Prediction method of drying shrinkage crack in reinforced concrete walls

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Abstract

Thin plate reinforced concrete members, such as walls and slabs, are greatly influenced by drying shrinkage. In these members, cracks often occur due to the restraint of the volume change caused by drying shrinkage. Therefore, the control of cracking due to drying shrinkage is very important in building construction where thin plate members are frequently used. However, few researches on estimating shrinkage cracking in RC walls have been executed, and the cracking control design of RC walls has been conducted based on experience rather than a quantitative design method.

In this study, a practical cracking prediction method using equivalent bond-loss length L_b was proposed for the quantitative drying shrinkage crack control of RC walls. Number of cracks and crack width were estimated using the proposed method. Those values were compared with the results from the experiment and the investigative values from the field study. In general, results from the new prediction method matched well with both the experiment and the field study.

Keywords: Drying shrinkage, Shrinkage cracking, RC walls, Cracking control design, Equivalent bond-loss length.

1. Introduction

In South Korea and Japan, when RC building is commonly constructed, concrete is poured into walls, beams, and columns at the same time. Consequently, cracks occur in the walls due to the restraint of the volume change by drying shrinkage because the walls in the RC building are restrained by the beams and columns [1]. After the crack develops, tensile force is transferred from reinforcement to concrete by the bond stress, and this causes new cracks [2, 3]. The new cracks are generated continuously until drying shrinkage deformation of the wall stabilizes. Because these cracks shorten the service life and increase the maintenance cost of RC structures, crack control is imperative. However, few researches on estimating shrinkage cracking in RC walls have been executed, and the cracking control design of RC walls has been conducted based on know-how and experience rather than a quantitative design method.

For the quantitative cracking control design in a RC wall, Ohno et al. [4] proposed the cracking estimation method based on the bond analysis. In this analysis, a uniaxially restrained RC wall was used, and the validity of the method was confirmed from the uniaxially restrained shrinkage cracking test [5, 6].

However, because this estimation method was very complex and required an extensive calculation process, a simple and practical estimation method was demanded.

In this study, a practical cracking prediction method using equivalent bond-loss length (L_b) was proposed for the quantitative drying shrinkage crack control of RC walls. Number of cracks and crack width were estimated using the proposed method. Comparison between the estimated values with the experimental results from the uniaxially restrained shrinkage cracking specimens and the investigative values from the field study were executed.

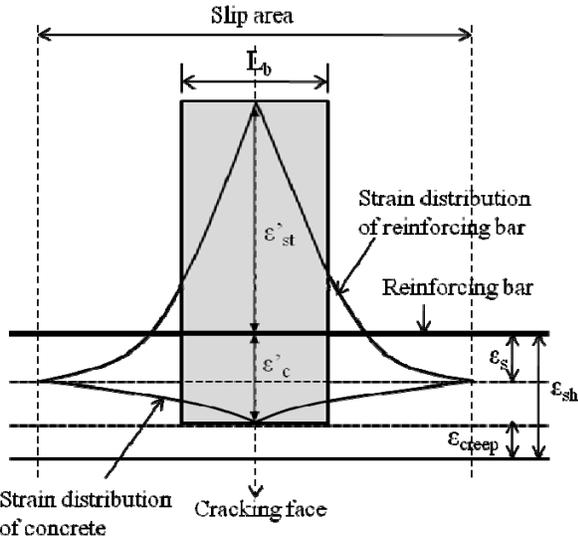## 2. Estimation of Drying Shrinkage Crack in RC Wall

2.1. Definition of L_b

Figure 1 [7] shows the strain distribution of reinforcement and concrete at the drying shrinkage cracking region of a RC wall. When the drying shrinkage crack occurs in a RC wall, the strain of reinforcement at the crack face is ε_{st}' and that of the concrete becomes ε_{sh}-ε_{creep} due to the release of the restraint. The crack width is calculated by the area enclosed by the strain curves of the reinforcement and concrete. To ease the estimation of crack width, equivalent bond-loss length (L_b) is adopted. L_b is decided so that the equivalent area using L_b has the same area as the area enclosed by the strain curves. As shown in Eqs. 1, if L_b is decided, the crack width is easily calculated by the multiplication of L_b and the sum of the strains of reinforcement and concrete at the crack face. In the previous study [7], the estimation equation of L_b was

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proposed based on the area enclosed by the strain curves of the reinforcement and concrete obtained from the bond analysis. The validity of the equation was confirmed using the results from the uniaxially restrained shrinkage cracking test [8]. This equation is shown in Eqs. 2.

$$L_b = K \cdot L_b(0), \quad K = K_{sh}K_{f_c}K_d K_{p_t}K_\sigma K_\varphi \tag{2}$$

Where

$$K_{sh} = 700\varepsilon_{sh} + 0.733$$
$$K_{f_c} = -0.019 f_c + 1.46$$
$$K_{p_t} = -13.14 p_t + 1.077$$
$$K_\sigma = 0.003\sigma_s + 0.560$$
$$K_\varphi = -0.013\varphi + 1.02$$
$$K_d = 0.78(D10), 1.00(D13), 0.89(D10 + D13)$$

Where σ_s is reinforcement stress at the crack face (MPa), E_s is Young's modulus of reinforcement (MPa), ε_{sh} is free drying shrinkage strain, ε_{creep} is creep strain, L_b is equivalent bond-loss length (mm), K is effect factor, $L_b(0)$ is equivalent bond-loss length in standard condition (300mm), f_c is compressive strength of concrete (MPa), p_t is reinforcement ratio, φ is creep coefficient, and D10 denotes deformed bars with a 10mm diameter.

2.2. Estimation of drying shrinkage crack using L_b

A RC wall is usually restrained by beams and columns surrounding the wall. However, as shown in Figure 2, only the horizontal restraint of the wall can be considered in this analysis for the following reasons. First, the horizontal length of the wall is significantly longer than that of the vertical length. Second, the columns are subjected to vertical loads.

L_b: Equivalent bond-loss length
ε_{sh}: Free drying shrinkage strain
ε_{creep}: Creep strain of concrete
ε_{st}': Strain of reinforcement at the crack face
ε_s: Strain of reinforcement at the stress continuity region
Fig. 1 Strain distribution of reinforcement and concrete at crack face [7]

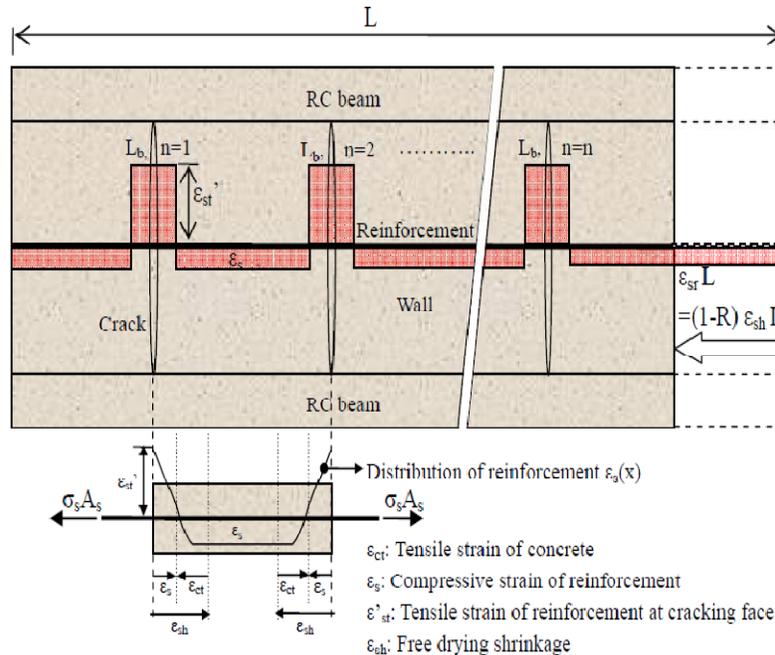$$w = \{\sigma_s / E_s + (\varepsilon_{sh} - \varepsilon_{creep})\}L_b \tag{1}$$

ε_{ct}: Tensile strain of concrete
ε_s: Compressive strain of reinforcement
ε'_{st}: Tensile strain of reinforcement at cracking face
ε_{sh}: Free drying shrinkage
Fig. 2 Uniaxial behavior of RC wall due to drying shrinkage

The restraint ratio (R) is defined as the degree of restraint from drying shrinkage deformation of a RC wall and represented as Eqs. 3. R=1 means the fully restrained condition, and R=0 means restraint free condition. According to the AIJ standard [9], the restraint ratio of 0.5~0.6 is considered for the first story wall members which are the most restrained members, while the restraint ratio of 0.3~0.4 is considered for the rest of the wall members. By adopting R into the equations, the restraint shrinkage deformation of the RC wall due to drying shrinkage can be expressed as $\varepsilon_{sr}L = (1-R)\varepsilon_{sh}L$, and by using L_b the shrinkage deformation of reinforcement is represented as $nL_b\varepsilon_{st}' - (L - nL_b)\varepsilon_s (= \int_0^L \varepsilon_s(x)dx)$. Eqs. 4 was obtained from the fact that the deformation of the RC wall and that of reinforcement due to drying shrinkage are the same.

$$R = (\varepsilon_{sh} - \varepsilon_{sr}) / \varepsilon_{sh} \tag{3}$$

$$n \cdot L_b \cdot \varepsilon_{st}' - (L - n \cdot L_b) \cdot \varepsilon_s = -(1-R) \cdot \varepsilon_{sh} \cdot L \tag{4}$$

Where R is the restraint ratio, ε_{sh} is free drying shrinkage strain, ε_{sr} is the restraint strain of RC wall due to the external restraint such as beams, and reinforcements, n is the number of cracks, L_b is equivalent bond-loss length, ε_{st}' is reinforcement tensile strain at the crack face, L is length of the wall, and ε_s is reinforcement compressive strain at the stress continuity region.

Based on equilibrium, the following equations are obtained (see Figure. 2).

$$\sigma_s A_s = P_c - P_s \tag{5}$$

$$P_c = \varepsilon_{ct} E_c' A_c = (\varepsilon_{sh} - \varepsilon_s) E_c' A_c \tag{6}$$

$$P_s = \varepsilon_s E_s A_s \tag{7}$$

Where σ_s is the tensile stress of reinforcement at the crack face, A_s is the sectional area of reinforcement, P_c is the tensile force of concrete at the stress continuity region, P_s is the compressive force of reinforcement at the stress continuity region, ε_{ct} is concrete tensile strain, E_c' is the effective Young's modulus of concrete, A_c is the sectional area of concrete, ε_{sh} is free drying shrinkage strain, ε_s is the reinforcement compressive strain at the stress continuity region, and E_s is Young's modulus of reinforcement.

By substituting Eqs. 6, 7 into Eqs. 5, the strain of reinforcement (ε_s, Eqs. 8) and the tensile stress of concrete (σ_c, Eqs. 9) at the stress continuity region are obtained.

$$\varepsilon_s = \frac{\varepsilon_{sh} - p_t \sigma_s / E_c'}{n' p_t + 1} \tag{8}$$

$$\sigma_c = \frac{(\sigma_s + \varepsilon_{sh} E_s) p_t}{n' p_t + 1} \tag{9}$$

Where p_t is reinforcement ratio, and $n' = E_s / E_c'$.
L_b is expressed as follows,

$$L_b = X \cdot K_\sigma = X \cdot (0.003\sigma_s + 0.56) \tag{10}$$

$$X = L_b(0) \cdot K_{sh} \cdot K_{f_c} \cdot K_d \cdot K_{p_t} \cdot K_\varphi \tag{11}$$

By substituting Eqs. 8 and Eqs. 10 into Eqs. 4, and then by rearranging the equation, the quadratic equation (Eqs. 12) of σ_s is obtained.

$$0.003 \cdot n \cdot X \cdot \sigma_s^2 + \{n' \cdot L \cdot p_t + n \cdot X \cdot (0.56 + 0.003 \cdot E_s \cdot \varepsilon_{sh})\} \cdot \sigma_s$$
$$+ \{0.56 \cdot n \cdot X - R \cdot L + n' \cdot p_t \cdot L \cdot (1-R)\} \cdot E_s \cdot \varepsilon_{sh} = 0 \tag{12}$$

Where n is the number of cracks, X is equation 11, $n' = E_s / E_c'$, L is length of the wall, p_t is the reinforcement ratio, E_s is Young's modulus of reinforcement, ε_{sh} is free drying shrinkage strain, and R is the restraint ratio.

When the tensile stress of reinforcement at the crack face (σ_s) is calculated from Eqs. 12, the crack width (w) can be estimated from Eqs. 1. The flowchart of cracking estimation using L_b is shown in Figure 3. The calculation of reinforcement stress (σ_s) using Eqs. 12 is repeatedly carried out by increasing the number of cracks, until the tensile stress of concrete (σ_c, Eqs. 9) is smaller than the tensile strength of concrete (f_{cr}, Eqs. 13). When σ_c is smaller than f_{cr}, the crack width (w) is calculated by Eqs. 1. To consider the creep effect of concrete, $\varepsilon_{creep} = \varepsilon_{sh} / 3$ was used for the mid-to-long term aged RC members [10], and Eqs. 13 [9] was used as the criteria for judgment of crack occurrence. The reduction factor (k) in Eqs. 13 was decided based on the experimental study [9, 11, 12, 13]. According to the study, shrinkage crack occurred when the tensile stress of concrete due to the restraint of free drying shrinkage reached about 60% of the split strength of concrete.

$$f_{cr} = 0.291 \cdot f_c^{0.637} \cdot k \tag{13}$$

Where f_c is the compressive strength of concrete (MPa), and k is the reduction factor (k=0.6).
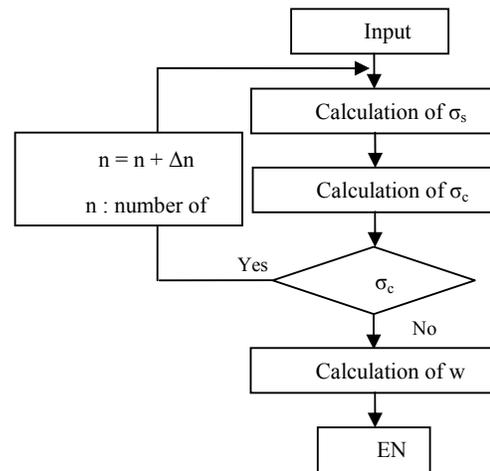

Fig. 3 Flowchart

2.3. Numerical examples

By using the proposed method of this study, the number of cracks and the crack width of a RC wall for the

given condition are estimated as follows.

 * Given condition

Length of wall (L): 6000 mm, Kind of reinforcement: D13, Reinforcement ratio (p_t): 0.5 %, Compressive strength of concrete (f_c): 21 MPa, Young's modulus of concrete (E_c): 21000 MPa, Young's modulus of steel (E_s): 200000 MPa, Creep coefficient (φ): 1.5, Shrinkage strain (ε_{sh}): 0.0006, Restraint ratio (R): 0.6

 * Results of calculation

n = 1, σ_s = 273 MPa, σ_c = 1.76 MPa > f_{cr} = 1.21 MPa, N.G

n = 2, σ_s = 190 MPa, σ_c = 1.38 MPa > f_{cr} = 1.21 MPa, N.G

n = 3, σ_s = 145 MPa, σ_c = 1.18 MPa < f_{cr} = 1.21 MPa, O.K, L_b=369 mm.

Three cracks are estimated, and the crack width is calculated from Eqs. 1

w={145/200000+(0.0006-0.0006/3)} 369=0.415 mm

According to the results, using the analysis method from the previous studies [4], the number of cracks was 3.2 and the crack width was 0.447 mm, while the tensile stress of reinforcement (σ_s) was 149 MPa. Both prediction methods resulted in similar results.
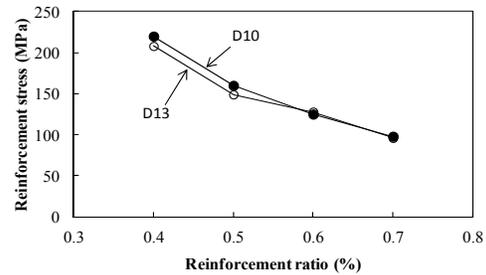
2.4. Parameter study

The parameter study was executed to investigate the influence of each parameter on the prediction method proposed in this study. The calculation condition used for the parameter study is shown in Table 1, and the results of the parameter study on the tensile stress of reinforcement (σ_s) is shown in Figure 4. It was observed that when the compressive strength of concrete (f_c) increased, the tensile stress of the reinforcement also increased. However, the opposite result was observed when comparing the reinforcement ratio (p_t) and the drying shrinkage stain (ε_{sh}) with the tensile stress of the reinforcement. The restraint ratio (R), the length of the wall (L), the creep coefficient (φ), and the size of the bar (D10, D13) hardly influenced the tensile stress of the reinforcement in the parameter study.
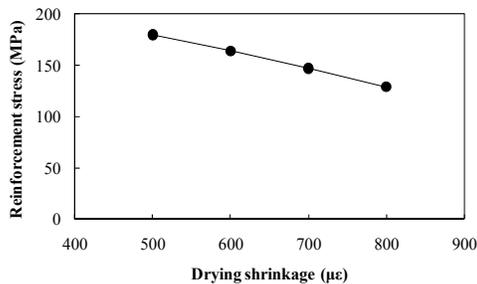
Table 1 Calculation condition

| Parameter	Variable	Constant
f_c	15, 27, 40 MPa	p_t=0.5%, D10, R=0.5, ε_{sh}=600μ, φ=2, L=8000mm
p_t, D	0.4, 0.5, 0.6, 0.7 %, D10, D13	f_c=24MPa, R=0.5, ε_{sh}=600μ, φ=2, L=8000mm
R	0.5, 0.6, 0.7, 0.8	p_t=0.5%, D10, f_c =24, ε_{sh}=600μ, φ=2, L=8000mm
L	5, 7, 9, 11 m	p_t=0.5%, D10, f_c =24, ε_{sh}=600μ, φ=2, R=0.5
ε_{sh}	500, 600, 700, 800μ	p_t=0.5%, D10, f_c =24, L=8000mm, φ=2, R=0.5
φ	1.0, 2.0, 3.0	p_t=0.5%, D10, f_c=24, ε_{sh}=600μ, L=8000mm, R=0.5
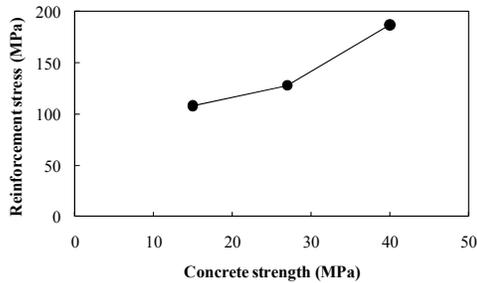
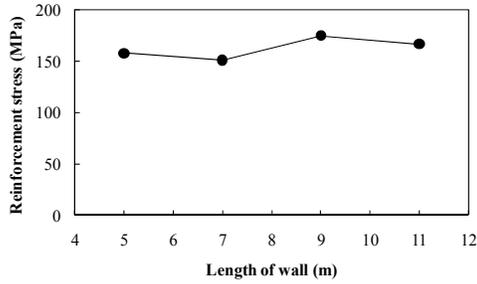(a) Effect of concrete strength

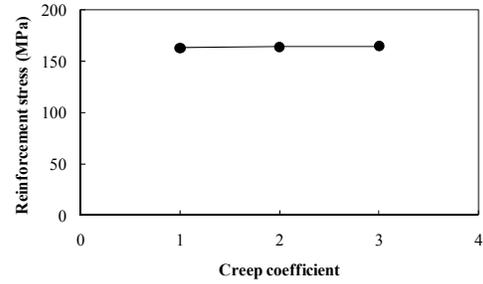(b) Effect of reinforcement ratio

(c) Effect of Drying shrinkage
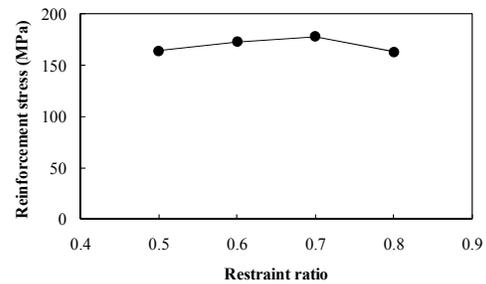
(d) Effect of restraint ratio
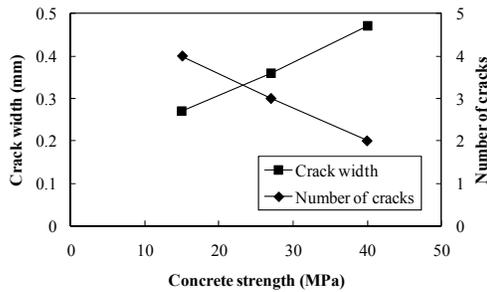
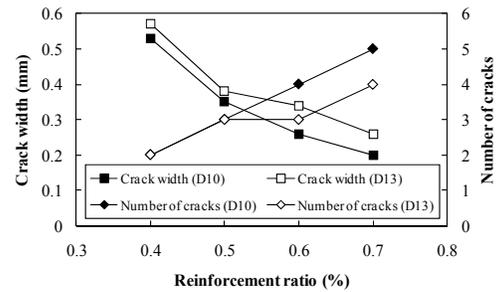(e) Effect of wall length   (f) Effect of creep coefficient

Fig. 4 Results of parameter study (σs)

Figure 5 shows the results of the parameter study of the crack width (w) and the number of cracks (n). The crack width decreased, when the compressive strength of the concrete and the diameter of the reinforcement decreased, while the reinforcement ratio increased. The crack width is highly influenced by the compressive strength of the
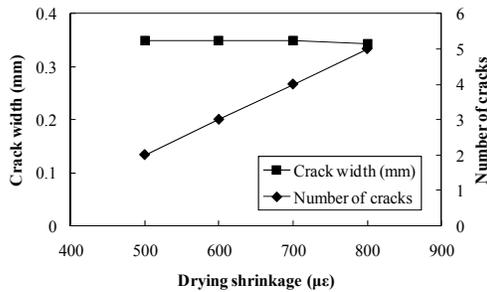
concrete, the diameter of the reinforcement, and the reinforcement ratio, but it is barely influenced by the drying shrinkage, the restrained ratio, the length of the wall, and the creep coefficient. The number of cracks is influenced by all parameters.
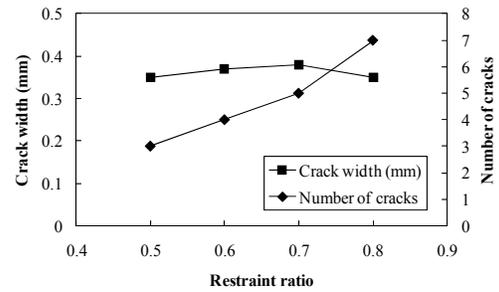

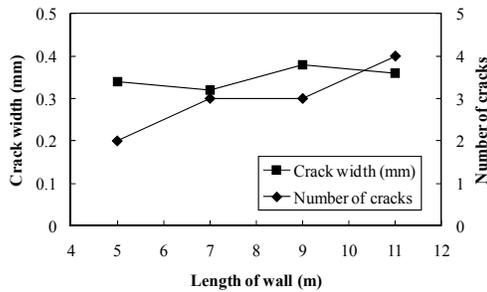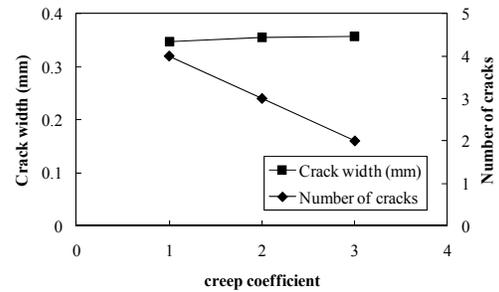(a) Effect of concrete strength   (b) Effect of reinforcement ratio

(c) Effect of Drying shrinkage   (d) Effect of restraint ratio

(e) Effect of wall length   (f) Effect of creep coefficient

Fig. 5 Results of parameter study (w, n)

3. Verification of Validity of Estimation Equations

3.1. Comparison between experimental values and calculated values

The experimental values from the uniaxially restrained shrinkage cracking specimens [5] and the estimated values from the equations proposed by Gilbert [2] and Base and Murray [10] for the cracking estimation due to the drying shrinkage of the restrained RC member were compared with the new estimation method proposed in this study. The size of the specimens [5] is shown in Figure 6, and the variables considered in the calculation are summarized in Table 2. These values were based on the experimental results. Contacting strain gauges (C.S.G) were used to measure the drying shrinkage strain of concrete, the creep strain of concrete, and crack width in the specimens. Figure 7 shows the comparison results of the crack width and the number of cracks. In general, results from the new prediction method matched well with the experimental values, but the crack widths calculated by other equations were about 50% smaller than the experimental values. For the number of cracks, results from Gilbert's equation were overestimated, while results from Base & Murray's equation were close to the experimental results.
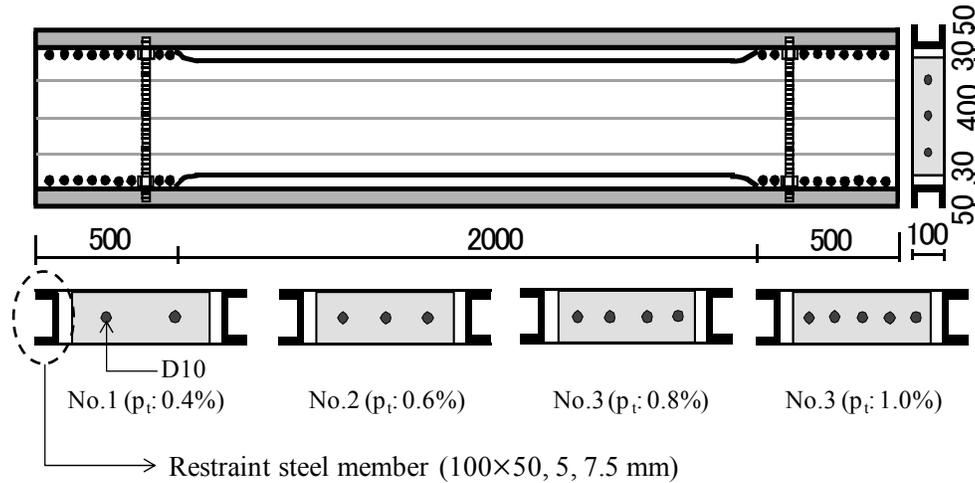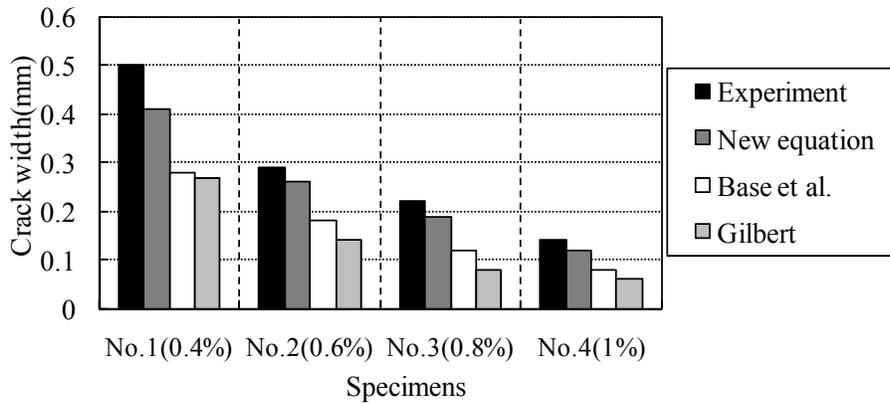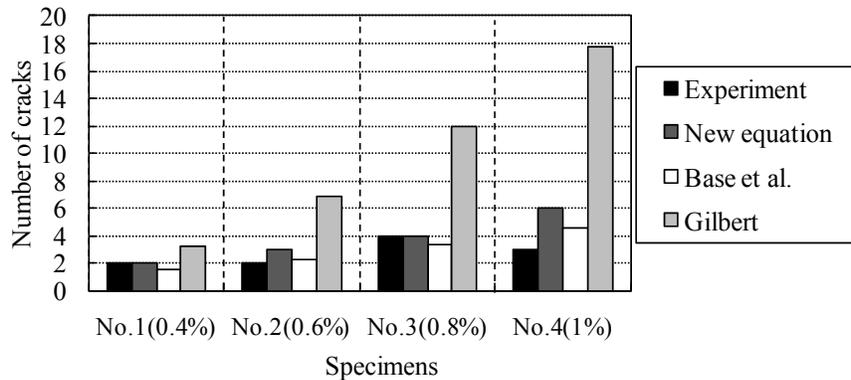

Fig. 6 Size of Specimen

(a) Crack width

(b) Number of cracks

Fig. 7 Comparison between calculation value and experimental value

**Table 2** Variables Considered in the Calculation

| Item	Values
f_c(MPa)	26
E_c(GPa)	21
p_t (%)	0.4, 0.6, 0.8, 1.8
ε_{sh} (μ)	700
φ	3.3
R	0.95

3.2. Comparison between investigative values from the field study and calculated values

The investigative values of cracking in the RC walls from the field study [9] were compared with the calculated values. The details of the RC walls are shown in Table 3, and the results are shown in Figure 8. The values of the drying shrinkage strain (ε_{sh}) and restraint ratio (R) were based on the report [9], and the creep coefficient was decided from the equation introduced in ACI209 [14]. Overall, results from the new prediction method were close to the investigative values from the field study. Figure 8 (a) shows the comparison of the crack width between the investigative results and the calculated values. The results from the new prediction method and Base & Murray's equation were close to the maximum crack width from the field study. However, the calculated values from Gilbert's equation were smaller than the crack width from the field study. Figure 8(b) shows the comparison of the number of cracks between the investigative results and the calculated values. The new prediction method and Base & Murray's equation almost corresponded to the investigation values. However, the number of cracks calculated by Gilbert' equation was overestimated.
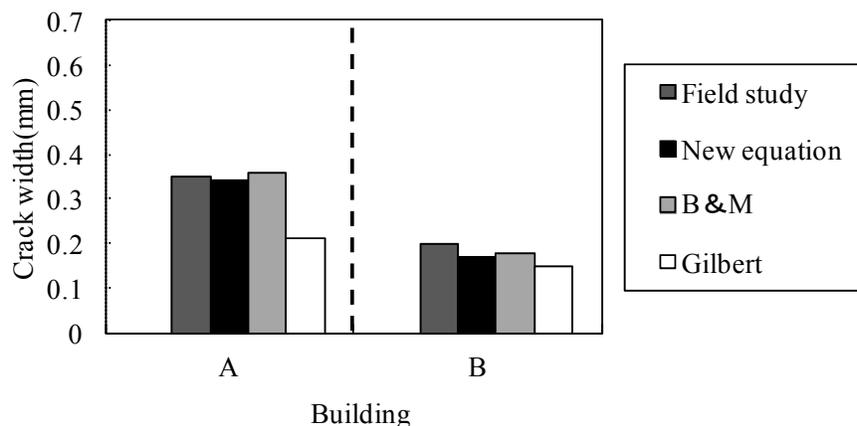
(a) Crack width
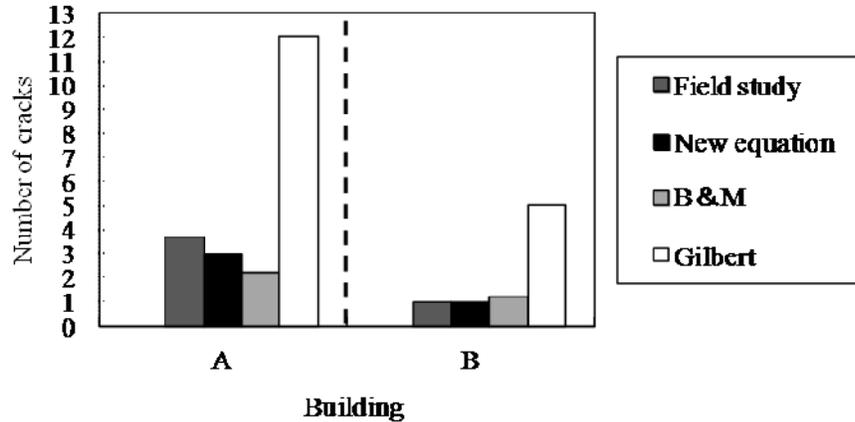
(b) Number of cracks
Fig. 8 Comparison between calculation value and investigative value

Table 3 Details of RC walls [9]

Buildings(Days)	L(mm)	t(mm)	p_t(%)	Reinforcement	f_c (MPa)	ε_{sh}(μ)	R
A(500-day)	8500	200	0.49	D10+D13	24	500	0.55
D(300-day)	3000	180	0.53	D10	24	430	0.33

L: Length of wall, t: Thickness of wall, p_t: Reinforcement ratio,
f_c: Compressive strength of concrete, ε_{sh}: Shrinkage strain, R: Restraint ratio

4. Crack Control Design of RC Wall

For crack control by the reinforcement, the crack width is calculated by Eqs. 1, and this value is compared with the allowable crack width of 0.30 mm [9]. If the calculated value exceeds the allowable value, crack control design is executed by increasing the reinforcing steel until the calculated value is less than 0.3mm. For crack control by the control joint the crack spacing is determined by the number of cracks, and the control joint is installed based on crack spacing. When the calculation is conducted, the shrinkage strain and creep coefficient can be obtained by the equations proposed in ACI 209 [14], CEB-FIP [15], and elsewhere [9]. By using the proposed method of this study, crack control design of a RC wall for the given condition is executed in sections 4.1 and 4.2.

* Given condition

Length of wall (L): 6000 mm, Kind of reinforcement: D10, Reinforcement ratio (p_t): 0.4%, Compressive strength of concrete (f_c): 24 MPa, Young's modulus of concrete (E_c): 21000 MPa, Young's modulus of steel (E_s): 200000 MPa, Creep coefficient (φ): 1.5, Shrinkage strain (ε_{sh}): 0.0006, Restraint ratio (R): 0.5

* Results of calculation

n = 1, σ_s = 288 MPa, σ_c = 1.49 MPa > f_{cr} = 1.32 MPa, N.G

n = 2, σ_s = 203 MPa, σ_c = 1.18 MPa < f_{cr} = 1.32 MPa, O.K, L_b=324 mm

Two cracks are estimated, and the crack width is calculated from Eqs. 1.

w={203/200000+(0.0006-0.0006/3)} 324=0.46 mm

4.1. Crack control by reinforcing bar

Since the calculated crack width of 0.46 mm is larger than the allowable crack width of 0.30 mm, the reinforcement ratio should be increased so that the crack width is smaller than the allowable crack width (0.30 mm). For this example, to maintain the crack width below 0.30 mm the reinforcement ratio of 0.5% or larger is required. When the reinforcement ratio is 0.5%, the calculated crack width is 0.30 mm (w={143/200000+(0.0006-0.0006/3)}271=0.30 mm).

4.2. Crack control by control joint

Since two cracks are estimated from this example, two control joints which induce cracks were installed using 2 meter spacing.

5. Conclusion

In this study, the practical cracking prediction method using equivalent bond-loss length L_b was proposed for the quantitative drying shrinkage crack control of the RC wall, and the validity of the proposed method was verified by comparing the estimated values from the new method with the results from the experiment and the field study.

The results are summarized as follows:

1)   The proposed prediction method from this study was compared with the analysis method from the previous studies, and both prediction methods resulted in similar results.

2) The crack width is highly influenced by the compressive strength of the concrete, the diameter of the reinforcement, and the reinforcement ratio, but it is barely influenced by the drying shrinkage, the restrained ratio, the length of the wall, and the creep coefficient. The number of cracks is influenced by all parameters.

3) In general, the predicted number of cracks and width of shrinkage cracks were close to the values of the experiment and the field study.

4) It is expected that the crack control design method proposed in this study enables quantitative crack control design by reinforcing bar and control joint.

The limit of application for the proposed equation is as follows,

* Compressive strength of concrete: 21~40MPa
* Reinforcement: D10, D13
* Reinforcement ratio: 0.4~0.7%
* RC wall without opening with typical horizontal wall length.

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