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Vehicle routing problem with multiple decision-makers for construction material transportation in a fuzzy random environment

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Abstract

In this paper, a bi-level decision making model is proposed for a vehicle routing problem with multiple decision-makers (VRPMD) in a fuzzy random environment. In our model, the objective of the leader is to minimize total costs by deciding the customer sets, while the follower is trying to minimize routing costs by choosing routes for each vehicle. Demand for each item has considerable uncertainty, so customer demand is considered a fuzzy random factor in this paper. After setting up the bi-level programming model for VRPMD, a bi-level global-local-neighbor particle swarm optimization with fuzzy random simulation (bglnPSO-frs) is developed to solve the bi-level fuzzy random model. Finally, the proposed model and method are applied to construction material transportation in the Yalong River Hydropower Base in China to illustrate its effectiveness.

Keywords: *Vehicle routing optimization, Multiple decision-makers, Construction material transportation, Fuzzy random variable, Particle swarm optimization.*

1. Introduction

Construction material transportation plays an important role in construction projects, especially in large-scale construction projects. In recent years, along with economic globalization and the rapid development of the logistics industry, transportation and distribution has been gradually paid serious attention in practice. The vehicle routing problem (VRP) which is the key to transportation and distribution requires an economic distribution line for a vehicle starting from the distribution center, servicing all customers and returning to the distribution center so that goods are delivered to customers at the lowest logistics cost.

In recent years, VRP has attracted more attention and been studied both in scientific and practical fields. During the last fifty years, many different formulations have been proposed. Since the VRP was first proposed by Danzig and Ramser [1], it has been furthered by many other scholars. At present, there are three main variants of the classical vehicle routing problem: VRPs with backhauls [2, 3], VRPs with pickup and delivery [4, 5] and VRPs with time window [6, 7, 8]. All the new development or extensions based on the classic VRP are significant on dealing with the complicated practical problems.

It can be seen that most studies made before have no more than one decision-maker, using multiple objective programming. However, in the real world, it is obvious that there are many participants typically involved in construction material transportation, such as supplier factories, logistics companies or transport companies, customers and so forth. In the actual construction material transportation projects, participants involved in can inevitably have all kinds of conflicts. These conflicts may have a big influence on the total construction material transportation costs, because all the participants belong to different stakeholder and they decide the implementation of the project based on their own interests. If their mutual influences are neglected, it will certainly affect the eventual results. In addition, one of the most important formulations for VRP is the formulation introduced by Fisher and Jaikumar who proved that VRP constraints can be divided into two sets [9]. According to the theory of Fisher and Jaikumar, we can use bi-level programming to deal with the VRP. Thus, based on previous studies, with the consideration of more decision-makers in practice, we proposed a new model using bi-level programming for the VRP with multiple decision-makers (VRPMD).

Much of the past research on VRP has been limited to a deterministic model. However, there are many uncertainties in the real world. For example, weather delays play an important role for projects carried out in harsh environmental conditions and therefore can be treat as fuzzy variables [10]. Thus, in order to constantly get closer to actual production, uncertainty in the VRP has been paid more attention in recent years. Teodorović and Pavković developed a model for vehicle routing when demand at the nodes is fuzzy [11]. Zheng and Liu also

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designed a fuzzy optimization model for the VRP where travel times were assumed to be fuzzy variables [12]. Many scholars have also studied randomness in the VRP. Based on previous studies, Gendreau et al did a literature review on stochastic vehicle routing problems and provided a scientific research summary on stochastic vehicle routing problems [13]. However, there is also other uncertain information in the VRP, which has seldom been considered in the past. Fuzzy random theory has been applied in many fields as in [14, 15, 16, 17], but there has been little research which focused on the fuzzy random factors which exist in practical VRP.

In summary, based on previous studies, a multiple decision-maker vehicle routing problem (VRPMD) in a fuzzy random environment is proposed. In the proposed formulation, the VRPMD is considered a bi-level problem with two decision-makers. The VRPMD model has two layers, in which the leader deals with the generalized assignment problem, and the follower deals with the optimal route selection problem. In the model, all costs involved in the VRPMD have been considered and classified clearly using bi-level programming. Fuzzy random theory is used to describe VRPMD customer demands. The reason for the use of fuzzy random theory in the VRPMD is outlined in section 2.1.2. From this investigation it can be concluded that the VRPMD with fuzzy random variables is closer to reality and can deal with complicated practical problems.

The remainder of this paper is organized as follows: In section 2, the key problems in the bi-level VRPMD are described, including the classical VRP, multiple subjects and uncertain environment. Then the VRPMD mathematical formulation model is presented in section 3. In section 4, a bi-level global-local-neighbor particle swarm optimization with fuzzy random simulation (bglnPSO-frs) is advanced to solve the model. In section 5, an application of the model to a construction material transportation problem at the Yalong River Hydropower Base in China is presented. Concluding remarks are in Section 6.

2. Key Problems Statement

2.1. Classical VRP

The VRP is a well-known NP-hard problem in combinatorial optimization problems. Generally, in the classical VRP, a set of customers located in various cities is given with each customer having their own demands. Vehicles of the same condition at the depot deliver goods to these customers with the requirement that they start and end at the depot. The objective of the classical VRP is to minimize total costs by designing an optimal delivery route for each vehicle. Delivery vehicles usually need to meet the following conditions: (1) Serve all customers using the least vehicles; (2) Each customer is served by only one vehicle once; (3) Each vehicle starts and ends at the depot; (4) Total customer demand on each route cannot exceed the load capacity of the vehicle. A general diagram of the classical VRP is in Fig. 1.

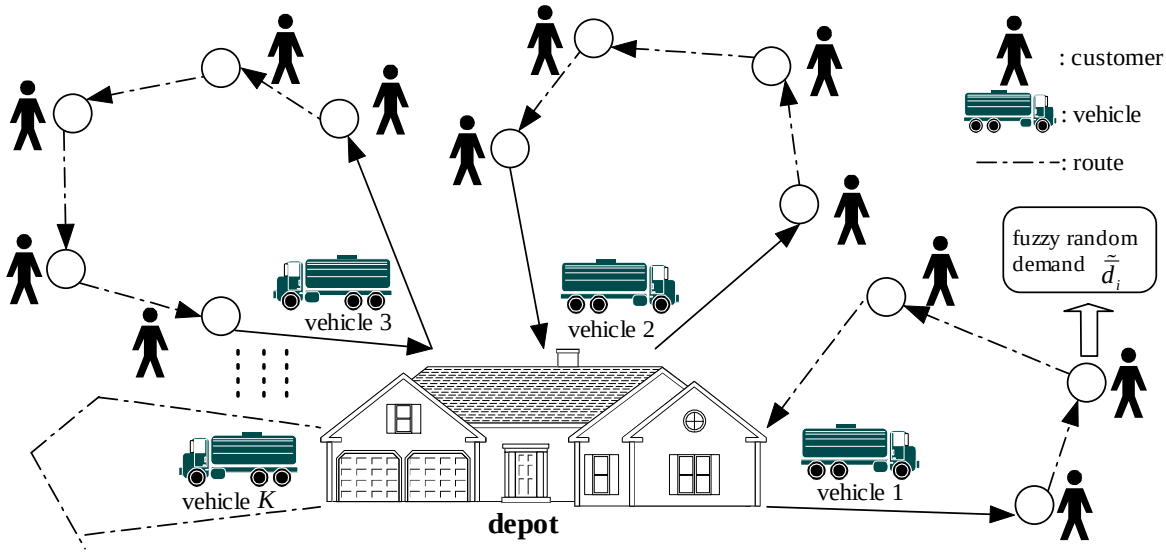

Fig. 1 A general illumination of the classical VRP

These days, the VRP is a common problem in almost every industry such as supply chain and the transport industry but it is even more important in construction projects, because unsuitable transportation routes can lead to significant losses, especially in large scale construction projects, such as the Yalong River Hydropower Base in China.

2.2. Multiple decision-makers

In Fisher and Jaikumar's study, they prove that the constraints of VRP can be divided into two sets. The first set are the constraints of a generalized assignment problem, which ensure that all the vehicles begin and end at the depot, each customer is served by some vehicle, and

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the load assigned to a vehicle is within capacity. The second set of constraints corresponds to a traveling salesman problem for finding an optimal route for each vehicle to serve all the customers [9].

Developed from Fisher and Jaikumar's theory, we find there can be more than one decision-maker in the VRP, and bi-level programming can help deal with the interactive influence from two decision-makers in one model. Bi-level programming problems were introduced by Von Stackelberg (1952) [18] and involve two optimization problems where the constraint region of the 2rst level problem is implicitly determined by another optimization problem. In this paper, supplier factories and transport companies (or logistics companies) are considered as the two independent decision-makers for the VRPMD. Their bi-level relationship in practice can be explained as follows: in practical construction material transportation, (1) one supplier factory employs one transport company to supply goods for its customers; (2) the supplier factory pursues a total cost minimization, including the serving cost (e.g. uploading cost, unloading cost) and the traffic expense for the transport company, while (3) the transport company is only concerned with his traffic expense including (e.g. driver's pay, vehicle expenses and gas); (4) the supplier factory deals with the generalized assignment problem, while the transport company deals with the optimal route selection problem; (5) he can only influence, but not control the transport company's route selection, and at the same time the transport company can have to make their route selection for each vehicle based on customer clusters decided by the supplier company. This interaction game is represented as a bi-level programming problem [19]. In this paper, all the costs involved in the VRPMD have been considered and classified clearing using bi-level programming. This bi-level relationship can be seen clearly in Fig. 2.

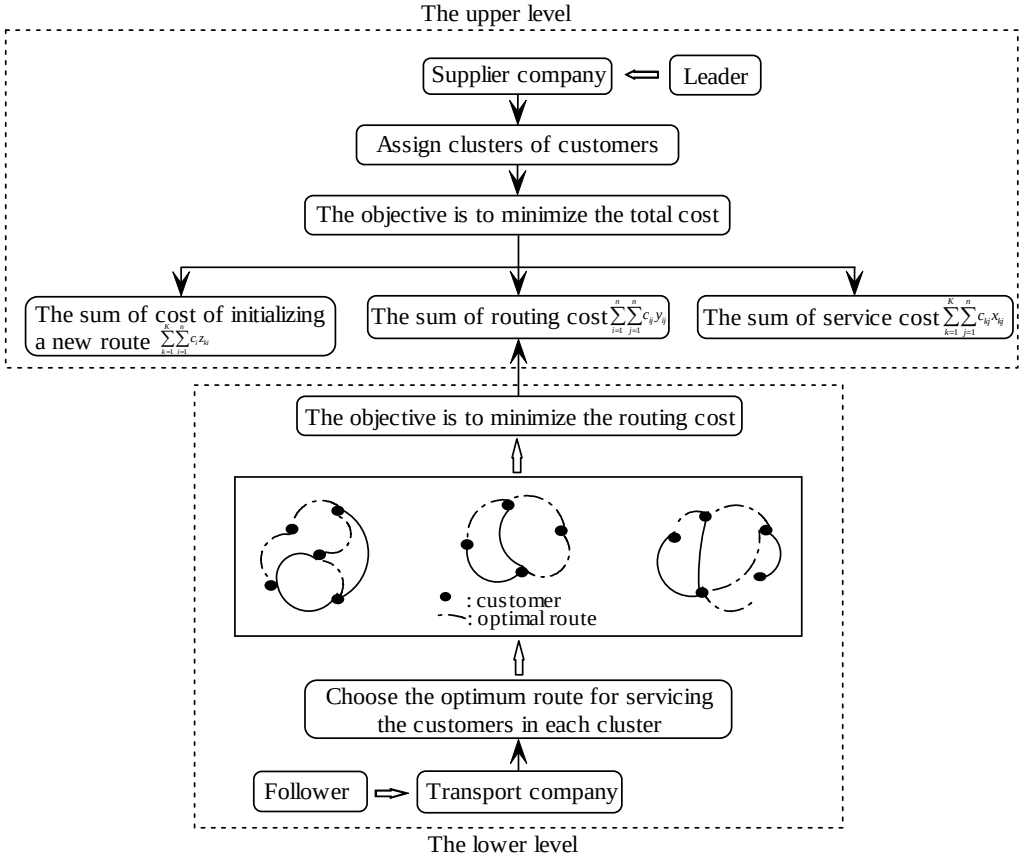

Fig. 2 The structure of bi-level relationship in the VRPMD

2.3. Uncertain environment

There are many uncertainties in the real world. In construction engineering projects, the uncertainties are especially rich and diverse, as can be seen in [10, 20, 21, 22, 23]. Thus, in order to move closer to actual production, uncertainties must be considered in the VRPMD for construction material transportation.

Though a great of research has considered uncertainty in the VRP, there are some uncertainties that have seldom been considered. For example, customer demand is usually determined using surveys or interviews, or described using ambiguous linguistic statements, such as "it is about 3 ton" or "it is no less than 1 ton". Stochastic factors are also involved in the VRPMD: (1) if one point usually has more than one person in charge, the choice of respondents is stochastic; (2) because of special circumstances such as the season, the weather, and the attitude of respondents (optimistic or pessimistic); the customers usually give different demand quantities. That is to say, the customer demand statements include both fuzzy and stochastic factors. Because of this, VRPMD needs to be studied in a

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fuzzy random environment.

In recent years, more fuzzy random theory studies have been conducted [24, 25, 26, 27]. In the VRPMD, the demand for each item is the most common factor that has considerable uncertainty. In this paper, demand is considered a fuzzy random variable. Fuzzy random theory is a useful tool for dealing with the type of VRPMD uncertainties under a fuzzy random environment. In this paper, when considering the uncertainties in the VRPMD, Kwakernaak's [28, 29] theory and the further developments by Kruse and Meyer [30] were chosen to describe and deal with the uncertainty.

3. Modelling

3.1. Notations and assumptions

In order to facilitate the description of the problem, the following notions are introduced.

Sets

V : set of vertex, $V = \{0,1,\ldots,n\}$ and vertex 0 refers to the depot.

C : set of customers, $C = V / \{0\}$.

S : subset of V, and $S \neq \Phi$.

E : set of index pairs, such $(i, j) \in E$ means customer i must precede customer j in the route.

H : set of vehicles, $H = \{1, 2, ...K\}$.

Indices and parameters

n : number of customers.

i/j : customer index, $i/j = 1, ..., n$.

K : number of vehicles.

k : vehicle index, $k = 1, 2, ..., K$.

$\tilde{\tilde{d}}_i$: the demand of customer i, and it is assumed to be fuzzy random.

c_i : cost of the seed customer i from the depot.

c_{kj} : cost of vehicle k for serving customer j.

c_{ij} : the routing cost between customer i and customer j.

Q : the vehicle capacity.

Decision variables

z_{ki} : a binary variable indicating that whether customer i is a seed customer. If customer i is a seed customer, then $z_{ki} = 1$; else, $z_{ki} = 0$.

x_{kj} : a binary variable indicating that whether customer is served by vehicle k. If $x_{kj} = 1$, then customer j is served by vehicle k; otherwise, $x_{kj} = 0$.

y_{ij} : a binary variable indicating that whether edge (i, j) is in the route. If $y_{ij} = 1$, then edge (i, j) is in the route; otherwise, $y_{ij} = 0$.

In the VRPMD, a route is defined as a sequence of locations that a vehicle must visit along with the service it provides [31]. Customer orders cannot be split. Generally, until the routes reach capacity or time limits, customers are assigned a single route. Then a new customer is selected as a seed customer for another new route and the process continues. A seed customer is defined as a customer who is not yet assigned a route and is used to initialize a new route. To model the bi-level formulation for the vehicle routing problem in a fuzzy random environment, the following assumptions are made:

(1) The capacity of each vehicle is the same;

(2) The demand of each customer is considered a fuzzy random variable;

(3) The vehicle must start and finish at the depot and there is only one depot;

(4) Each customer is served by a single vehicle and a seed customer is the start of a new route;

(5) Time is enough for each vehicle serving all its customers;

(6) Different vehicles are assigned different labor levels.

3.2. Model formulation

Objective functions

In general, construction material transportation involves a great deal of human, material, and financial resources. Thus, decision makers try to minimize total construction material transportation costs in large scale construction projects. The mathematical problem is to construct a low cost, feasible set of routes for each vehicle, so, the objective of the leader to find the lowest cost with a feasible set of routes in the VRPMD bi-level formulation in a fuzzy random environment is met. The mathematical objective to minimize total construction material transportation costs is as follows:

$$\min_{x,z} \sum_{k=1}^{k}\sum_{i=1}^{n} c_i z_{ki} + \sum_{k=1}^{k}\sum_{j=1}^{n} c_{kj} x_{kj} + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} y_{ij} \quad (1)$$

c_i is cost of the seed customer i from the depot. z_{ki} is a binary variable indicating that whether customer i is a seed customer served by vehicle k. If customer i is a seed customer served by vehicle k, then $z_{ki} = 1$; else, $z_{ki} = 0$. In construction material transportation, the first part $\sum_{k=1}^{k}\sum_{i=1}^{n} c_i z_{ki}$ represents the sum of the seed customers' cost from the depot, including the loading costs (labor charges) and the transport costs (oil consumption and driver cost). c_{kj} is cost of vehicle k for serving customer j. x_{kj} is a binary variable indicating that whether customer is served by vehicle k. If $x_{kj} = 1$, then customer j is served by vehicle k; otherwise, $x_{kj} = 0$.

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$\sum_{k=1}^{k}\sum_{j=1}^{n} c_{kj}x_{kj}$ is the sum of the service cost of vehicle k serving customer j, most of which is unloading costs (labor charges). Finally, c_{ij} is the routing cost between customer i and customer j and y_{ij} is a binary variable indicating that whether edge (i, j) is in the route. If $y_{ij}=1$, then edge (i, j) is in the route; otherwise, $y_{ij}=0$. $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}y_{ij}$ is the sum of the routing cost between customer i and customer j, which are mainly transport costs (oil consumption and driver cost).

Notice: Firstly, according to assumption (6), different truck is assigned with different labor level, for example, truck 1 may have two workers for the unloading jobs, while truck 2 may have six. Thus, the cost of different truck serving the same customer is different, namely c_{kj} is different when customer j is fixed and vehicle k is unfixed. Secondly, c_i associated with binary variable z_{ki} is the seed customers' cost from the depot, including the uploading cost (labor charges) and the transport costs (oil consumption and driver cost), while c_{kj} associated with binary variable x_{kj} refers to service cost of vehicle k for serving customer j, most of which is the unloading cost (labor charges). Thus, there is a need to set two binary variable z_{ki} and x_{kj} which have different meaning. Finally, there is not only serving cost in the transportation network, but also transport costs. Thus, the routing cost between edges (i, j) in the route c_{ij} is necessary.

Leader constraints:

Chance constrained programming is a useful tool for the handling of fuzzy random variables. In practical decision-making processes, decision makers usually choose a satisfactory solution with an allowed certain deviation rather than the optimum solution. For the VRPMD, because dynamic changes continually happen, decision makers have to make decisions based on a certain possibility level. According to assumption (2), the demand of each customer is a fuzzy random variable. Here then a chance-constrained operator is used to deal with this constraint. The theory concerning $\eta_j(\eta_j \geq 0.5)$ can be found in [32, 33]. Further in reference to [17] and [34], this customer demand constraint can be written as a set of chance-constraints as follows:

$$\Pr\left\{\omega \mid \Pr\left\{\sum_{j=1}^{n} \tilde{\tilde{d}}_j(\omega)x_{kj} \leq Q\right\} \geq \theta_j\right\} \geq \eta_j, \quad \forall k \in H, \tag{2}$$

$\tilde{\tilde{d}}_j$ is the demand of customer j, and it is assumed to be fuzzy random. x_{kj} is a binary variable indicating whether customer j is served by vehicle k. Q is the vehicle capacity. H is set of vehicles. This constraint ensures that all customers served by vehicle k cannot be beyond vehicle capacity.

A seed customer is the start of a new route. Each vehicle can have only one seed customer (one start) and the number of seed customers must be equal to the number of vehicles. Thus, among all customers, there is one and only one seed customer for each vehicle, and the sum of z_{ki} should be equal to the number of vehicles, mathematically:

$$\sum_{i=1}^{n} z_{ki}=1, \quad \forall k \in H. \tag{3}$$

$$\sum_{k=1}^{K}\sum_{i=1}^{n} z_{ki}=K, \tag{4}$$

z_{ki} is a binary variable indicating that whether customer i is a seed customer served by vehicle k. If customer i is a seed customer served by vehicle k, then $z_{ki}=1$; else, $z_{ki}=0$. k is the number of vehicles.

There are two circumstances which may occur when serving construction material transportation customers. In the first, each customer demands more than the vehicle's capacity, so every customer is served by no less than two vehicles, while the other is that each customer demands less than the vehicle's capacity. In this paper, the second circumstance is considered. The company decides on one vehicle to serve more than one customer to reduce overall costs, and each customer is served by only one vehicle. Mathematically:

$$\sum_{k=1}^{K} x_{kj}=1, \quad \forall j \in C. \tag{5}$$

C is the set of customers.

Since z_{ki} and x_{kj} are binary variables, thus:

$$z_{ki}=\{0,1\}, x_{kj}=\{0,1\}, \quad \forall k \in H, \quad \forall i/j \in C. \tag{6}$$

H and C are the set of vehicles and customers, respectively. z_{ki} and x_{kj} are binary variables, and also decision variables of the upper decision makers. z_{ki} helps decide the seed customers' cost from the depot, namely the initial costs c_i the uploading costs and the transport costs from leaving the depot to the seed customer. x_{kj} helps decide the serving cost, namely c_{kj} the unloading costs for serving each customer.

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Contractor constraints:

The second set of constraints is the TSP constraints for the customers of each vehicle. The main interest of the decision maker on the second level is to find optimal routes from these assignments. The dispatcher or the transport company can be treated as the lower-level decision makers. They are seriously concerned with the transportation cost, including driver's pay, vehicle expenses, gas and so forth. After the customer assignment is decided by the upper level, their objective is to minimize the transportation cost. Mathematical formulation for this objective is as follows:

$$\min_y \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} y_{ij} \tag{7}$$

c_{ij} is the routing cost between customer i and customer j and y_{ij} is a binary variable indicating that whether edge (i, j) is in the route. If $y_{ij} = 1$, then edge (i, j) is in the route; otherwise, $y_{ij} = 0$. $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} y_{ij}$ is the sum of the routing cost between customer i and customer j, which are mainly transport costs (oil consumption and driver cost).

The route selection job starts after customers have been assigned:

$$y_{ij} \le x_{kj}, \quad \forall k \in H, \quad \forall i/j \in C \tag{8}$$

x_{kj} is a binary variable indicating whether customer j is served by vehicle k. H and C are the set of vehicles and customers, respectively.

In construction material transportation, each customer is served by only one vehicle on the route. It is necessary that each node is entered once and is left once. The mathematical formulation is as follows:

$$\sum_{i=1}^{n} y_{ij} = 1, \quad \forall j \in C, \tag{9}$$

$$\sum_{j=1}^{n} y_{ij} = 1, \quad \forall i \in C. \tag{10}$$

Sub tour elimination constraints are needed:

$$\sum_{i\in V}\sum_{j\in V} y_{ij} \le |S| - 1, \quad \forall S \in V, \quad S \ne \Phi \tag{11}$$

V is set of vertex, $V = \{0,1,\ldots,n\}$ and vertex 0 refers to the depot. S is subset of V, and $S \ne \Phi$.

The same with z_{ki} and x_{kj}, y_{ij} is binary variable:

$$y_{ij} = \{0,1\}, \quad \forall i/j \in C \tag{12}$$

3.3. General global model

We propose the vehicle routing problem with multiple decision-makers (VRPMD). The model of the VRPMD has two layers, in which the upper level decision-maker, namely the leader, with the generalized assignment problem, and the follower deals with the optimal route selection problem. Thus, based on the above, a mathematical formulation for the construction material transportation in fuzzy random environment VRPMD as follows:

$$M_0 \begin{cases} \min_{x,z} \sum_{k=1}^{k}\sum_{i=1}^{n} c_i z_{ki} + \sum_{k=1}^{k}\sum_{j=1}^{n} c_{kj} x_{kj} + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} y_{ij} \\ s.t. \begin{cases} \sum_{i=1}^{n} z_{ki} = 1, \quad \forall k \in H, \\ \sum_{k=1}^{K}\sum_{i=1}^{n} z_{ki} = K, \\ \Pr\left\{\omega \mid \Pr\left\{\sum_{j=1}^{n} \tilde{\tilde{d}}_j(\omega) x_{kj} \le Q\right\} \ge \theta_j\right\} \ge \eta_j, \quad \forall k \in H., \\ \sum_{k=1}^{K} x_{kj} = 1, \quad \forall j \in C, \\ z_{ki} = \{0,1\}, x_{kj} = \{0,1\}, \quad \forall k \in H, \quad \forall i/j \in C, \\ \min_y \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} y_{ij}, \\ s.t. \begin{cases} y_{ij} \le x_{kj}, \quad \forall k \in H, \quad \forall i/j \in C, \\ \sum_{i=1}^{n} y_{ij} = 1, \quad \forall j \in C, \\ \sum_{j=1}^{n} y_{ij} = 1, \quad \forall i \in C, \\ \sum_{i\in V}\sum_{j\in V} y_{ij} \le |S| - 1, \quad \forall S \in V, \quad S \ne \Phi, \\ y_{ij} = \{0,1\}, \quad \forall j \in V. \end{cases} \end{cases} \end{cases}$$

In our model, we have considered all the costs involved in the VRPMD in a better way and classified them in a clear way by the bi-level programming. As for the leader's objective, the first part $\sum_{k=1}^{k}\sum_{i=1}^{n} c_i z_{ki}$ represents the sum of the seed customers' cost from the depot, including the loading costs (labor charges) and the transport costs (oil consumption and driver cost), the second part $\sum_{k=1}^{k}\sum_{j=1}^{n} c_{kj} x_{kj}$ is the sum of the service cost of vehicle k serving customer j, most of which is unloading costs (labor charges), and the final part $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} y_{ij}$ is the sum of the routing cost between customer i and customer j, which are mainly transport costs (oil consumption and driver cost) decided by the follower. The leader can choose the seed customers and assign clusters of customers to decide first part and the second part of the total cost by his decision variables, z_{ki} and x_{kj}, but he cannot control the third part. Since decisions must be feasible and all the constraints must be met, the leader has to consider the constraints from its own and from follower's perspective. On the other hand, the

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main interest of the follower is to find optimal routes from these assignments to minimize the transport cost. They are seriously concerned with the transportation cost, including driver's pay, vehicle expenses, gas and so forth. After the customer assignment is decided by the leader, they will choose an optimal route for each vehicle to minimize the transportation cost by deciding y_{ij}.

4. Bi-level Glnpso with Fuzzy Random Simulation (Bglnpso-Frs)

Many heuristic algorithms are used in construction engineering and several more new heuristic algorithms have been proposed, see [35, 36, 37]. A new evolutionary heuristic algorithm, called the particle swarm optimization (PSO), was proposed recently and has proved to be a powerful competitor in the field of NP-hard problem optimization [38, 39]. The PSO method has been widely used to solve NP-hard problems, as well as bi-level problems [40, 41]. However, after observation, the basic PSO was found to have a very definite weakness in that the particles in the swarm tend to cluster rapidly toward the global best particle which means that the swarm is frequently trapped in a local optimum and can no longer move.

As we known, the VRP is a NP-hard problem and the VRPMD which is using bi-level programming is more difficult and complicated. What's more, the uncertainty makes the problem even more difficult. Therefore, using traditional algorithm to solve the problem is really difficult. Solving NP-hard discrete optimization problems to optimality is often an immense job requiring very efficient algorithms. To deal with this premature convergence of the classic PSO, a modified approach is to reinitialize some or all of the particles except the global best particle. In this section, we use the strategy adopted in the glnPSO method to develop a bi-level glnPSO with fuzzy random simulation algorithm (bglnPSO-frs) to reach solutions for problems defined by Model M_0. A case study is then provided to prove the practicality of the proposed VRPMD model and allow for a brief comparison to prove the efficiency of the proposed algorithm.

4.1. Fuzzy random simulation

For the following constraints, $\Pr\left\{\omega \mid \Pr\left\{\sum_{j=1}^{n} \tilde{\tilde{d}}_j(\omega) x_{kj} \le Q\right\} \ge \theta_j\right\} \ge \eta_j$ in order to check the feasibility, for given x_{kj} and Q, we first generate M random vectors $\omega^j = \left(\omega_1^j, \omega_2^j, \cdots \omega_j^j\right)^T$, $j=1,2,\cdots M$ independently from Ω according to the probability measure $\Pr$. For any given sample $\omega^j \in \Omega$, the technique of fuzzy random simulation can be applied to check the random constraint $\sum_{j=1}^{n} \tilde{\tilde{d}}_j(\omega) x_{kj} \le Q$. First, generate $d_j(\omega^j)$ from $\tilde{\tilde{d}}_j(\omega^j)$ according to the probability measure $\Pr$, respectively. If $\sum_{j=1}^{n} \tilde{\tilde{d}}_j(\omega) x_{kj} \le Q$, then we can believe that the stochastic constraint is feasible. After a given number of cycles, if no feasible $d_j(\omega^j)$ are generated, then we say that the fuzzy random constraint is infeasible. Let M' be the number of occasions on which $\Pr\left\{\sum_{j=1}^{n} \tilde{\tilde{d}}_j x_{kj} \le Q\right\} \ge \theta_j$. By the definition of probability measure, $\Pr\left\{\omega \mid \Pr\left\{\sum_{j=1}^{n} \tilde{\tilde{d}}_j(\omega) x_{kj} \le Q\right\} \ge \theta_j\right\} \ge \eta_j$ can be estimated by M'/M provided that M is large enough. If $M'/M \ge \eta_j$, then we say x_{kj} and Q are feasible. We summarize it as follows:

Step 1: Generate $\omega^j = \left(\omega_1^j, \omega_2^j, \cdots \omega_j^j\right)^T$ from Ω according to the probability measure $\Pr$.

Step 2: Randomly generate $d_j(\omega^j)$ from $\tilde{\tilde{d}}_j(\omega^j)$ according to the probability measure $\Pr$, $r=1,2,\cdots P$ respectively.

Step 3: If $\sum_{j=1}^{n} \tilde{\tilde{d}}_j(\omega) x_{kj} \le Q$, return feasible and go to Step 5.

Step 4: Repeat the second to third steps for M' cycles.

Step 5: Repeat the first to fourth steps for M cycles.

Step6: Let M' be the feasible number. If $M'/M \ge \eta_j$, return x_{kj} and Q are feasible

4.2. Solution representation and decoding method

In this paper, two vectors are used to represent a solution: the first vector is called the vehicle vector, and the second vector the ranking vector. Following is an example to describe the coding method.

Example Suppose a company has 3 vehicles numbered 1, 2, 3 that serve 10 customers numbered 1, 2, · · ·, 10. Then one solution as follows:

customers	1	2	3	4	5	6	7	8	9	10
vehicle vector	2	3	1	3	1	1	3	1	2	2
ranking vector	2	3	2	2	3	1	1	4	1	3

This implies the following routes for the 3 vehicles:

vehicle 1	$6 \to 3 \to 5 \to 8$
vehicle 2	$9 \to 1 \to 10$
vehicle 3	$7 \to 4 \to 2$

4.3. Update

The basic elements of the PSO technique are particle, population, velocity, inertia weight, individual best, global, learning coefficients, and stopping criteria best [42]. In this paper, the bglnPSO-frs algorithm is used to solve the bi-level model, namely M_0. The glnPSO which was first proposed by Ai and Kachitvichyanukul (2009) [43], the component for social learning behavior includes not only the global best but also the local best and near neighbor best. The local best particle is the best one among several adjacent particles. In

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bglnPSO-frs, the update the inertia weight, velocity and position can be seen in Eq. (12) as below:

$$\begin{aligned} w(\tau) &= w(T)+\frac{\tau-T}{1-T}\left[w(1)-w(T)\right] \\ v_{id}(\tau+1) &= w(\tau)v_{id}(\tau)+c_p r_1[p_{id}^{best}(\tau)-p_{id}(\tau)]+c_g r_2[p_{gd}^{best}(\tau)-p_{id}(\tau)] \\ &\quad +c_l r_3[p_{id}^{lbest}(\tau)-p_{id}(\tau)]+c_n r_4[p_{id}^{nbest}(\tau)-p_{id}(\tau)] \\ p_{id}(\tau+1) &= p_{id}(\tau)+v_{id}(\tau+1) \end{aligned} \tag{13}$$

The near neighbor best is a social learning behavior concept proposed by Veeramachaneni et al. (2003) [44], and it is determined by a fitness-distance-ratio (FDR) as follows:

$$FDR=[Fitness(P_i)-Fitness(P_o)]\Big/\left|p_{id}-p_{od}\right| \tag{14}$$

4.4. Overall procedure of the bglnPSO-frs

In summary, due to the uncertainties and the bi-level structure, we propose a bi-level global-local-neighbor particle swarm optimization based on fuzzy random simulation procedure (bglnPSO-frs) to solve this MDPSP model with fuzzy random variables. The details of this algorithm are specified as follows:

Step 1. Initialize the swarm I.

Step 2. Constraints check based on the fuzzy random simulation. If in the feasible region, go to Step 3, otherwise, go back to Step 1.

Step 3. For particle $i=1,2,\ldots,I$, generate the response from the follower.

Step 3.1 For particle $i=1,2,\ldots,I$, calculate the optimal route assignment y_{ij} for the follower, namely $\min_y \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}y_{ij}$

Step 3.2 For particle $i=1,2,\ldots,I$, return the optimal route of each particle to the leader.

Step 4. Update the particles positions and velocities.

Step 4.1 For $i=1,2,\ldots,I$, decode each particle to an instalment group. Calculate the fitness value and set the position of the $i-th$ particle as its personal best. Choose the best one as the global best position. The fitness function is as follows:

$$\min_{x,z} \sum_{k=1}^{k}\sum_{i=1}^{n} c_i z_{ki} + \sum_{k=1}^{k}\sum_{j=1}^{n} c_{kj}x_{kj} + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}y_{ij}$$

Step 4.2 Update pbest, gbest, lbest. Generate nbest according to Eq. (14).

Step 4.3 Update the velocity and the position of each $i-th$ particle according to Eq. (13).

Step 4.4 Check whether the particles beyond the mark.

Step 5. Based on instalment group, group the ranking vector and number that in one group, the smallest one is numbered 1, the second smallest is numbered 2 and so on. Replace the ranking vector by using these new numbers.

Step 6. If the stopping criterion is met, stop; otherwise, $\tau=\tau+1$ and return to Step 3.

The bglnPSO-frs has proved to be effective in avoiding the particles being trapped into a local optimum. It has also proved to be very effective for solving the VRPMD in this paper. In Fig. 3, it shows the complete procedure for the bglnPSO-frs algorithm.

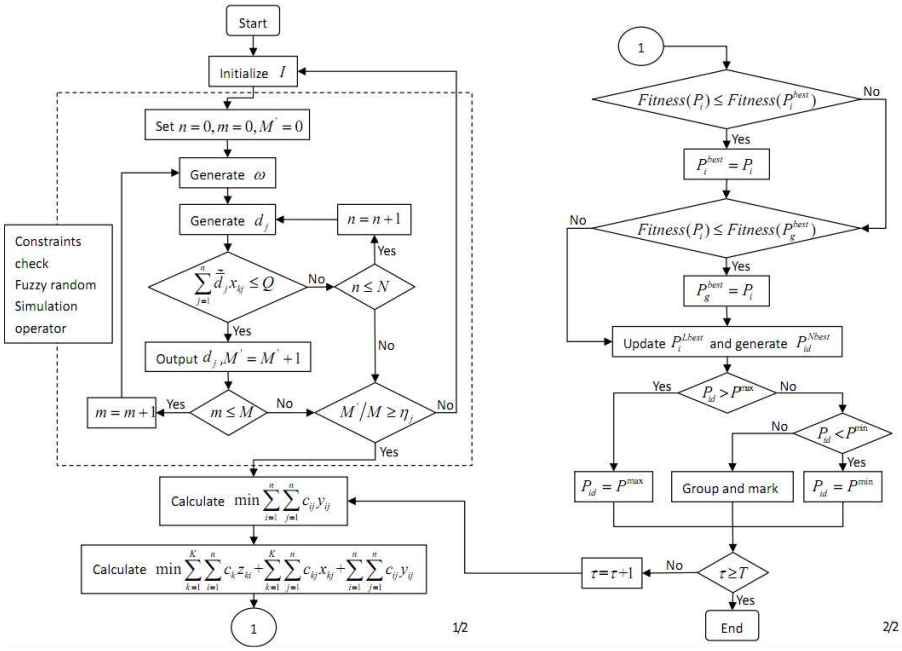

Fig. 3 Overall procedure of the bglnPSO-frs algorithm

5. Case Study

5.1. Project presentation

To prove the efficiency and practicality of the advanced model and methods, the Yalong River Basin which is considered as one of the most favorable

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development bases in China's twelve hydropower bases is taken as an application example. The Yalong River is in the west of Sichuan Province, China.

The first project on the Yalong River Basin is the Ertan Hydropower Station which is on the lower reaches of the Yalong River, about 40 km from Panzhihua City. The Ertan Hydropower Station is a super project next only in size to the Three Gorges Hydropower Station in China. The main works and diversion works of Ertan project consist of about 8.1472 million cubic meters of earth-rock cut and cover, 3.3683 million cubic meters of rock holes dug, about 1.4 million cubic meters of earth and stone filling, a concrete capacity of about 5.98 million cubic meters, and about 19,000 tons of metal structures installation. There are 4 borrow areas which are the main source of the rockfill. The location and detailed information is in Fig. 4.

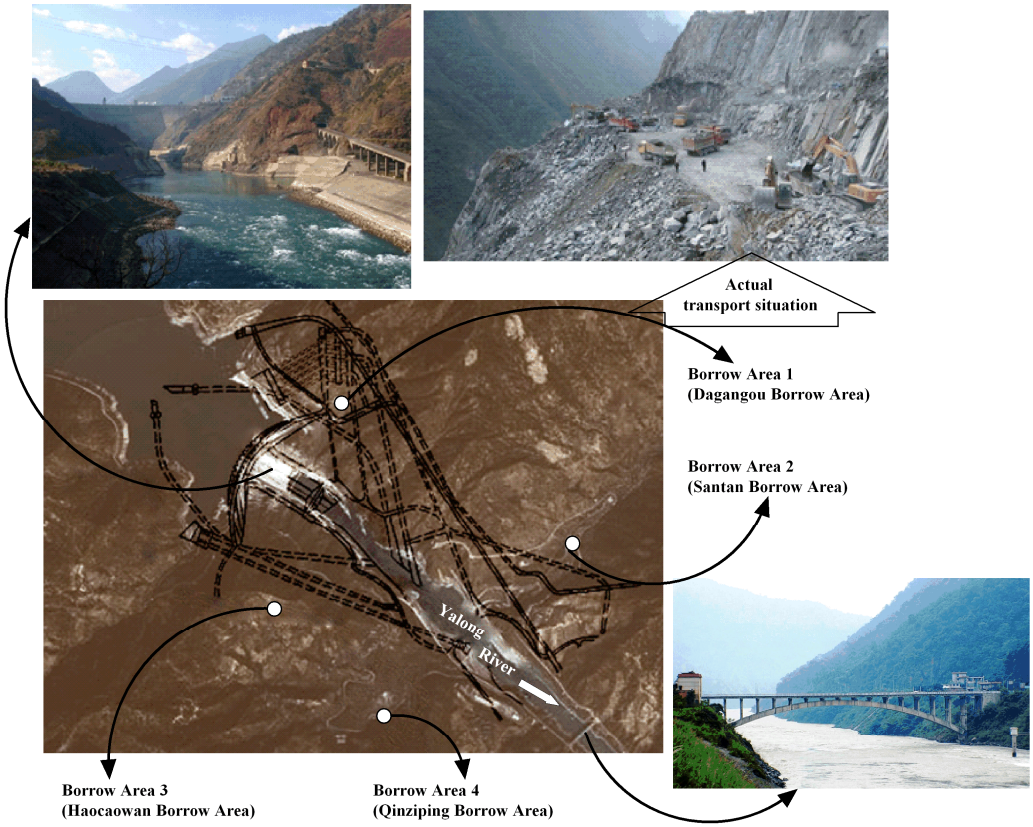

Fig. 4 The location and detailed information of Ertan Hydropower Station

Many kinds of materials are needed and must be transported to certain places in the cascade hydropower station projects of the Yalong River Basin. To further complicate the problem, the Yalong River is in the inland area of western China, where both the climate and the traffic are poor. Hence material transportation is one of the most important elements in these projects. In this application project, there are 18 customer nodes, most of which belong to the Ertan Hydropower Station and other construction projects in the cascade hydropower station projects of the Yalong River Basin. The customer node data are shown in the following, most of which were obtained using surveys. The distance from the depot and each customer's loading time are shown in Table 1. The fuzzy random demand for each customer is shown in Table 2. The distance between any two customers is shown in Table 3.

Table 1 The distance from the depot and the uploadign time of each node

Node	Uploading Time(h)	Distance (km)	Node	Uploading time (h)	Distance (km)
1	1	65.8752	10	0.8	36.9709
2	1	43.2000	11	0.5	67.9896
3	1.5	25.7406	12	1.5	56.6464
4	0.5	65.9438	13	0.5	59.6096
5	0.7	42.7286	14	0.7	49.5464
6	1.2	45.3194	15	0.75	61.9395
7	1	32.4168	16	1.25	40.1323
8	0.75	38.1215	17	1.75	54.6631
9	1.25	42.0777	18	0.75	52.7555

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Table 2 The fuzzy random demand of each customer

Node	Demand	Parameter	Node	Demand	Parameter
1	$(1.8, \rho_1, 2.2)$	$\rho_1 \sim N(1.8, 2.2)$	10	$(1.25, \rho_{10}, 1.7)$	$\rho_{10} \sim N(1.25, 1.7)$
2	$(1.7, \rho_2, 2.0)$	$\rho_2 \sim N(1.7, 2.0)$	11	$(1.1, \rho_{11}, 1.5)$	$\rho_{11} \sim N(1.1, 1.5)$
3	$(2.3, \rho_3, 2.7)$	$\rho_3 \sim N(2.3, 2.7)$	12	$(2.0, \rho_{12}, 2.6)$	$\rho_{12} \sim N(2.0, 2.6)$
4	$(1.0, \rho_4, 1.4)$	$\rho_4 \sim N(1.0, 1.4)$	13	$(0.8, \rho_{13}, 1.2)$	$\rho_{13} \sim N(0.8, 1.2)$
5	$(1.0, \rho_5, 2.0)$	$\rho_5 \sim N(1.0, 2.0)$	14	$(1.0, \rho_{14}, 1.5)$	$\rho_{14} \sim N(1.0, 1.5)$
6	$(1.5, \rho_6, 2.0)$	$\rho_6 \sim N(1.5, 2.0)$	15	$(1.3, \rho_{15}, 1.7)$	$\rho_{15} \sim N(1.3, 1.7)$
7	$(1.9, \rho_7, 2.1)$	$\rho_7 \sim N(1.9, 2.1)$	16	$(1.2, \rho_{16}, 2.0)$	$\rho_{16} \sim N(1.2, 2.0)$
8	$(1.5, \rho_8, 2.0)$	$\rho_8 \sim N(1.5, 2.0)$	17	$(2.8, \rho_{17}, 3.5)$	$\rho_{17} \sim N(2.8, 3.5)$
9	$(2.3, \rho_9, 2.8)$	$\rho_9 \sim N(2.3, 2.8)$	18	$(1.6, \rho_{18}, 1.7)$	$\rho_{18} \sim N(1.6, 1.7)$

Table 3 The distance between any two customers

Note	1	2	3	4	5	6	7	8	9
1	0	35.427	40.44	31.972	34.797	32.404	46.639	28.383	28.094
2	35.427	0	27.209	56.257	41.602	41.643	44.639	25.827	33.245
3	40.44	27.209	0	42.957	20.995	22.769	18.278	12.381	17.204
4	31.972	56.257	42.957	0	23.217	20.679	35.145	33.418	26.196
5	34.797	41.602	20.995	23.217	0	2.953	13.242	15.778	8.9196
6	32.404	41.643	22.769	20.679	2.953	0	16.191	15.942	8.4119
7	46.639	44.639	18.278	35.145	13.242	16.191	0	21.541	18.604
8	28.383	25.827	12.381	33.418	15.778	15.942	21.541	0	7.7279
9	28.094	33.245	17.204	26.196	8.9196	8.4119	18.604	7.7279	0
10	36.744	38.122	15.4	29.002	5.8873	8.3487	9.9005	12.947	8.7144
11	21.8	50.743	43.108	11.18	26.852	23.908	40.015	31.667	26.005
12	21.645	42.432	31.845	13.846	16.305	13.379	29.531	20.787	14.682
13	7.2028	33.029	33.961	27.688	27.676	25.36	39.437	21.69	20.906
14	16.369	25.307	24.075	31.264	22.69	21.227	32.337	12.202	14.174
15	37.205	57.378	40.643	7.4108	19.795	17.89	30.099	32.82	25.139
16	49.159	52.286	26.788	31.387	14.416	16.776	8.8527	27.64	22.572
17	40.071	54.821	35.007	14.912	14.16	13.285	22.418	29.22	21.597
18	50.585	61.173	37.802	25.121	19.725	20.441	21.101	35.424	28.56
Note	10	11	12	13	14	15	16	17	18
1	36.744	21.8	21.645	7.2028	16.369	37.205	49.159	40.071	50.585
2	38.122	50.743	42.432	33.029	25.307	57.378	52.286	54.821	61.173
3	15.4	43.108	31.845	33.961	24.075	40.643	26.788	35.007	37.802
4	29.002	11.18	13.846	27.688	31.264	7.4108	31.387	14.912	25.121
5	5.8873	26.852	16.305	27.676	22.69	19.795	14.416	14.16	19.725
6	8.3487	23.908	13.379	25.36	21.227	17.89	16.776	13.285	20.441
7	9.9005	40.015	29.531	39.437	32.337	30.099	8.8527	22.418	21.101
8	12.947	31.667	20.787	21.69	12.202	32.82	27.64	29.22	35.424
9	8.7144	26.005	14.682	20.906	14.174	25.139	22.572	21.597	28.56
10	0	31.782	20.718	29.543	22.653	25.66	14.705	19.616	23.303
11	31.782	0	11.345	19.016	25.543	17.998	38.46	23.903	34.814
12	20.718	11.345	0	15.649	17.445	16.56	29.443	18.425	29.033
13	29.543	19.016	15.649	0	10.259	31.966	42.07	33.878	44.097
14	22.653	25.543	17.445	10.259	0	33.408	36.732	32.779	41.562
15	25.66	17.998	16.56	31.966	33.408	0	25.134	8.2006	17.769
16	14.705	38.46	29.443	42.07	36.732	25.134	0	16.943	12.644
17	19.616	23.903	18.425	33.878	32.779	8.2006	16.943	0	10.99
18	23.303	34.814	29.033	44.097	41.562	17.769	12.644	10.99	0

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The decision makers decide that four trucks, whose deadweight is 10 tons and driving speed is 40 km/h, will be used in this project. Generally, in China, the labor charge is from 200 to 300 RMB/8h for one person, oil consumption for each truck is about 300 to 500 RMB/h, and driver costs are about 200 RMB/h. Different trucks have different labor levels, for example, truck 1 may have two workers for the loading or unloading, while truck 2 may have six. Thus, the cost of a different truck serving the same customer is different.

5.2. Result analysis

Now, consider Model M_0 with the above data and use the bglnPSO-frs algorithm to deal with it. The parameters in the environment for the problem are set as follows: Population size: popsize = 20; Maximum generation: maxGen = 200; Inertia weight: $\omega(1)=0.9$, $\omega(\tau)=0.1$ and $\omega(\tau)$ is linearly decreasing from 0.9 to 0.4; Acceleration constant: $c_p=c_g=c_l=c_n=2$. In this paper, MATLAB 7.0 on a Pentium 4, 1.83GHz clock pulse with 1024 MB memory was used, and the performance of the method was test with the data in section 5.1.

After running the program 10 times, Table 4, the best satisfactory solution was found. Fig. 5 (1/2) shows the detailed distribution of the objective value obtained by the bglnPSO-frs in different generations. It shows that the total cost of the upper level gets gradually smaller from one generation to another, which is consistent with the evolutionary idea of the bglnPSO-frs. The objective value is 20773.9 RMB and the relevant solution is as follows:

vehicle 1 $7\rightarrow16\rightarrow9$

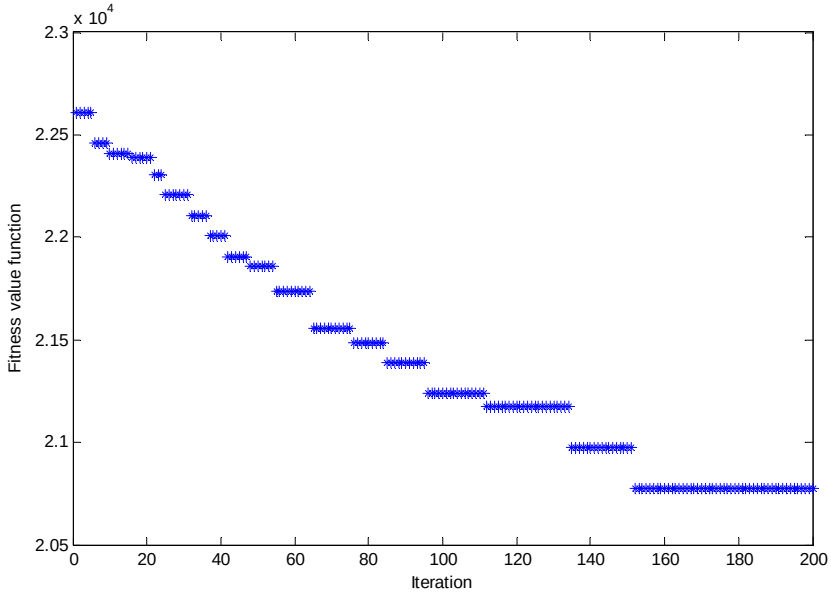
vehicle 2 $17\rightarrow18\rightarrow15\rightarrow14\rightarrow11$

vehicle 3 $3\rightarrow6\rightarrow12\rightarrow10\rightarrow5$

vehicle 4 $8\rightarrow2\rightarrow14\rightarrow1\rightarrow13$

The objective of the leader is to minimize total costs. However, the leader is only able to control two parts. Using the model and the method proposed in this paper can solve this problem. Since the proposed bi-level model is interactive, the leader can influence the follower's decision behavior through their own decision making process. The leader chooses customer nodes 7, 17, 3, 8 as the seed customers for each vehicle respectively, which makes the sum of the cost of initializing the new routes, including the loading costs and the transport costs, to be 9532.3 RMB. The customer sets are also decided by the leader to be as follows: node 7, 9, 16 served by vehicle 1, node 11, 14, 15, 17, 18 served by vehicle 2, node 3, 5, 6, 10, 12 served by vehicle 3 and node 1, 2, 8, 13, 14 served by vehicle 4. This customer cluster assignment brings the service cost to be vehicle 1 1890 RMB, vehicle 2 2040 RMB, vehicle 3 2508 RMB, and vehicle 4 2370 RMB. Thus, the total service cost, most of which is the unloading cost, is 8808 RMB.

The follower's goal is to minimize total routing cost. Therefore, the leader's decision has a large influence. When the leader decides the seed customer and customer sets for each vehicle, the follower is only able to make their decision within the seed customer and customer sets. From the leaders' decisions in the above section, the follower chooses the optimal route to minimize total routing costs. The routing cost is 353.53 RMB, 532.68 RMB, 705.99 RMB, and 840.43 RMB for each vehicle, respectively. Thus, the best total routing cost is 2432.63 RMB.



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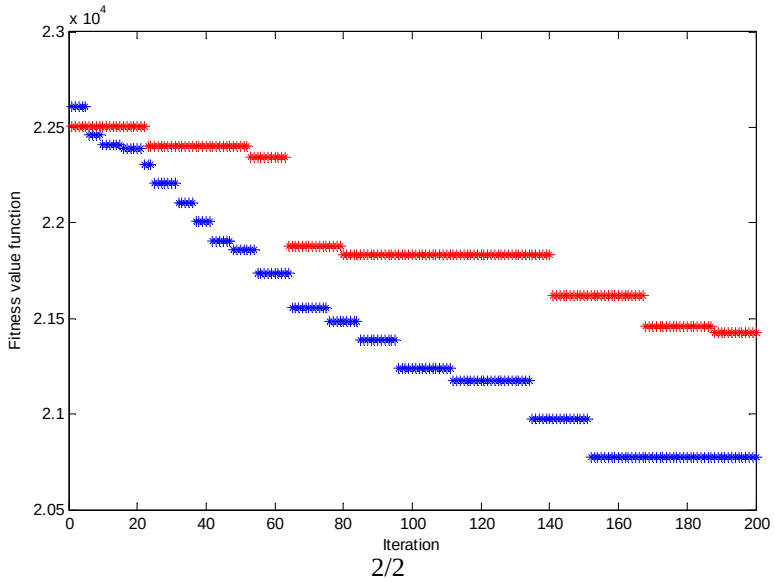

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Fig. 5 The iterative process of application by the bglnPSO-frs and the classic PSO

Table 4 Computer generated results of the bglnPSO-frs

No.	Vehicle	Leader's decisions		Follower's decisions	Best results	
		Seed customer	Customer set	Route selection	Leader	Follower
1	1	16	(4 11 12 16 18)	16→18→12→11→4	21007.6	2564.9
	2	14	(7 8 9 14)	14→8→9→7		
	3	10	(5 6 10 15 17)	10→17→15→6→5		
	4	3	(1 2 3 13)	3→13→1→2		
2	1	7	(7 9 16)	7→16→9	20773.9	2432.6
	2	17	(4 11 15 17 18)	17→18→15→14→11		
	3	3	(3 5 6 10 12)	3→6→12→10→5		
	4	8	(1 2 8 13 14)	8→2→14→1→13		
3	1	7	(7 9 16)	7→16→9	20773.9	2432.6
	2	17	(4 11 15 17 18)	17→18→15→14→11		
	3	3	(3 5 6 10 12)	3→6→12→10→5		
	4	8	(1 2 8 13 14)	8→2→14→1→13		
4	1	16	(4 11 12 16 18)	16→18→12→11→4	21007.6	2564.9
	2	14	(7 8 9 14)	14→8→9→7		
	3	10	(5 6 10 15 17)	10→17→15→6→5		
	4	3	(1 2 3 13)	3→13→1→2		
5	1	3	(1 2 3)	3→2→1	21182.8	2729.5
	2	9	(7 8 9 10 14)	9→8→10→7→14		
	3	16	(11 12 13 16 17)	16→17→12→11→13		
	4	18	(4 5 6 15 18)	18→15→4→5→6		
6	1	7	(7 9 16)	7→16→9	20773.9	2432.6
	2	17	(4 11 15 17 18)	17→18→15→14→11		
	3	3	(3 5 6 10 12)	3→6→12→10→5		
	4	8	(1 2 8 13 14)	8→2→14→1→13		
7	1	16	(4 11 12 16 18)	16→18→12→11→4	21007.6	2564.9
	2	14	(7 8 9 14)	14→8→9→7		
	3	10	(5 6 10 15 17)	10→17→15→6→5		
	4	3	(1 2 3 13)	3→13→1→2		
8	1	7	(7 9 16)	7→16→9	20773.9	2432.6
	2	17	(4 11 15 17 18)	17→18→15→14→11		
	3	3	(3 5 6 10 12)	3→6→12→10→5		
	4	8	(1 2 8 13 14)	8→2→14→1→13		
9	1	2	(2 3 8 9 14)	2→14→8→3→9	21550.5	3010.8
	2	6	(1 6 11 13)	6→11→1→13		
	3	5	(4 5 10 12 16 18)	5→4→16→18→12→10		
	4	7	(7 15 17)	7→17→15		
10	1	7	(7 9 16)	7→16→9	20773.9	2432.6
	2	17	(4 11 15 17 18)	17→18→15→14→11		
	3	3	(3 5 6 10 12)	3→6→12→10→5		
	4	8	(1 2 8 13 14)	8→2→14→1→13		

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5.3. Comparison analysis

To show better the effectiveness of the proposed algorithm, here a brief comparison is made between the bglnPSO-frs and the classic PSO. The parameters of the basic version of the classic PSO algorithm: Population size: popsize = 50; Maximum generation: maxGen = 200; Inertia weight: $\omega(1) = 0.9$, $\omega(\tau) = 0.1$, and $\omega(\tau)$ is linearly decreasing from 0.9 to 0.4; Acceleration constant: $c_p = c_g = 2$. For this algorithm MATLAB 7.0 on a Pentium 4, 1.83GHz clock pulse with 1024 MB memory is also used, and the performance of the method is tested using the actual data in section 5.1.

To explore the reasons why the bglnPSO-frs is superior, the dynamic of the swarm is studied by recording the dispersion and velocity indices in every iteration step. Fig. 5 (1/2) shows the detailed distribution of the objective value obtained by the bglnPSO-frs in different generations. Fig. 5 (2/2) shows both the convergence of the best in history of the bglnPSO-frs and the classic PSO. From Fig. 5 (2/2), both profiles show the general tendency of the particle movements in the swarm: all particles move towards the global best position, so all particles are laid close to each other and the results become better and better at the end of each iteration.

It is also observed that the dynamic of the swarms are different between the swarm in the basic version of the classic PSO and those in the bglnPSO-frs. In the basic version, the dispersion indices plotted in Fig. 5 (2/2) (the red profile) shows that the swarm is shrinking slowly over the iterations which means that the coverage of the search area by the swarm is decreasing slowly over the iteration. Hence, the swarm could sufficiently explore various regions of the problem space, but at the end of the iteration process the dispersion index is still not stable or the swarm size is not yet small enough. Further, it is not possible to confirm if the best satisfactory objective value has been achieved as the iterative process is unstable. This implies that there is sufficient time (or iteration steps) for exploration but not enough time for exploitation.

The blue profile shows the convergence of the best in history of the bglnPSO-frs. From Fig. 5 (2/2) (the blue profile) it can seen that the results are poor in the first period because the results may be infeasible and have a punishing function. As the program continues running, the swarm is shrinking more rapidly and the results become stable after about the 100th generation. In the first half of the iteration, while the swarm size is big enough, the swarm focuses on exploring various regions in the problem space. Then, during the second half of the iteration, as the swarm is clustered in a very small area, the swarm is more concentrated and is able to locate the optimum more precisely which implies that there is enough time for both exploration and exploitation. Hence, it can be concluded that there is a good balance between exploration and exploitation which may be a contributing factor for the bglnPSO-frs to yield better solution than those obtained using the classic PSO version.

The bglnPSO-frs program and the classic PSO were both run 10 times and a comparison is made. Table 5 shows the differences between the bglnPSO-frs and the classic PSO. From Table 5 the predominance of our algorithm can be clearly seen compared with the classic PSO.

Table 5 The comparison between the bglnPSO-frs and the calssic PSO

Item	The bglnPSO-frs	Classic PSO
Best result	20773.9	21333.2
Worst result	21550.5	22229.5
Average total cost	20962.56	21827.85
Difference between the best and the worst	776.6	896.3
Difference between the average and the best	188.66	494.65

5.4. Model analysis

From the data features, the customer demand is described as a fuzzy random variable. Since the definition of a fuzzy random variable is the refining and expansion of the fuzzy variables, the results between the fuzzy random model and the fuzzy model are compared. Fuzzy data are derived which ignore the randomness phenomenon and only consider the fuzzy environment. This fuzzy data is put into the bglnPSO-frs, and the program run 10 times, the results of which are shown in Table 6. From the results, it can be seen that considering fuzzy random factors may bring more economic benefits, and the cost saving can reach 1392.4 RMB or 6.6 %. Considering randomness and fuzziness at the same time may help the decision makers learn more about the problems. More detail about the problems could result in more successful decisions. As the fuzzy data are somewhat divorced from the facts, fuzzy random variables have been shown to be effective and efficient. From the results here, it is clearly seen that data translated into fuzzy random numbers is closer to reality, and has a much better performance.

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Table 6 The comparison between VRPMD in fuzzy random enviroment and fuzzy enviroment

Type	Best result	Worst result	Average total cost
Fuzzy random environment	20773.9	21550.5	20962.56
Certain environment	22161.7	22508.2	22354.975

6. Conclusion

In this paper, a vehicle routing problem with multiple decision-makers under a random environment and its application to the construction material transportation in the Yalong River Hydropower Base in the southwest region of China has been discussed. For this problem, a new mathematical model was proposed, the bi-level decision making model, in which every kind of cost is fully considered. To solve this problem, the bglnPSO-frs algorithm was presented. Then, the proposed model and method were applied to the Yalong River Hydropower Base. The results indicated that the proposed model and method is viable and efficient in handling such complex problems. At the end, a brief comparison is made between the bglnPSO-frs and the classic PSO to further illustrate the merits of the algorithm.

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References

[1] Dantzig GB, Ramser JH. The truck dispatching problem, Management science, 1959, pp. 80-91.

[2] Garcia-Najera A. The vehicle routing problem with backhauls: a multi-objective evolutionary approach, Evolutionary Computation in Combinatorial Optimization, 2012, pp. 255-266.

[3] Zachariadis EE, Kiranoudis CT. An effective local search approach for the Vehicle Routing Problem with Backhauls, Expert Systems with Applications, 2012, No. 3, Vol. 39, pp. 3174-3184.

[4] Nagy G, Wassan NA, Salhi S. The vehicle routing problem with restricted mixing of deliveries and pickups, Journal of scheduling, 2012, pp. 1-15.

[5] Goksal FP, Karaoglan I, Altiparmak F. A hybrid discrete particle swarm optimization for vehicle routing problem with simultaneous pickup and delivery, Computers & Industrial Engineering, 2012.

[6] Pavone M, Bisnik N, Frazzoli E, Isler V. A stochastic and dynamic vehicle routing problem with time windows and customer impatience, Mobile Networks and Applications, 2009, No. 3, Vol. 14, pp. 350-364.

[7] Kallehauge B, Larsen J, Madsen OBG, Solomon MM, Vehicle routing problem with time windows, Column generation, 2005, pp. 67-98.

[8] Xu J, Yang Y. A class of multiobjective vehicle routing optimal model under fuzzy random environment and its application, World Journal of Modelling and Simulation, 2008, No. 2, Vol. 4, pp. 112-119.

[9] Fisher ML, Jaikumar R. A generalized assignment heuristic for vehicle routing, Networks, 1981, No. 2, Vol. 11, pp. 109-124.

[10] Sebt MH, Rajaei H, Pakseresht MM. A fuzzy modeling approach toweather delays analysis in construction projects, International Journal Of Civil Engineering, 2007, No. 3, Vol. 5, pp. 169-181.

[11] Teodorović D, Pavković G. The fuzzy set theory approach to the vehicle routing problem when demand at nodes is uncertain, Fuzzy sets and systems, 1996, No. 3, Vol. 82, pp. 307-317.

[12] Zheng Y, Liu B. Fuzzy vehicle routing model with credibility measure and its hybrid intelligent algorithm, Applied Mathematics and Computation, 2006, No. 2, Vol. 176, pp. 673-683.

[13] Gendreau M, Laporte G, Séguin R. Stochastic vehicle routing, European Journal of Operational Research, 1996, No. 1, Vol. 88, pp. 3-12.

[14] Li J, Xu J. A class of multiobjective linear programming model with fuzzy random coefficients, Mathematical and Computer Modelling, 2006, Nos. 11-12, Vol. 44, pp. 1097-1113.

[15] Liu Y, Xu J. Multi-objective decision making model under fuzzy random environment and its application to inventory problems. Information Sciences, 2008, No. 14, Vol. 178, pp. 2899-2914.

[16] Näther W. Regression with fuzzy random data, Computational Statistics & Data Analysis, 2006, No. 1, Vol. 51, pp. 235-252.

[17] Xu J, Wei P. A bi-level model for location-allocation problem of construction & demolition, International Journal Of Civil Engineering, 2012, No. 1, Vol. 10, pp. 1-12.

[18] Von Stackelberg H. The Theory of the Market Economy, William Hodge, 1952.

[19] Kim TJ, Suh S. Toward developing a national transportation planning model: a bilevel programming approach for Korea, The Annals of Regional Science, 1988, Vol. 22, pp. 65-80.

[20] Tao Z, Xu J. A class of rough multiple objective programming and its application to solid transportation problem, Information Sciences, 2011.

[21] Xu J, Yan F. A multi-objective decision making model for the vendor selection problem in a bifuzzy environment, Expert Systems with Applications, 2011.

[22] Xu J, Liu Q, Wang R. A class of multi-objective supply chain networks optimal model under random fuzzy environment and its application to the industry of Chinese liquor, Information Sciences, 2008, No. 8, Vol. 178, pp. 2022-2043.

[23] Xu J, Zhao L. A multi-objective decision-making model with fuzzy rough coefficients and its application to the inventory problem, Information Sciences, 2010, No. 5, Vol. 180, pp. 679-696.

[24] Chang HC, Yao JS, Ouyang LY. Fuzzy mixture inventory model involving fuzzy random variable demand and fuzzy total demand, European Journal of Operational Research, 2004, No. 1, Vol. 169, pp. 65-80.

[25] Das K, Roy TK, Maiti M. Multi-item stochastic and fuzzy stochastic inventory models under two restrictions, Computers and Operations Research, 2004, Vol. 31, pp. 1793-1806.

[26] Dutta P, Chakraborty D, Roy AR. A single period

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inventory model with fuzzy random variable demand, Mathematical and Computer Modelling, 2005, Vol. 41, pp. 915-922.
[27] Dutta P, Chakraborty D, Roy AR. An inventory model for single period products with reordering opportunities under fuzzy demand, Computers and Mathematics with Applications, 2007b, Vol. 53, pp. 1502-1517.
[28] Kwakernaak H. Fuzzy random variables. part i: definitions and theorems, Information Science, 1978, Vol. 15, pp. 1-29.
[29] Kruse R, Meyer KD. Statistics with Vague Data. D. Reidel Publishing Company, 1987.
[30] Kwakernaak H. Fuzzy random variables. part ii: algorithms and examples for the discrete case, Information Science, 1979, Vol. 17, pp. 253-278.
[31] Bodin L, Golden B, Assad A, Ball M. Routing and scheduling of vehicles and crews: The state of the art, Computers & Operations Research, 1983, No. 2, Vol. 10, pp. 63-211.
[32] Ağpak K, Gökcen H. A chance-constrained approach to stochastic line balancing problem, European Journal of Operational Research, 2007, Vol. 180, pp. 1098-1115.
[33] Urban TL, Chiang WC. An optimal piecewise-linear program for the U-line balancing problem with stochastic task times, European Journal of Operational Research, 2006, Vol. 168, pp. 109-120.
[34] Xu J, Zhou X, Li S. A class of chance constrained multi-objective portfolio selection model under fuzzy random environment, Journal of Optimization Theory and Applications, 2011, Vol. 150, pp. 530-552.
[35] Kaveh A, Sabzi O. A comparative study of two meta-heuristic algorithms for optimum design of reinforced concrete frames, International Journal Of Civil Engineering, 2011, No. 3, Vol. 9, pp. 193-206.
[36] Khanzadi M, Tavakkoli SM. Optimal plastic design of frames using evolutionary structural optimization (ESO), International Journal of Civil Engineering, 2011, No. 3, Vol. 9, pp. 165-170.
[37] Afshar A, Zahraei SA, Mariňo MA. Ocyclic storage design and operation optimization; hybrid GA decomposition approach, International Journal Of Civil Engineering, 2008, No. 1, Vol. 6, pp. 34-47.
[38] Kennedy J, Eberhart R. Particle swarm optimization, IEEE International Conference on Neural Networks, 1995, Vol. 4, pp. 1942–1948.
[39] Kennedy J, Eberhart RC, Shi Y. Swarm intelligence, Morgan Kaufmann Publishers, 2001.
[40] Koh A. A coevolutionary particle swarm algorithm for bi-level variational inequalities: applications to competition in highway transportation networks, Natural intelligence for scheduling, planning and packing problems, 2009, pp. 195–217.
[41] Gao Y, Zhang G, Lu J, Wee HM. Particle swarm optimization for bi-level pricing problems in supply chains, Journal of Global Optimization, 2010, doi: 10.1007/s10898-010-9595-8.
[42] Abido MA. Optimal power flow using particle swarm optimization, International Journal of Electrical Power & Energy Systems, 2002, No. 7, Vol. 24, pp. 563-571.
[43] Ai T, Kachitvichyanukul V. A particle swarm optimization for the vehicle routing problem with simultaneous pickup and delivery, Computers & Operations Research, 2000, Vol. 36, pp. 1693–1702.
[44] Veeramachaneni K, Peram T, Mohan C, Osadciw L. Poptimization using particle swarms with near neighbor interactions, Proceedings of Genetic and Evolutionary Computation Conference, 2003, pp. 110–121.

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