random constraints in chance-constrained models respectively [35]. Thus in this case the program below is given. The detailed program is in the Appendix B: Programs.

3.2 Update and improvement against a fall into a local optimal

When a particle finds a current group optimum location, other particles quickly draw close to it. If the particle is at the local optimum solution, the particle swarm cannot search the solution space, so the algorithms fall into local optimization, recognized as premature convergence. To avoid this, a random disturbance is added to the current whole PSO solution, which is helpful in determining the local optimum [17].

Hypothesis κ is a random variable which obeys standard normal distribution, κ ~ N(0,1), so

$$\Psi_{g(h+1)} = \Psi_{gh}\,(1+\kappa) \tag{11}$$

So, by updating the Ψ_{gh} to $\Psi_{g(h+1)}$ with Eq. 11, the velocity and position of each particle can be updated as shown below. The particle positions in each dimension are held to a maximum position θ^{max} and a minimum position θ^{min} , and position of which in that dimension is limited to θ^{max} and θ^{min} [19, 20]. The detailed program is in the Appendix B: Programs.

For this LA BLP, an IPSO-based FRS is used to search for optimum results and a random disturbance is added to avoid premature convergence. To describe the algorithmic model, the procedure is presented as follows.

The overall algorithm program flow diagram is presented at Figure 2. Full details of the IPSO-based FRS program are in Appendix B.

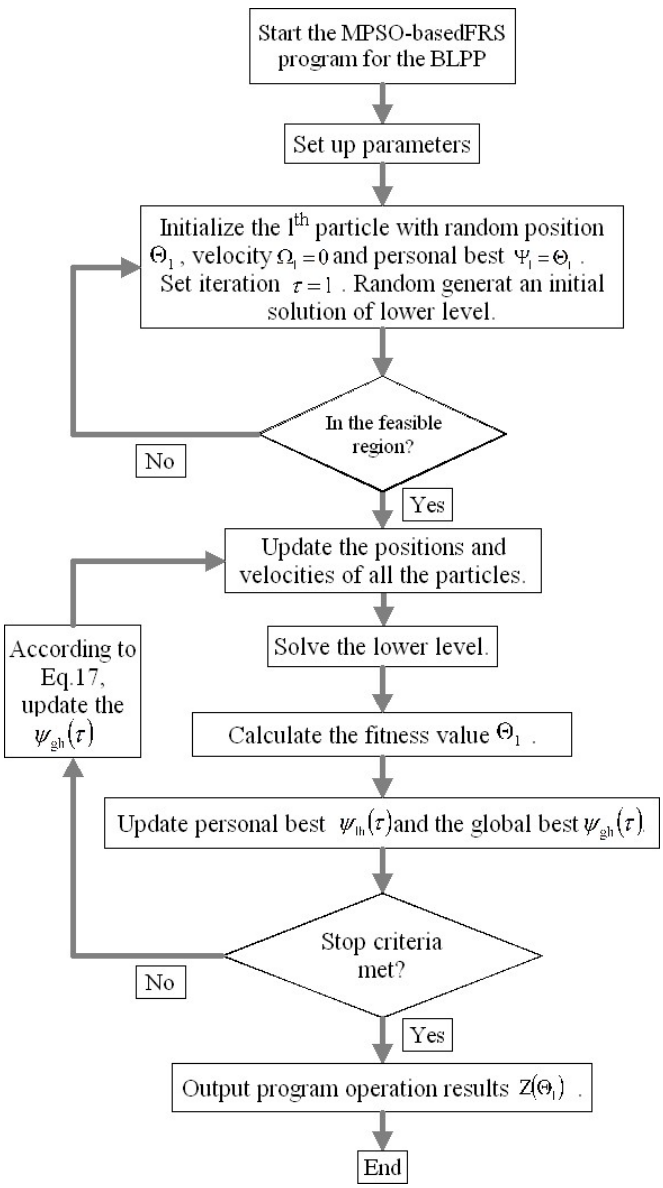


Fig. 2. Overall procedure of the IPSO-based FRS

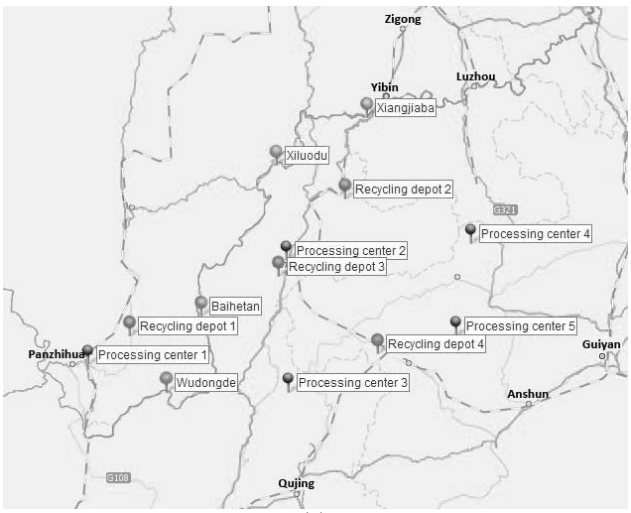
4. Case study

The government's criteria for the construction of waste processing centers are to improve social living conditions, regional economic development, and environmental protection. Large-scale water conservancy and hydropower construction projects produce large amounts of C&D waste, so it is necessary to locate processing centers near these projects.

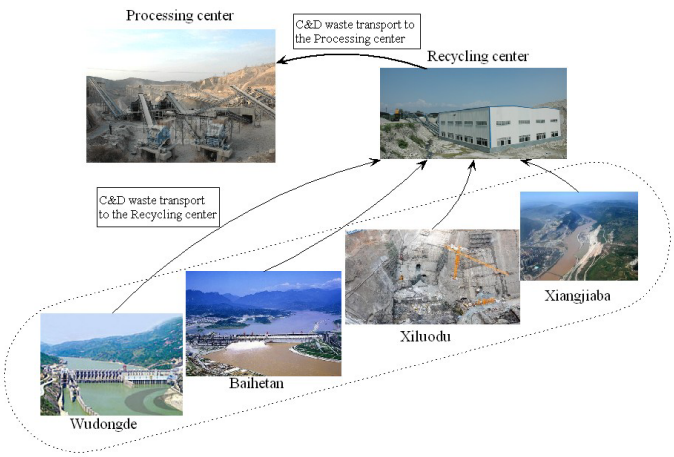
The case study here examines the conclusion of four large-scale water conservancy and hydropower construction projects; the Xiangjiaba Hydroelectric Station, the Xiluodu Hydropower Station, the Baihetan Hydropower Station and the Wudongde Hydropower Station. These four hydropower stations consitute the four hydropower cascades downstream on the Jinsha River. Among them, the Xiangjiaba Hydropower Station is at the last level of the Jinsha River. Construction commenced in 2006, and is planned to be completed in 2015, with project duration of 9 years. The Xiluodu Hydropower 11 Station is the biggest project of the four with commencement in 2003 and a planned completion and commission date in 2015. On 27th October, 2010, the government launched the preparatory work for the Baihetan Hydropower Station and the project is estimated to be formally completed in 2022. The Wudongde Hydropower Station construction commenced in 2010, with a the total construction duration of approximately 8 years enabling C&D waste to be collected, recycled and processed to achieve a reasonable recycling resource.

The four large-scale water conservancy and hydropower construction projects are located in southwest China. There were some potential locations for the processing centers around the region which were chosen by experts according to geological characteristics, government regulations and the development of the surrounding cities, etc. The specific locations are in Figure 3 (A) and the picture of them shows in Figure 3 (B).

The input data for this case is presented in the tables below, most of which were obtained using surveys from the existing processing center management s. Because of limited space, here only part of the data is presented in Table 1. Table 1 (A) presents basic information about the processing centers and recycling depots such as the capacity limits, basic construction costs and unit processing costs. However, with a project like this, exact determinations are difficult such that when gathering data there are many statements such as "it is about 0.2 ton"," it is few, but may increase a little in the future" or



(A)



(B)

Fig. 3. The location and picture of the four hydropower construction projects

Table 1. The input data for this case study
(A) Basic information about processing center and recycling depots

Potential processing center k	Capacity limit W_k (in m3)	Unit processing cost D_k (in Yuan)	Basic construction cost B_k (in 106 Yuan)
P1	450	18.8	1.8
P2	350	18.8	1.2
P3	450	18.8	1.8
P4	450	18.8	1.8
P5	350	18.8	1.2
Recycling depots j	Capacity limit S_j (in m3)	Unit recycling cost D_j (in Yuan)	——
R1	650	10.3	——
R2	650	10.7	
R3	650	10.8	
R4	650	10.8	

"P" and "R" represent the processing center and recycling center.

Table 1. The input data for this case study
(B) Fuzzy random variables

Demand note i	Demand $\tilde{\tilde{Q}}_i$	Parameter $d(\omega)$
1	(206, $d(\omega)$, 264)	$d(\omega) \sim \mathrm{N}(235,23)$
2	(180, $d(\omega)$, 220)	$d(\omega) \sim \mathrm{N}(200,14)$
3	(210, $d(\omega)$, 240)	$d(\omega) \sim \mathrm{N}(230,25)$
4	(260, $d(\omega)$, 300)	$d(\omega) \sim \mathrm{N}(280,18)$
Items	Unit cost of transport	Parameter $d(\omega)$
$\tilde{\tilde{C}}_{ij}$	(8, $d(\omega)$, 13)	$d(\omega) \sim \mathrm{N}(10,16)$
$\tilde{\tilde{C}}_{jk}$	(12, $d(\omega)$, 16)	$d(\omega) \sim \mathrm{N}(14,7)$
$\tilde{\tilde{C}}_{ki}$	(8, $d(\omega)$, 12)	$d(\omega) \sim \mathrm{N}(10,25)$

"busy period and idle period transport costs are a little different". Here these descriptions have been converted into FRVs, as shown in Table 1 (B) and which shows the fuzzy random parameters of demand at each node and the unit transport costs.

It should be noted that the FRVs in Table 1 (B) were obtained using the following steps: 1) collect previous data and divide according to predetermined periods; 2) the lower bound of the fuzzy random number (i.e., α) is the minimum value of all groups; 3) the upper bound of the fuzzy random number (i.e., β) is the maximum value of all groups; 4) suppose the median value of every group to be a random variable (i.e., $d(\omega)$) and follow a normal distribution, and use a maximum likelihood method to estimate the distribution parameters ; 5) use goodness-of-fit testing to justify the appropriateness of the normal distribution in modeling the observed data; 6) finally, the fuzzy random number, (α, $d(\omega)$, β) , is derived.

Table 1 (C) shows that the distance in kilometers from one point to another is not straight as the path bypasses the town centers.

4.2 Results and analysis

Now, considering Model (A1) with the above data and using the IPSO-based FRS to deal with it. In this paper, MATLAB 7.0 on a Pentium 4, 1.83GHz clock pulse with 2048 MB memory was used with the following set parameters: Population Size , Iteration number T=300, c_p=2, c_g=3 , Inertia weight $w(1)$=0.9 and $w(T)$=0.1, respectively. After running the program 10 times the best solution was achieved as shown in Table 2.

Figure 4 (A) shows the detailed distribution of the objective value obtained by the IPSO-based FRS in different generations. It shows that the total cost of upper level f gets gradually smaller from one generation to another, which is consistent with the evolutional idea of an IPSO-based FRS.

The optimal solution shows that only 2 of the 5 potential processing centers need to be built as this would be enough to deal with the C&D waste from the 4 hydropower stations construction projects. The 2 processing centers are in the best location which means the solution offers significant cost saving, including the savings accrued from redundant processing centers' construction fee and the transportation costs of choosing an unsuitable location. After the processing centers deal with the C&D waste, some of which can be converted into available construction materials, the four hydropower stations will be a sustainable development system ultimately allowing for problem free construction of the hydropower stations and a reduction in environmental pollution.

Table 1. The input data for this case study
(c) The distance from one pot to another

	Xiangjiaba Hydroelectric Station	Xiluodu Hydroelectric Station	Baihetan Hydroelectric Station	Wudongde Hydroelectric Station
R 1	709.2	840.3	440.7	400.5
R 2	120.2	334.6	379.2	495.9
R 3	227.7	358.8	253.6	383.5
R 4	402.8	533.9	359.7	479.0
	Xiangjiaba Hydroelectric Station	Xiluodu Hydroelectric Station	Baihetan Hydroelectric Station	Wudongde Hydroelectric Station
P 1	664.2.	424.8	297.7	257.8
P 2	222.1.	353.3	258.7	408.4
P 3	472.2.	612.7	299.8	404.4
P 4	400.3.	641.6	543.2	689.1
P 5	473.9	683.7	539.0	985.8
	Recycling depot 1	Recycling depot 2	Recycling depot 3	Recycling depot 4
P 1	104.4.	771.3	826.7	702.0
P 2	404.1.	134.4	25.6	160.8
P 3	405.2.	354.5	224.2	205.9
P 4	1057.7.	564.2	346.8	312.8
P 5	967.8	480.2	317.7	167.6

"P" and "R" represent the processing center and recycling center.

Table 2. The results of the IPSO-based FRS

Gen	y	x_{ij}	x_{jk}	x_{ki}	f	$H\left(\tilde{\tilde{C}}\right)$
1	(11111)	xj1 : (200; 20; 20; 20) xj2:(20;2.75041;200; 20) xj3 : (20; 20; 20; 20) xj4 : (20; 20; 20; 20) xj5 : (20; 20; 20; 20)	x1i : (20; 20; 80 84; 200) x2i : (112 56; 120; 70 44; 20) x3i : (20; 20; 38 72; 20) x4i : (38 71; 20; 20; 20) x5i : (38 71; 20; 20; 20)	x1 j : (20; 200; 160; 20) x2 j: (20; 85 35772; 20; 20) x3 j : (20; 20; 200; 160) x4 j: (20; 20; 82 85714; 20)	16,936, 946	9,110, 637
50	(10111)	xj1 : (200; 0; 0; 0) xj2 : all = 0 xj3:(80;0;200;117.9254) xj4 : (0; 0; 1.750405; 0) xj5 : (0; 0; 0; 162.0746)	x1i : (0; 136.8; 30; 80) x2i : all = 0 x3i : (27 84; 63.2; 200; 200) x4i : (2.16; 0; 0; 0) X5i : (200; 0; 0; 0)	x1 j : (0; 200; 200; 0) x2 j : all = 0 x3 j : (0; 0; 200; 88.41286) x4 j : (200; 0; 200; 0)	15,075, 872	8,649, 770
100	(11101)	xj1 : (200; 0; 0; 0) xj2 : (0; 150; 200; 0) xj3 : (0; 0; 80; 107.4392) xj4 : all = 0 xj5 : (0; 0; 0; 24.31118)	x1i : (0; 0; 46.8; 200) x2i : (200; 200; 31 9; 0) x3i : (0; 0; 151.3; 80) x4i : all = 0 x5i : (30; 0; 0; 0)	x1 j : (0; 200; 85.1429; 0) x2 j : (0; 200; 14.8571; 0) x3 j : (0; 0; 200; 200) x4 j : (0; 0; 188.2149; 0)	13,014, 045	6,995, 048
150	(11100)	xj1 : (200; 0; 0; 0) xj2 : (0; 150; 200; 0) xj3: (0; 0; 80; 131.7504) xj4 : all = 0 Xj5 : all = 0	x1i : (0; 0; 46.8; 200) x2i : (200; 200; 31.9; 0) x3i : (30; 0; 151. 3; 80) x4i : all = 0 x5i : all = 0	x1 j : (0; 200; 85.71429; 0) x2 j : (0; 200; 14.28571; 0) x3 j : (0; 0; 200; 200) x4 j : (0; 0; 188.2149; 0)	11,832, 060	7,005, 711
200	(11001)	xj1 : (200; 0; 0; 0) xj2 : (0; 150; 200; 0) xj3 : all = 0 xj4 : all = 0 xj5:(0;0; 11.75041; 200)	x1i : (0; 0; 46.8; 200) x2i : (30; 200; 121.9; 80) x3i : all = 0 x4i : all = 0 x5i : (200; 0; 61.3; 0)	x1 j : (0; 200; 85.71429; 0) x2 j : (0; 200; 14.28571; 0) x3 j : (0; 0; 200; 102.5006) x4 j : (85.71429; 0; 200; 0)	11,721, 329	7,491, 173
250	(01100)	xj1 : all = 0 xj2 : (0; 150; 200; 0) xj3:(140;0;131.7504; 140) xj4 : all = 0 x j5: all = 0	x1i : all = 0 x2i : (121 9; 200; 30; 80) x3i : (108 1; 0; 200; 200) x4i : all = 0 X5i : all = 0	x1 j : (0; 200; 0; 0) x2 j : (0; 200; 14.28571; 0) x3j:(0;73.92915; 200; 200) x4 j : (0; 0; 200; 0)	11,246, 112	8,219, 784
300	(01100)	xj1 : all = 0 xj2 : (0; 150; 200; 0) xj3:(140;0;131.7504; 140) xj4 : all = 0 x j5: all = 0	x1i : all = 0 x2i : (121 9; 200; 30; 80) x3i : (108 1; 0; 200; 200) x4i : all = 0 X5i : all = 0	x1 j : (0; 200; 0; 0) x2 j : (0; 200; 14.28571; 0) x3j:(0;73.92915; 200; 200) x4 j : (0; 0; 200; 0)	11,246, 112	8,219, 784

4.3 Model analysis

As data was collected, the descriptions were translated into FRVs according to the data characteristics. The definition of an FRV is the refining and expansion of the fuzzy variables, so the results from the fuzzy random model to the fuzzy model were compared. From this fuzzy data were derived, which ignored randomness and only considered the fuzzy environment. The two kinds of data were put into the IPSO-based FRS, and the program run 10 times each, the results of which are shown in Table 3 (B).

From the results it can be seen that because fuzzy numbers can relax limitations and extend the solution space, considering fuzzy random factors may bring economic benefit, with a cost saving in this case of around 0.4 million. Considering randomness and fuzziness at the same time when making decisions can give decision makers more and better information and thus assist in making more controlled, informed decisions. From the results, it can be clearly seen that data translated into fuzzy random numbers conforms more to reality, and has a much better performance. As the fuzzy data is somewhat divorced from the facts, FRV is shown to be effective and efficient.

4.4 Algorithm evaluation

For better illumination of our algorithm, here a brief comparison is made between an IPSO-based FRS with a classic PSO using the same data and parameters: Population Size L=30, Iteration number T=300, c_p=2, c_g=3, Inertia weight $w(1)$=0.9 and $w(T)$=0.1, respectively. Figure 4 (B) shows a best in history convergence comparison from the IPSO-based FRS to the classic PSO with the same parameters. From Figure 4 (B), the optimum result begins to ameliorate after the 250*th* generation, and is not very steady. After the program is run 10 times, the results are quite different so from Table 3 (A) the predominance of our algorithm can be clearly seen compared to the classic PSO. The results and convergence from an IPSO-based FRS are shown to be better than those derived from the classical method.

The PSO parameters were determined from the results of preliminary experiments that were carried out to observe the behavior of an algorithm in different parameter settings. By comparing several sets of parameters, including population size, iteration number, acceleration constant, initial velocity,

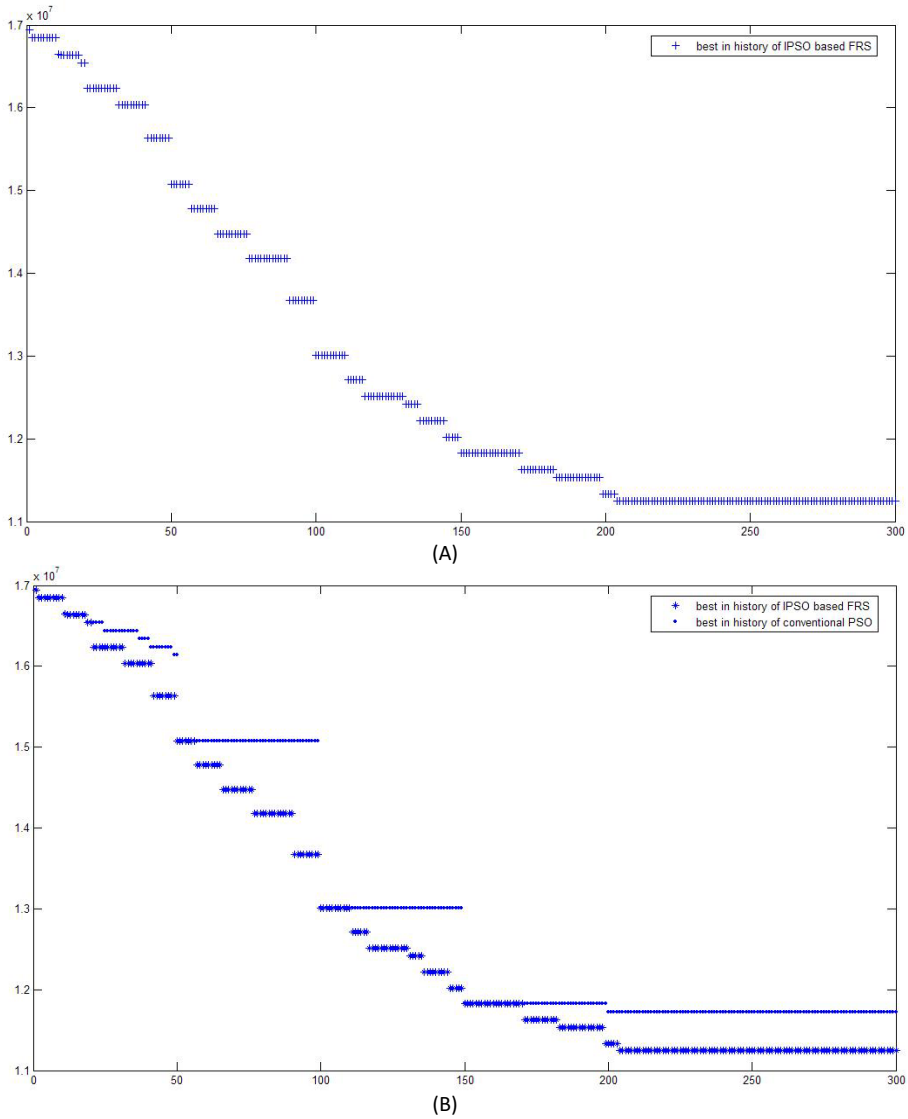


fig. 4. The convergence of best in history of the IPSO-based FRS and classical PSO

and inertia weight, the most reasonable parameters for this BLP model were identified. Note that population size (i.e., the number of particles) determined the evaluation run, and thus impact optimization cost [25].

After testing the solution algorithm, 30 particles were selected as the population size and 300 times as the iteration number. Through further experiments, $w(1)=0.9$ and $w(T)=0.1$ were found to be the most suitable for controlling the impact of the previous velocities and influencing the trade-off between global and local experience. Other parameters were selected by comparing the results with the observations of the swarm's dynamic search behavior. Since various learning factors, i.e., c_p and c_g , may lead to a little difference in the PSO's performance [25], they were set at values 2 and 3 respectively in this study. The values for initial velocity were selected based on the magnitude of the decision variables.

Table 3. The results of comparison

A

Item	IPSO-based FRS	classic PSO
Best results	11, 246, 112	11,856,363
Worst results	12, 936, 946	14,007,758

B

Item	FRVs	Fuzzy
Best results	11, 246, 112	13,435,382
Worst results	12, 936, 946	14,546,864

4.5 Effectiveness analysis

In the proposed fuzzy random BLP model, directed at the different personalities of the decision makers, the parameters in the model can be adjusted to accord to their styles. Because different valued parameters yield different results, decision makers can adjust these parameters to obtain different solutions.

The solutions reflect the different optimistic-pessimistic attitudes for uncertainty and the different requests for the probability and possibility levels. For example, if different values for the parameters θ_i are set in the dependence chance operator, different best fitness values are found which reflect

the different attitudes of decision makers, so, depending on the circumstances, the parameter can be adjusted to produce information to support our decisions.

5. Conclusion

This paper studies an LA problem in a C&DWM, and considers environmental elements and reverses logistics in a fuzzy random environment. This paper also detailed the fuzzy random environment which exists in C&D waste management LA problems and explained the necessity of using fuzzy random theory to handle such problems. To describe the relationship between the two hierarchies in this C&DWM system, a BLP model was established and an IPSO-based FRS proposed to solve it.

A case was studied and the results showed that the proposed model is simple, applicable and can be used as starting point in practical projects as it is valuable for helping make C&DWM decisions. Lastly, a brief comparison between an IPSO-based FRS and a classic PSO was made to highlight the merits of the presented algorithm, and another comparison between fuzzy random data and fuzzy data was conducted to explain the necessity of using fuzzy random variables.

Although the BLP model and IPSO-based FRS algorithm discussed in this paper are helpful in solving some real world problems, a detailed analysis and further research is necessary to reveal further properties.

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Appendix A: Notation

Index:

I : index of demand node, i=1,2,...,I;
J : index of recycling depots, j=1,2,...,J ;
K : index of processing centers, k=1,2,...,K;

Variables:

t_{ij} : the transit distance from demand node i to recycling depot j;
t_{jk}: the transit distance from recycling center j to processing center k;
t_{ki}: the transit distance from processing center k to demand node i;
$\tilde{\tilde{C}}_{(\square)}$: fuzzy random unit total cost for transporting from point to point ;
D_j : unite cost for C&D waste rough handling in recycling depot j;
D_k : unite cost for processing C&D waste in processing center k ;
$\tilde{\tilde{Q}}_i$: fuzzy random demand of demand node for materials;
B_k : the basic construction cost of processing center k ;
S_j : recycling capacity of recycling depot j;
W_k : processing capacity of processing center k;
N_k : the ceiling number of processing centers;

Decision variables:

x_{ij} :amount of waste material supplied to recycling depot j by demand node i;
x_{jk}: amount of rough handling waste material supplied to processing center k by recycling depot j ;
x_{ki} : amount of already processed available materials supplied to demand node i by processing center k;

$$y_k = \begin{cases} 1; \text{represents the k-th processing center is built;} \\ 0; \text{otherwise.} \end{cases}$$

Appendix B: Programs

Fuzzy random simulation for expected value model

Step 1: Set $E=0$;
Step 2: Sample ω from Ω according to the probability measure Pr;
Step 3: $E \leftarrow e+E(\tilde{\tilde{C}}(\omega))$, where $E(\tilde{\tilde{C}}(\omega))$ may be calculated by the fuzzy simulation as following sub-steps;
Step 3.1: Set $E=0$;
Step 3.2: Randomly generate $u_{1j},u_{2j},...,u_{nj}$ from the ε-level sets of $\omega_1,\omega_2,...,\omega_n$, and denote $u_j=(u_{1j},u_{2j},...,u_{nj})$, $j=1,2,...,n$ respectively, where ε is a sufficiently small number.
Step 3.3: Set
Step 3.3: Set $a=f(u_1)\wedge f(u_2)\wedge\cdots\wedge f(u_m),\ b=f(u_1)\vee f(u_2)\vee\cdots\vee f(u_m)$.
Step 3.4: Randomly generate r from $[a,b]$.
Step 3.5: if $r\geqq 0$, then $E\leftarrow E+C_r\{\tilde{\tilde{C}}(\omega)\geqq r\}$;
Step 3.6: If $r<0$, then $E\leftarrow E-C_r\{\tilde{\tilde{C}}(\omega)\geqq r\}$;
Step 3.7: Repeat the 3.4 to 3.6 steps for N times.
Step 3.8 : $E[\tilde{\tilde{C}}(\omega)]=a\vee 0+b\wedge 0+E\cdot(b-a)/N$.
Step 4: Repeat the second to fourth steps N times;
Step 5: $E_d[\tilde{\tilde{C}}]=e/N$;

Fuzzy random simulation for chance constrained model

Step 1: Generate $\omega=(\omega_1, \omega_2, ... \omega_N)^T$ from Ω according to the probability measure ;
Step 2: Generate a determined vector $\tilde{\tilde{Q}}_i$ uniformly from the θ_i-*cut* of fuzzy vector $\tilde{\tilde{Q}}_i(\omega)$, $i=1,2,..I$.

Step 3: If $\sum_{k=1}^{K} x_{ki} \geq Q_i$ then output that $\tilde{\tilde{Q}}_i$ is feasible, turn to step 5;
Step 4: Return step 2, and repeat M times;
Step 5: Return step 1, and repeat N times;
Step 6: Let $N`$ is the time when $Q_i$ is feasible. And $N$ is a sufficient large number. If $N`/N\geqq\eta_i$, output that Q_i is feasible, otherwise Q_i is not feasible, $i=1,2,...,I$.

Update the velocity of particle

Step 1: calculate the inertia weight in the τth iteration use:

$$w(\tau)=w(T)+\frac{\tau-T}{1-T}\left[w(1)-w(T)\right]$$

Step 2: update the velocity of particle:

$$\omega_{lh}(\tau+1)=w(\tau)\omega_{lh}(\tau)+c_p u\left(\psi_{lh}-\theta_{lh}(\tau)\right)+c_g u\left(\psi_{l(h+1)}-\theta_{lh}(\tau)\right)$$

Update the position of particle

Step 1: update the position of particle use:

$\theta_{lh}(\tau+1)=\theta_{lh}(\tau)+\omega_{lh}(\tau+1)$;

Step 2: judge the position as to whether it is in the feasible region or not:

if $\theta_{lh}(\tau+1)>\theta^{max}$, then: $\theta_{lh}(\tau+1)=\theta^{max}$, $\omega_{lh}(\tau+1)=0$

if $\theta_{lh}(\tau+1)<\theta^{min}$, then: $\theta_{lh}(\tau+1)=\theta^{min}$, $\omega_{lh}(\tau+1)=0$

The overall procedure for an IPSO-based FRS:

Step 1: Set up parameters including population size L(the number of particles), maximum and minimum position value θ^{max} and θ^{min}, inertial weight (ω). Two acceleration constants, C_p, C_g and a uniform random number u is in the interval [0,1].

Step 2: Initialize the *lth* particle with random positionΘ_l in the range [θ^{max},θ^{min}], velocity $\Omega_l=0$ and personal best $\Psi_{l=}\Theta_l$ for $l=1,2,...,L$. Set iteration $\tau=1$. Randomly generate an initial solution for the lower level.

Step 3: Constraints check. If in the feasible region, go to Step 4, otherwise, go back to Step 2.

Step 4: Perform the following actions on all the particles.

Step 4.1: Update the velocity and position for each *lth* particle.

Step 4.2: Substitute the initialization value for the upper level y into the lower level program and get the optimum x;

Step 4.3: Substitute the y and x into the objective function of upper level. For $l=1,2,...,L$ compute the performance measurement of R_i, and set this as the fitness value of Θ_l, represented by $Z(\Theta_l)$.

Step 4.4: Update the : For $l=1,2,...,L$, update $\Psi_{l=}\Theta_l$, if $Z(\Theta_l)<Z(\Psi_l)$.

Step 4.5: Update the : For $l=1,2,...,L$, update $\Psi_{g=}\Psi_g$, if $Z(\Psi_l)<Z(\Psi_g)$.

Step 4.6: Constraints check. If in the feasible region, go to Step 5, otherwise, back to Step 4.1.

Step 5: If the stopping criterion is met, i.e., $\tau = T$, go to Step 6. Otherwise, $\tau = \tau + 1$ and go to step 6.

Step 6: According to Eq.11 update the Ψ_{gh} to $\Psi_{g(h=1)}$, solve the lower level to get the optimum solution. Go to Step 4.

Step 7: Output program operation results $Z(\Theta_l)$. Algorithm goes to the end.