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Design

Relative performances of composite conoidal shell roofs with parametric variations in terms of static, free and forced vibration behavior

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Abstract

A review of literature reveals that although singly curved conical shells applicable in many fields of mechanical engineering have been studied by many researchers but doubly curved conoidal shells which are very popular as civil engineering roofing units have not received due attention. Hence relative performances of composite conoidal shells in terms of displacements and stress resultants are studied in this paper under static and dynamic loadings. Free vibration frequencies are also reported. A curved quadratic isoparametric eight noded element is used to model the shell surface. Clamped and simply supported boundary conditions are considered. Results obtained from the present study are compared with established ones to check the correctness of the present approach. A number of additional problems of composite conoidal shells are solved for eight different stacking sequences of graphite-epoxy composite for each of the edge conditions. Uniformly distributed load for static bending analysis and step load of infinite duration for solution of forced vibration problem are considered. The results are interpreted from practical application standpoints and findings that are important for a designer to note, before he decides on the shell combination he will finally adopt among a number of possible options, are highlighted.

Keywords: *Conoidal shell, Composite material, Finite element method, Forced vibration, Newmark's method.*

1. Introduction

Conoidal shells (Fig. 1) are used in civil engineering industry to cover large column free areas such as in stadiums, airports and shopping malls. Being a doubly curved and singly ruled surface, it satisfies aesthetic and ease of casting requirements of the industry. Moreover, conoidal shell allows entry of daylight and natural air which is preferred in food processing and medicine units. To use these doubly curved, singly ruled surfaces efficiently, the behavior of these forms under bending, buckling, vibration, impact etc are required to be understood comprehensively.

The use of laminated composites to fabricate shells became preferred to civil engineers form second half of the last century. The reasons were high strength/stiffness to weight ratio, low cost of fabrication and better durability. Moreover, the stiffness of laminated composites can be altered by varying the fiber orientations and lamina thicknesses which gives designer flexibility. As a result, laminated shells were found more cost effective compared to the isotropic ones as application of laminated composites to fabricate shells reduces their mass induced seismic forces and foundation costs.

Ambartsumyan [1] was the first author to report the analysis that combined bending-stretching coupling and since then many researchers worked on different laminated composites single and doubly curved shell roofs. Among them, a group of researchers worked on laminated conoidal shells. Researchers like Dey et al [2], Chakravorty et al. [3], Nayak and Bandyopadhyay [4, 5], Das and Chakravorty [6, 7, 8, 9, 10], Kumari and Chakravorty [11,12] and Pradyumna and Bandyopadhyay [13,14] reported static bending and free vibration studies on laminated conoidal shells. Dey et al [2], Chakravorty et al. [3] and Nayak and Bandyopadhyay [4, 5] reported free vibration studies on laminated conoidal shells. Bending and free vibration characteristics of laminated conoidal shells with and without stiffeners were reported by Das and Chakravorty [6, 7, 8, 9, 10] while Kumari and Chakravorty [11, 12] worked on bending behavior of delaminated conoidal shells. Pradyumna and Bandyopadhyay [13] studied free vibration characteristics of laminated conoidal shells using higher order shear deformation theory. The authors [14] continued their research on laminated conoidal shells and reported dynamic instability behavior of these shell forms.

A shell surface may subject to short time dynamic forces in its service life by internal wind suction, snow loading in low temperature areas, air blast and seismic waves. Hence a thorough dynamic study including free

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and forced vibration studies is necessary to ensure long and uninterrupted service life of laminated shells. Feeling this necessity, a number of researchers like Reddy and Chandrashekhara [15], Lee and Han [16], Nanda and Bandyopadhyay [17, 18], Chakravorty et al. [19] and Nayak and Bandyopadhyay [20] published their research works on forced vibration analysis of singly and doubly curved shells. Reddy and Chandrashekhara [15] reported linear and nonlinear transient responses of laminated shells. Forced vibration behavior of laminated cylindrical and spherical shells was reported by Lee and Han [16]. Nanda and Bandyopadhyay [17, 18] worked on nonlinear transient responses of laminated cylindrical and spherical shells with and without initial geometric imperfections respectively. But similar studies on conoidal shells are really scanty compared to the other shell forms. Chakravorty et al. [19] and Nayak and Bandyopadhyay [20] are the only authors reported forced vibration of laminated composite and isotropic conoidal shells respectively. There are a number of published papers on singly curved conical shells but industrially important doubly curved conoidal shells need more detailed study. Hence the objective of the present paper is to provide detailed information on forced vibration characteristics of these shellforms to evolve meaningful engineering design guidelines regarding choice of a particular shell option when a designer has to decide one among many such combinations. Since the overall suitability of a shell form can be concluded only by giving due importance to static and free vibration characteristics as well, the present study takes a comprehensive approach to the problem.

2. Mathematical Formulation

2.1. Finite element formulation

Let us consider a composite conoidal shell (Fig. 1) of uniform thickness 'h' and radii of curvature 'R_{yy}' and 'R_{xy}'. Each of the thin laminae may be oriented at an angle 'θ' with reference to the x-axis (Fig. 2). An eight noded curved quadratic isoparametric element is used to model the conoidal shell. The element consists of five degree of freedoms at each node, which are u, v, w, α and β. The element strain vector is,

$$\{\varepsilon_x,\ \varepsilon_y,\ \gamma_{xy},\ \gamma_{xz},\ \gamma_{yz}\}^T = \{\varepsilon_x^0,\ \varepsilon_y^0,\ \gamma_{xy}^0,\ \gamma_{xz}^0,\ \gamma_{yz}^0\}^T + z\{k_x,\ k_y,\ k_{xy},\ k_{xz},\ k_{yz}\}^T \tag{1}$$

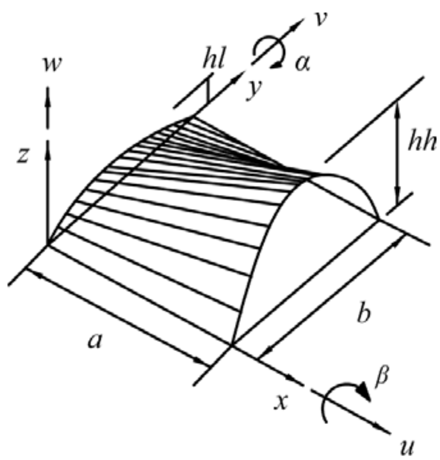


Fig. 1. Conoidal shell with degree of freedoms

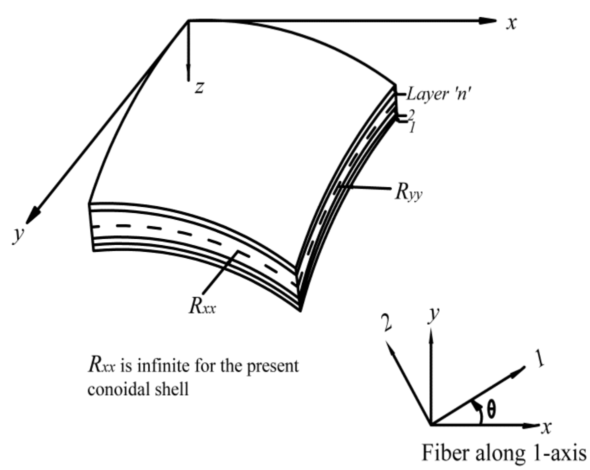


Fig. 2. Doubly curved laminated composite shell element

where
$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} = \begin{Bmatrix} \partial u_0/\partial x \\ \partial v_0/\partial y - w/R_{yy} \\ \partial u_0/\partial y + \partial v_0/\partial x - 2w/R_{xy} \\ \alpha + \partial w \quad /\partial x \\ \beta + \partial w \quad /\partial y \end{Bmatrix}$$
and
$$\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \\ k_{xz} \\ k_{yz} \end{Bmatrix} = \begin{Bmatrix} \partial\alpha/\partial x \\ \partial\beta/\partial y \\ \partial\alpha/\partial y + \partial\beta/\partial x \\ 0 \\ 0 \end{Bmatrix}$$

The strain vector is related to the nodal values of element degree of freedoms by the strain displacement matrix [*B*]. The strain displacement matrix [*B*] and the cubical shape functions [*N*] adopted in the present study are same as those were reported by Das and Chakravorty [6].

The laminate constitutive relationship is,

$$\{F\}=[D]\{\varepsilon\} \tag{2}$$

where $\{F\} = \{N_x \quad N_y \quad N_{xy} \quad M_x \quad M_y \quad M_{xy} \quad Q_x \quad Q_y\}$ (Fig. 3) and $\{\varepsilon\} = \{\varepsilon_x^0 \quad \varepsilon_y^0 \quad \gamma_{xy}^0 \quad k_x \quad k_y \quad k_{xy} \quad \gamma_{xz}^0 \quad \gamma_{yz}^0\}^T$ The laminate constitutive matrix [*D*] used in the present study is adopted from the paper reported by Das and Chakravorty [6].

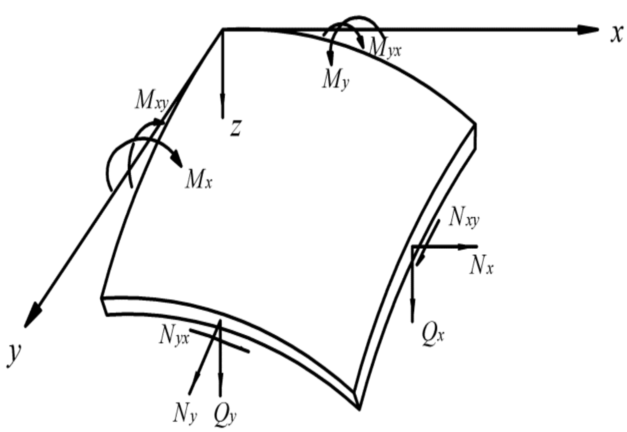


Fig. 3 Shell stress resultants

2.2. Derivation of static problem

The static displacements of the composite shell can be obtained through solution of Eq. (3),

$$[K]\{d\} = \{Q\} \tag{3}$$

The above equation is solved by the Gauss elimination method. [22]

2.3. Derivation of free vibration problem

The free vibration problem of the composite shell can be solved using Eq. (4)

$$\left|[K] - \omega^2[M]\right| = 0 \tag{4}$$

The free vibration problem is solved by subspace iteration algorithm. [22].

2.4. Derivation of forced vibration problem

Dynamic displacements of the composite shell can be obtained through solution of Eq. (5),

$$[M]\{\ddot{d}\} + [K]\{d\} = \{Q\} \tag{5}$$

The force vector {*Q*} in Eq. (5) is transient in nature and solved using Newmark's method. [22]

3. Numerical Problems

To establish the correctness of the static and free vibration formulations of the finite element code proposed in this paper, the authors compared their results with pre-established results reported by Reddy [21]. Static displacements and fundamental frequencies of cross ply simply supported spherical shell are evaluated using the present formulation and compared with Reddy [21]. The comparative results are presented in Table 1 and Table 2. The material and geometric properties of the spherical shell are presented with the tables as footnote. The radius of cross curvature of the present element is assigned a high value (10^{30}) to make it effectively zero to model a shell with no cross curvature.

Table 1 Nondimensional central displacements ($\overline{w}\times10^3$) of simply supported composite spherical shell under uniformly distributed load

Lamination	$0^0/90^0$	$0^0/90^0/0^0$	$0^0/90^0/90^0/0^0$
J.N Reddy [2]	16.980	6.697	6.833
Present FEM (2×2)	8.294	5.116	4.644
(4×4)	16.898	6.724	6.820
(6×6)	16.969	6.710	6.826
(8×8)	17.009	6.707	6.835

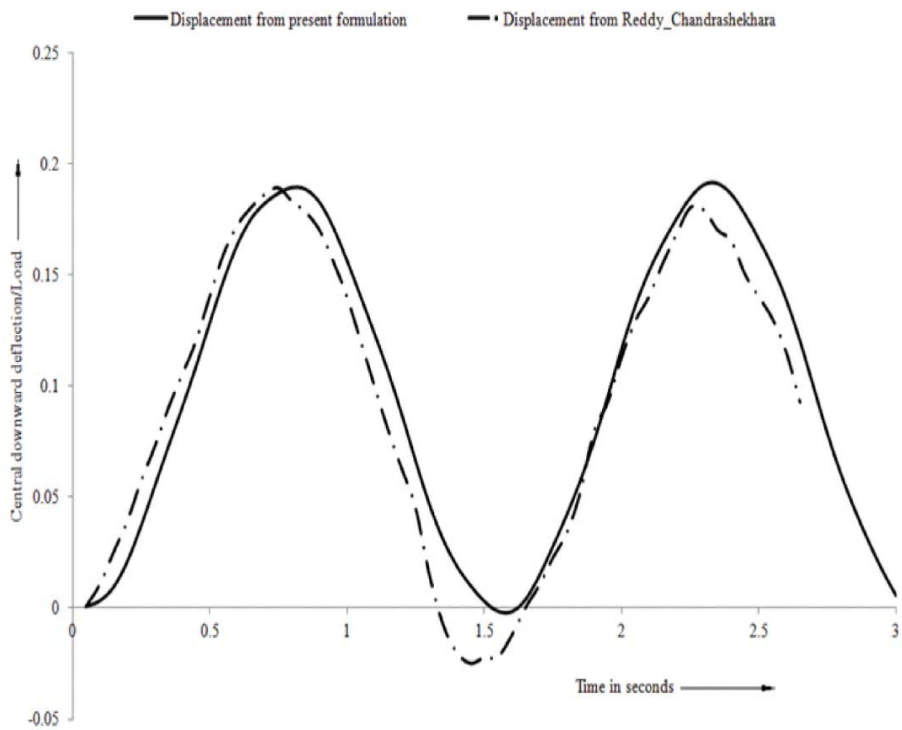
[a] a/b=1, a/h=100, $E_{11}=25E_{22}$, $G_{12}=G_{13}=0.5E_{22}$, $G_{23}=0.2E_{22}$, ν=0.25, $E_{22}=10^6 N/cm^2$, $R/a=10^{30}$

Table 2 Nondimensional fundamental frequencies (ϖ) of simply supported composite spherical shell

Lamination	$0^0/90^0$	$0^0/90^0/0^0$	$0^0/90^0/90^0/0^0$
J.N Reddy [2]	9.687	15.183	15.184
Present FEM (2×2)	14.897	15.209	15.221
(4×4)	9.722	15.209	15.222
(6×6)	9.691	15.179	15.195
(8×8)	9.681	15.180	15.183

[a] a/b=1, a/h=100, E_{11}=25E_{22}, G_{12}=G_{13}=0.5E_{22},G_{23}=0.2E_{22}, ν=0.25,E_{22}=10^6 N/cm^2, R/a=10^{30}

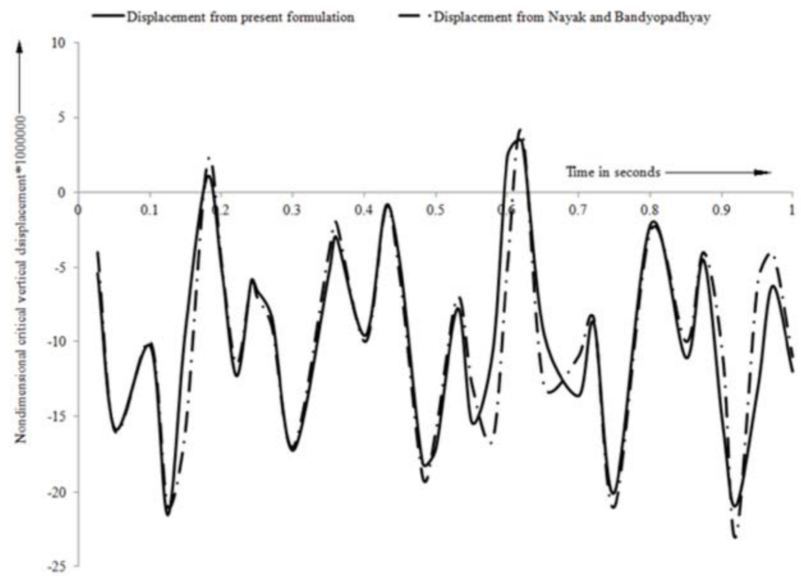
To validate the forced vibration formulation of the present code, the authors solve two additional problems as benchmark. The first problem evaluates the dynamic displacement of a simply supported laminated composite spherical shell under step load of infinite duration which was solved earlier by Reddy and Chandrashekhara [15]. The geometric and material details of the problem are furnished with Fig. 4 which shows the comparative response curves. The second problem was solved earlier by Nayak and Bandyopadhyay [20] and deals with transient response of clamped isotropic conoidal shell under rectangular step load of infinite duration. Present composite shell formulation is used for the isotropic material by making the elastic and shear modulii equal in all directions. The relevant parameters are furnished with Fig. 5 showing the response curves.



R=1000cm, E_{11}=25E_{22}, G_{12}= G_{13}=0.5E_{22}, E_{22}=10^6 N/cm^2, ν=0.25, ρ=1N-S^2/cm^6

Fig. 4 Dynamic response of laminated composite spherical shell, lamination=$0^0/90^0$

Apart from the benchmark problems, the authors solve additional problems of static and vibration responses of graphite-epoxy multilayered composite conoidal shells. Symmetric and antisymmetric stacking sequences of cross and angle ply laminations and two different boundary conditions (clamped and simply supported) are considered. The nondimensional values of static and dynamic displacements, static stress resultants, fundamental frequencies of different shell combinations are presented systematically in Table 3 to Table 11. The material and geometric properties of the conoidal shells for additional problems are presented below.



$a/b=1$, $b/h=250$, $b/h_h=0.15$, $h_l/h_h=0.25$, $b=25.0$cm, $E=25.491\text{X}10^9\text{N/m}^2$, $\nu=0.15$, $\rho=2500\text{kg/m}^3$

Fig. 5 Dynamic response of isotropic conoidal shell under uniformly distributed step load

Table 3 Nondimensional downward static deflections values of composite conoidal shells for different boundary conditions and laminations

Boundary condition	Lamination in degree	*hl/hh*=0.25	*hl/hh*=0.2	*hl/hh*=0.15	*hl/hh*=0.1	*hl/hh*=0.05	*hl/hh*=0.0
CCCC	0/90	0.0000319	0.0000387	0.0000474	0.0000588	0.0000736	0.0000927
	0/90/0	0.0000295	0.0000363	0.0000455	0.0000580	0.0000752	0.0000987
	45/-45	0.0000722	0.0000906	0.0001130	0.0001401	0.0001718	0.0002082
	45/-45/45	0.0000620	0.0000739	0.0000885	0.0001054	0.0001248	0.0001465
	0/90/0/90	0.0000289	0.0000341	0.0000404	0.0000485	0.0000586	0.0000713
	0/90/90/0	0.0000344	0.0000413	0.0000503	0.0000613	0.0000759	0.0000945
	45/-45/45/-45	0.0000566	0.0000682	0.0000814	0.0000971	0.0001154	0.0001362
	45/-45/-45/45	0.0000557	0.0000667	0.0000801	0.0000957	0.0001138	0.0001343
SSSS	0/90	0.0005628	0.0006439	0.0007630	0.0009229	0.0011327	0.001419
	0/90/0	0.0004651	0.0005601	0.0006868	0.0008708	0.0011301	0.0014694
	45/-45	0.0003121	0.0003535	0.0004093	0.0005062	0.0006451	0.0008203
	45/-45/45	0.0002367	0.0002719	0.0003164	0.0003864	0.0002367	0.0005823
	0/90/0/90	0.0004282	0.0004922	0.0005833	0.0007019	0.0008475	0.0010303
	0/90/90/0	0.0004494	0.0005351	0.0006509	0.0008014	0.0010064	0.0012671
	45/-45/45/-45	0.0001996	0.0002283	0.0002661	0.0003243	0.0003982	0.0004928
	45/-45/-45/45	0.0002031	0.0002334	0.0002739	0.0003348	0.0004119	0.0005146

Table 4 Nondimensional fundamental frequency values of composite conoidal shells for different boundary conditions and laminations

Boundary condition	Lamination in degree	*hl/hh*=0.25	*hl/hh*=0.2	*hl/hh*=0.15	*hl/hh*=0.1	*hl/hh*=0.05	*hl/hh*=0.0
CCCC	0/90	73.78	73.92	74.21	74.65	75.25	76.01
	0/90/0	102.89	103.48	104.17	104.96	105.85	106.85
	45/-45	81.09	77.94	74.76	71.61	68.54	65.62
	45/-45/45	96.94	93.32	89.69	86.12	82.67	79.41
	0/90/0/90	87.92	88.29	88.80	89.43	90.20	91.09
	0/90/90/0	101.16	101.72	102.40	103.17	104.05	105.03
	45/-45/45/-45	100.22	96.93	93.59	90.23	86.91	83.67
	45/-45/-45/45	100.06	96.68	93.27	89.88	86.55	83.33
SSSS	0/90	47.68	44.10	40.45	36.81	33.25	29.95
	0/90/0	53.07	49.05	44.84	40.52	36.23	32.27
	45/-45	53.17	52.61	51.37	48.60	44.24	39.49
	45/-45/45	70.27	68.23	64.28	59.10	53.65	48.45
	0/90/0/90	53.01	49.51	46.01	42.56	39.25	36.18

0/90/90/0	53.66	49.79	45.82	41.83	37.95	34.36
45/-45/45/-45	74.73	73.22	69.28	63.37	57.14	51.30
45/-45/-45/45	74.41	72.48	68.42	62.78	56.82	51.16

Table 5 Nondimensional downward dynamic displacement values of composite conoidal shells for different boundary conditions and laminations

Boundary condition	Lamination in degree	*hl*/*hh*=0.25	*hl*/*hh*=0.2	*hl*/*hh*=0.15	*hl*/*hh*=0.1	*hl*/*hh*=0.05	*hl*/*hh*=0.0
CCCC	0/90	7.048E-05	7.959E-05	9.747E-05	0.0001181	0.00015235	0.0002005
	0/90/0	6.145E-05	7.783E-05	0.0001004	0.0001379	0.00019138	0.0002436
	45/-45	0.0001639	0.0002192	0.0002635	0.0003100	0.00036732	0.0004450
	45/-45/45	0.0001362	0.0001775	0.0002092	0.0002475	0.00027601	0.0003178
	0/90/0/90	5.809E-05	7.048E-05	8.692E-05	0.0001058	0.00013841	0.0001620
	0/90/90/0	7.177E-05	9.245E-05	0.0001117	0.0001509	0.00018562	0.0002192
	45/-45/45/-45	0.0001362	0.0001563	0.0001867	0.0002270	0.00026515	0.0003123
	45/-45/-45/45	0.0001314	0.0001515	0.0001775	0.0002181	0.00025244	0.0003061
SSSS	0/90	0.0011933	0.0013367	0.0015652	0.0018370	0.0023135	0.0028758
	0/90/0	0.0010180	0.0012180	0.0014622	0.0018295	0.0023937	0.0030345
	45/-45	0.0006207	0.0007139	0.0008599	0.0010660	0.0013022	0.0016710
	45/-45/45	0.0004907	0.0005718	0.0006740	0.0007814	0.0010251	0.0012656
	0/90/0/90	0.0008794	0.0010703	0.0012940	0.0015675	0.0018835	0.0022374
	0/90/90/0	0.0009855	0.0011790	0.0014010[	0.0016929	0.0021074	0.0026711
	45/-45/45/-45	0.0004227	0.0004929	0.0005407	0.0006732	0.00082357	0.0009981
	45/-45/-45/45	0.0004328	0.0005174	0.0006261	0.0006868	0.0008552	0.0010676

Table 6 Nondimensional compressive static inplane force resultants ($\overline{N}_x$) of composite conoidal shell for different boundary conditions and laminations

Lamination in degree	*hl*/*hh*=0.25		*hl*/*hh*=0.15		*hl*/*hh*=0.0	
	CCCC	SSSS	CCCC	SSSS	CCCC	SSSS
0/90	0.11788	1.9148	0.1469	2.3198	0.31588	3.8064
0/90/0	0.08712	1.6665	0.1291	1.7918	0.29989	2.9941
45/-45	1.1268	8.6344	1.3224	11.052	1.816	15.317
45/-45/45	1.1118	1.6594	1.2855	2.0862	1.6146	3.1301
0/90/0/90	0.11618	1.6792	0.13724	1.8923	0.25545	2.7303
0/90/90/0	0.1216	1.6582	0.15394	1.813	0.32171	2.971
45/-45/45/-45	1.1237	2.1876	1.3071	2.6619	1.706	3.3897
45/-45/-45/45	1.1134	1.5976	1.2849	1.9512	1.6677	2.9567

Table 7 Nondimensional compressive static inplane force resultants ($\overline{N}_y$) of composite conoidal shell for different boundary conditions and laminations

Lamination in degree	*hl*/*hh*=0.25		*hl*/*hh*=0.15		*hl*/*hh*=0.0	
	CCCC	SSSS	CCCC	SSSS	CCCC	SSSS
0/90	1.5662	2.9112	1.9255	3.4402	2.6892	4.5765
0/90/0	1.755	3.6613	2.0225	4.5667	2.9008	5.222
45/-45	1.4549	8.3206	1.6247	10.895	1.9547	15.364
45/-45/45	1.3801	2.5068	1.4903	2.9314	1.8646	3.934
0/90/0/90	1.5358	2.2882	1.7906	2.7909	2.1843	3.3884
0/90/90/0	1.5537	2.987	1.88[	3.3641	2.5568	4.6688
45/-45/45/-45	1.3714	2.433	1.4937	3.0397	1.7329	4.039
45/-45/-45/45	1.36	2.40	1.4826	2.996	1.7598	3.9893

Table 8 Nondimensional hogging static moment resultants ($\overline{M}_x$) of composite conoidal shell for different boundary condition and lamination

Lamination in degree	*hl/hh*=0.25		*hl/hh*=0.15		*hl/hh*=0.0	
	CCCC	SSSS	CCCC	SSSS	CCCC	SSSS
0/90	0.003004	0.00415	0.004193	0.00586	0.007361	0.01131
0/90/0	0.001509	0.000506	0.002172	0.000739	0.004068	0.00122
45/-45	0.00295	0.01588	0.004340	0.01741	0.007362	0.02128
45/-45/45	0.00519	0.02269	0.00680	0.03037	0.00980	0.048
0/90/0/90	0.00457	0.00286	0.00601	0.004021	0.00941	0.006479
0/90/90/0	0.002715	0.000897	0.003697	0.001309	0.006169	0.00214
45/-45/45/-45	0.004033	0.00329	0.005556	0.003185	0.008673	0.004764
45/-45/-45/45	0.00444	0.01583	0.00603	0.021	0.00940	0.03280

Table 9 Nondimensional sagging static moment resultant ($\overline{M}_x$) of composite conoidal shell for different boundary conditions and laminations

Lamination in degree	*hl/hh*=0.25		*hl/hh*=0.15		*hl/hh*=0.0	
	CCCC	SSSS	CCCC	SSSS	CCCC	SSSS
0/90	0.00151	0.0101	0.00197	0.01461	0.00297	0.02462
0/90/0	0.00089	0.00359	0.00131	0.00599	0.00244	0.01307
45/-45	0.00188	0.01394	0.00256	0.01640	0.00372	0.01998
45/-45/45	0.00210	0.01577	0.00249	0.02038	0.00440	0.02952
0/90/0/90	0.00205	0.01373	0.00252	0.01805	0.00333	0.02756
0/90/90/0	0.00146	0.00645	0.00193	0.00993	0.00301	0.01817
45/-45/45/-45	0.00206	0.01343	0.00224	0.01635	0.00378	0.02130
45/-45/-45/45	0.00230	0.01465	0.00244	0.01883	0.00412	0.02697

Table 10 Nondimensional hogging static moment resultants ($\overline{M}_y$) of composite conoidal shell for different boundary conditions and laminations

Lamination in degree	*hl/hh*=0.25		*hl/hh*=0.15		*hl/hh*=0.0	
	CCCC	SSSS	CCCC	SSSS	CCCC	SSSS
0/90	0.000390	0.000980	0.000515	0.002463	0.000738	0.006048
0/90/0	0.003787	0.00652	0.00538	0.004850	0.009686	0.008106
45/-45	0.002466	0.015138	0.002912	0.01701	0.004981	0.021262
45/-45/45	0.00435	0.02172	0.00576	0.02978	0.008569	0.047973
0/90/0/90	0.001083	0.00245	0.001693	0.003034	0.003176	0.006473
0/90/90/0	0.00393	0.00586	0.00524	0.004795	0.008169	0.0075234
45/-45/45/-45	0.003434	0.003484	0.004756	0.002904	0.0074912	0.0039452
45/-45/-45/45	0.003772	0.01514	0.00523	0.02057	0.008290	0.032781

Table 11 Nondimensional sagging static moment resultant ($\overline{M}_y$) values of composite conoidal shell for different boundary condition and lamination

Lamination in degree	*hl/hh*=0.25		*hl/hh*=0.15		*hl/hh*=0.0	
	CCCC	SSSS	CCCC	SSSS	CCCC	SSSS
0/90	0.00377	0.01585	0.00470	0.01892	0.006746	0.025763
0/90/0	0.00094	0.02491	0.00148	0.03248	0.003305	0.048648
45/-45	0.00187	0.01390	0.00273	0.01640	0.004264	0.019561
45/-45/45	0.00204	0.01558	0.00241	0.02041	0.004382	0.029904
0/90/0/90	0.00206	0.01630	0.00252	0.01949	0.003495	0.026555
0/90/90/0	0.00093	0.02243	0.00145	0.02848	0.002991	0.039721
45/-45/45/-45	0.00195	0.01316	0.00214	0.01619	0.003803	0.021261
45/-45/-45/45	0.00223	0.01447	0.00240	0.01882	0.004139	0.027241

a/b=1, a/h=100, hh/a=0.2, hl/hh=0.25, E_{11}=25E_{22}, G_{12}=G_{13}=0.5E_{22}, G_{23}=0.2E_{22}, ν=0.25, ρ=100N-sec^2/m^4

Uniformly distributed step load of infinite duration is considered to act on the conoidal shells (Fig. 6) for forced vibration study. The magnitude of the static load is considered equal to the step load value of the load-time history.

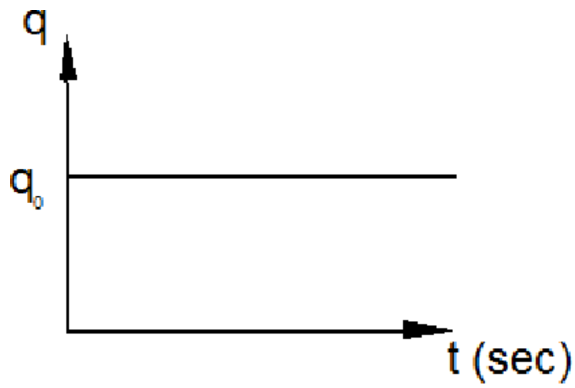


Fig. 6 Transient load case

4. Results and Discussion

4.1. Benchmark problems

Table 1 and Table 2 show very good agreement of the present results with the established ones and this validate the static and free vibration formulation. The tables also show the convergence of displacements and fundamental frequencies with increasingly finer mesh and an 8×8 division is taken up for further study since the values do not improve by more than 1% on further refining. The natures of published response curves and those obtained by present approach (Fig. 4 and Fig. 5) establish the correctness of the present forced vibration formulation for composite conoidal shell.

4.2. Additional problems

Nondimensional static displacements, fundamental frequencies and dynamic displacements of clamped and simply supported composite conoidal shells are presented in Table 3 to Table 5 for eight different stacking sequences of graphite-epoxy composite with five different curvatures achieved by changing the *hl*/*hh* ratio from 0.25 to 0.0 in discrete steps of 0.05. Governing static force and moment resultants (including the deflection, compressive inplane forces and bending moments which govern the shell thickness) are presented in Table 6 to Table 11 for the above laminations and three *hl*/*hh* ratios. Dynamic displacements of the conoidal shell are studied at the location of corresponding maximum static displacements.

4.2.1. Relative performances of composite conoidal shells for different hl/hh ratio

Cost of casting, ease of casting and fabrication of conoidal shells directly depend on their curvature and hence on *hl*/*hh* ratio. Hence from design point of view response of conoidal shells for varying curvature is an important aspect of study. It is evident from Table 3 and Table 5 that static and dynamic deflections are monotonically decreasing functions of *hl*/*hh* ratio for any given boundary condition and lamination. The clamped shell with highest curvature ratio (*hl*/*hh*=0.25) shows the best overall performance. But it is important to note here that locking all support degrees of freedoms or manufacturing a conoidal shell with highest curvature ratio may not be an economic option. It is further noted from the Table 3 and Table 5 that, for a constant lamination, a clamped shell loses its stiffness more by releasing its support degrees of freedoms rather than by reducing its curvature ratio. Hence to ensure a cost effective shell design, an engineer may vary the curvature ratio of the conoidal shell as in that case the loss of shell stiffness is relatively less then what one encounters when the clamped edges of the shell are made simply supported. The governing static stress resultants follow a similar variation with *hl*/*hh* ratio as evident from Table 6 to Table 11 though the reverse trend is exhibited by the fundamental frequency values (Table 4). The only exception to this general trend is exhibited by clamped cross ply shells where the frequency decreases as *hl*/*hh* ratio increases. This fact also establishes the truth that among two shell options the one better in terms of static force resultants and deflections may not be performing better in terms of free vibration behavior. Both the stiffness and mass change with change of curvature and the frequency of vibration depends on the ratio of stiffness to mass basically. This is why an increase of stiffness will result in a reduced deflection but may yield a lower value of frequency if the increase of mass is relatively more conspicuous.

4.2.2. Relative performances of composite conoidal shells for different boundary conditions

Close observation of static and dynamic deflections from Table 3 and Table 5 reveal that clamped shells show lower values when compared to simply supported shells for any given lamination and curvature. This is quite obvious as in clamped boundary condition, all the support degree of freedoms of a shell are locked which restrict its possible movements along the boundaries and makes the shell stiffer compared to the simply supported ones, where some of the support degree of freedoms are released. It is noted that for a given lamination scheme, boundary condition and *hl*/*hh* ratio, the static and dynamic deflections are by and large of the same order with minor variations because the inertia effect for dynamic deflection and hence the mass has a role to play as explained above.

Table 4 shows that for any given lamination, the clamped shell shows higher value of fundamental frequency than a simply supported one. It is very interesting to note that even some of the simply supported shells may exhibit higher frequency values then some clamped shell options by virtue of having different stacking orders. This is where the importance of tailoring the fiber orientations to optimize the frequency value may be realized as a unique advantage of laminated composites. For example a $0^0/90^0$ laminated clamped shell show less fundamental frequency than a number of simply supported shells with laminations $(45^0/\text{-}45^0)_2$ and $(45^0/\text{-}45^0)_s$ and *hl*/*hh* ratio 0.25. This observation also establishes the fact that the role of stacking sequence in determining the dynamic rigidity of a composite conoidal shell is no less important than its boundary condition.

Comparative study of governing static stress resultants show that clamped shells exhibit better performances than the simply supported shells for any given lamination and *hl*/*hh* ratio. But for hogging moment resultants only, a simply supported shell may show a lower value than a clamped one for few laminations. For example simply supported shells show lower static hogging M_x values than clamped shells for a number of laminates like $0^0/90^0/0^0$, $(0^0/90^0)_2$, $(0^0/90^0)_s$, $(45^0/-45^0)_2$. Hogging M_y values also shows the same trend for $0^0/90^0/0^0$, $(0^0/90^0)_s$, $(45^0/-45^0)_2$ when *hl*/*hh* ratio is less than or equal to 0.15. This finding again reinforces the fact that in composite shells, lamination plays a very important role along with the support condition to determine resultant stiffness. It is also evident that relative performance study of shells in terms of their deflections cannot be taken as the only basis of comparing their overall performance.

4.2.3. Relative performances of composite conoidal shells for different laminations

In civil engineering applications among two shell forms the one which exhibits lower deflection is accepted as a better option from serviceability point of view. Again the shell with high fundamental frequency value has lower possibility to fall in resonance with low frequency dynamic excitations where the frequency may gradually build up its value and hence should be preferred. It is evident from Table 3 and Table 5 that, for a given *hl*/*hh* ratio, both symmetric and antisymmetric cross ply laminates are convincingly better than angle ply ones for clamped edges while for simply supported ones the reverse trend is noted where the angle ply laminates exhibit lower static and dynamic deflections than cross ply ones. It is interesting to note from Table 4 that for simply supported shells among any two laminations the one which performs better in terms of deflection is also better in terms of free vibration exhibiting higher frequency values. For clamped shells, however, such unified behavior is not found to hold good.

Results of Table 3 and Table 5 show that symmetric stacking sequences of cross and angle ply laminates exhibit better performance compared to antisymmetric ones in terms of static/dynamic deflections and frequencies until the number of laminae is less than four in a laminate. As the number of laminae exceeds four, the bending-stretching coupling of the antisymmetric laminates reduces and the antisymmetric stacking orders prevail the symmetric ones for both the edge conditions considered here. In spite of the above trend being true by and large there are some sporadic exceptions, for example when static and dynamic deflections of $0^0/90^0/0^0$ and $0^0/90^0$ laminations are compared for different values of *hl*/*hh* ratio.

Time variation of deflections and stress resultants for a $0^0/90^0/0^0/90^0$ laminated clamped conoidal shell is studied upto one second and response curves are presented for three different *hl/hh* ratios (Fig. 7 to Fig. 11) to have a comprehensive idea about the dependence of deflections and stress resultants on curvature. The $0^0/90^0/0^0/90^0$ conoidal shell option is selected for response curve study because its dynamic deflections are least for all *hl*/*hh* ratios. Response curves for deflections are observed to oscillate at different points of time and the oscillation even reach to upward deflections at few points of time which represents reversal of stresses. Hence it can be concluded that static simplification of dynamic problems using a suitable factor is an incomplete dynamic study as it is unable to account for the reversal of the stresses. It is also observed from the response curves in Fig. 8 and Fig. 9 that the reversal of N_x is more pronounced than N_y and it is true for all three *hl*/*hh* ratios of the conoidal shell.

The response curves for deflections and stress resultants follow the same pattern for different values of *hl*/*hh* ratio taken up here. But amplitudes of the response curves for deflections and stress resultants are noted to increase as *hl*/*hh* ratio decreases. Dynamic amplification is more prominent for moment resultants compared to that for force resultants.

5. Conclusions

Following conclusions can be drawn from the present study,

1) The close agreement between the results obtained by the present approach and those appearing in the published literature establishes the correctness of the formulation.

2) The static stiffness of the conoidal shell surface increases with curvature and so as the surface gradually becomes deeper the static displacement and forces decrease. But since such increase in *hl*/*hh* ratio is also associated with an increase of mass, the frequency values exhibit a mixed behavior. The frequency increases with increase of *hl*/*hh* ratio in general but there are quite a number of exceptions.

3) For any given lamination, the clamped shells are better compared to simply supported ones in terms of displacement and fundamental frequency. But when their relative performances are judged practically considering all the force resultants , there are cases where the simply supported shells show better performances in terms of some of the force resultants. This indicates that to choose a boundary condition between the two, the overall performance must be considered.

It is also very important to note that if lamination is allowed to be varied then some of the simply supported shells may perform convincingly better than clamped ones for a given curvature.

4) It is evident from the dynamic response results that the values of dynamic load factors may exceed two and such factors for stress resultants may exceed those for displacements. Hence to estimate the dynamic stresses a comprehensive study is needed and using the loads factor for displacements to estimate the dynamic stresses may be misleading. It is also important to appreciate the stress reversal phenomenon which cannot be accounted for by

merely multiplying the static values by dynamic load factors.

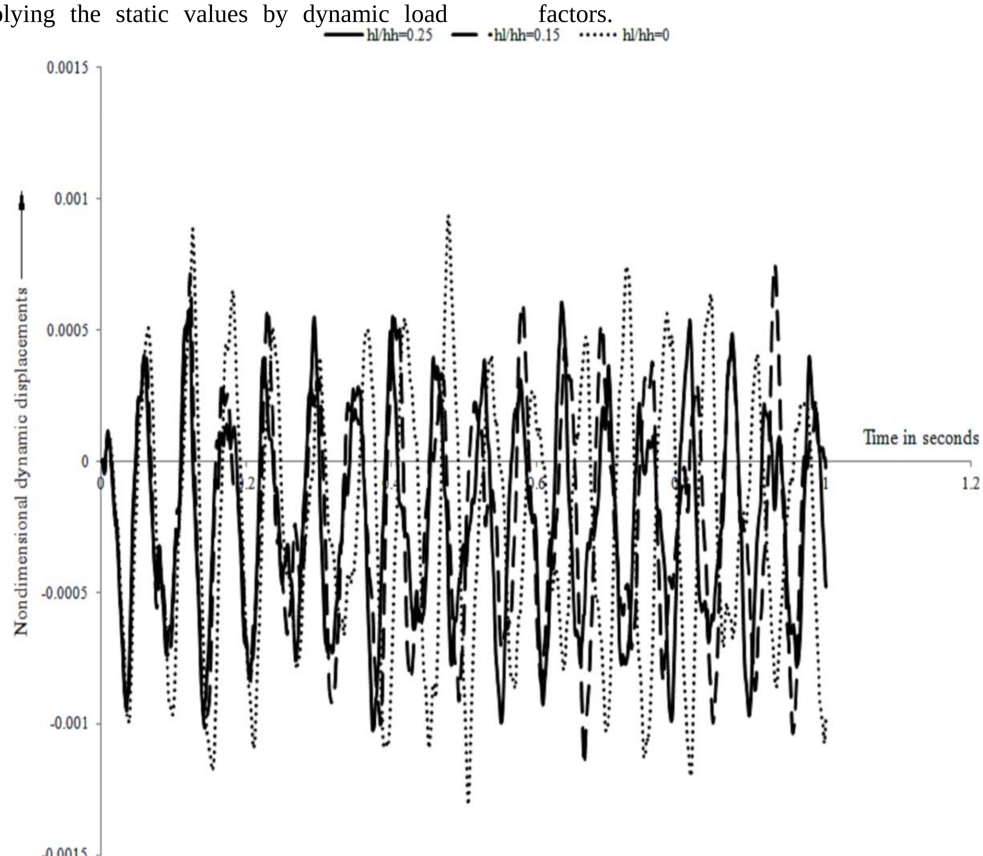


Fig. 7 Nondimensional dynamic deflections of $0^0/90^0/0^0/90^0$ shell for different *hl*/*hh* ratio

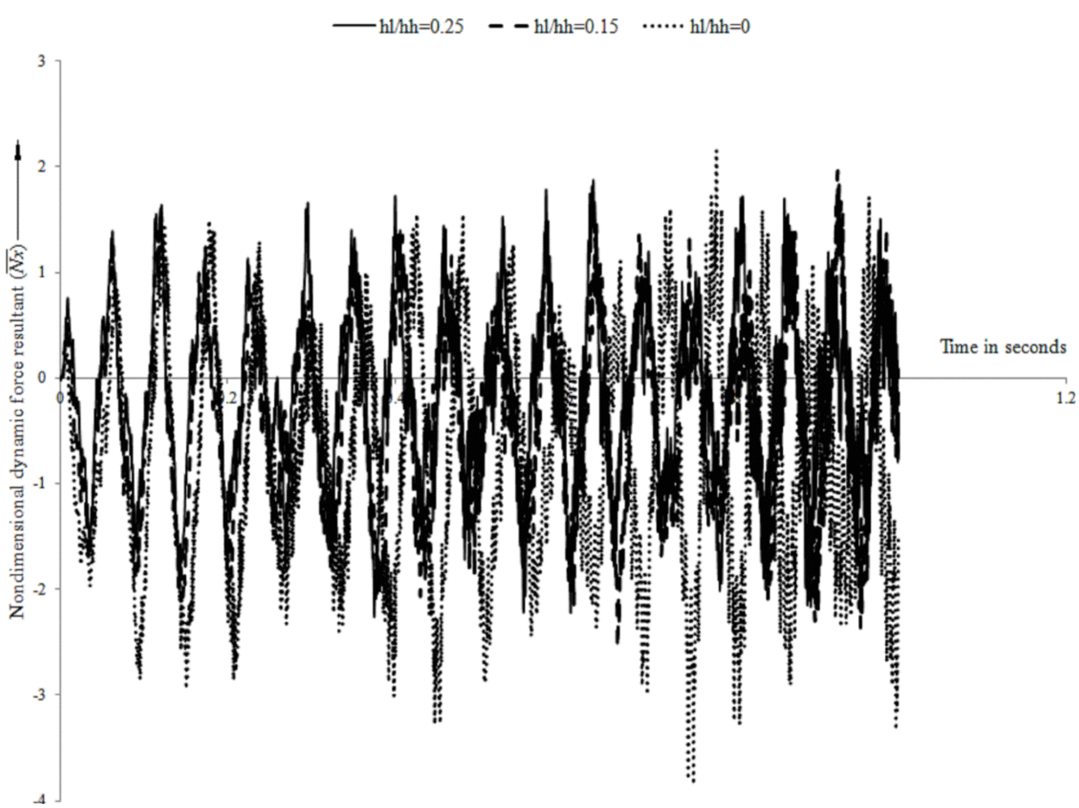


Fig. 8 Nondimensional dynamic force resultant ($\overline{N}_x$) of $0^0/90^0/0^0/90^0$ shell for different *hl*/*hh* ratio

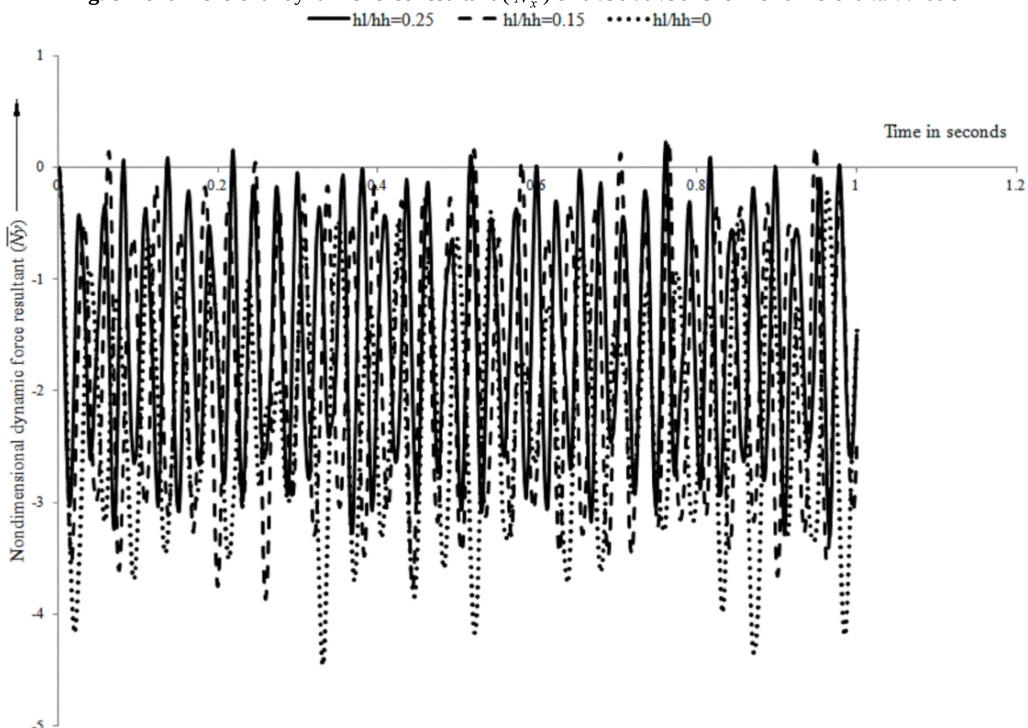


Fig. 9 Nondimensional dynamic force resultant ($\overline{N}_y$) of $0^0/90^0/0^0/90^0$ shell for different *hl*/*hh* ratio

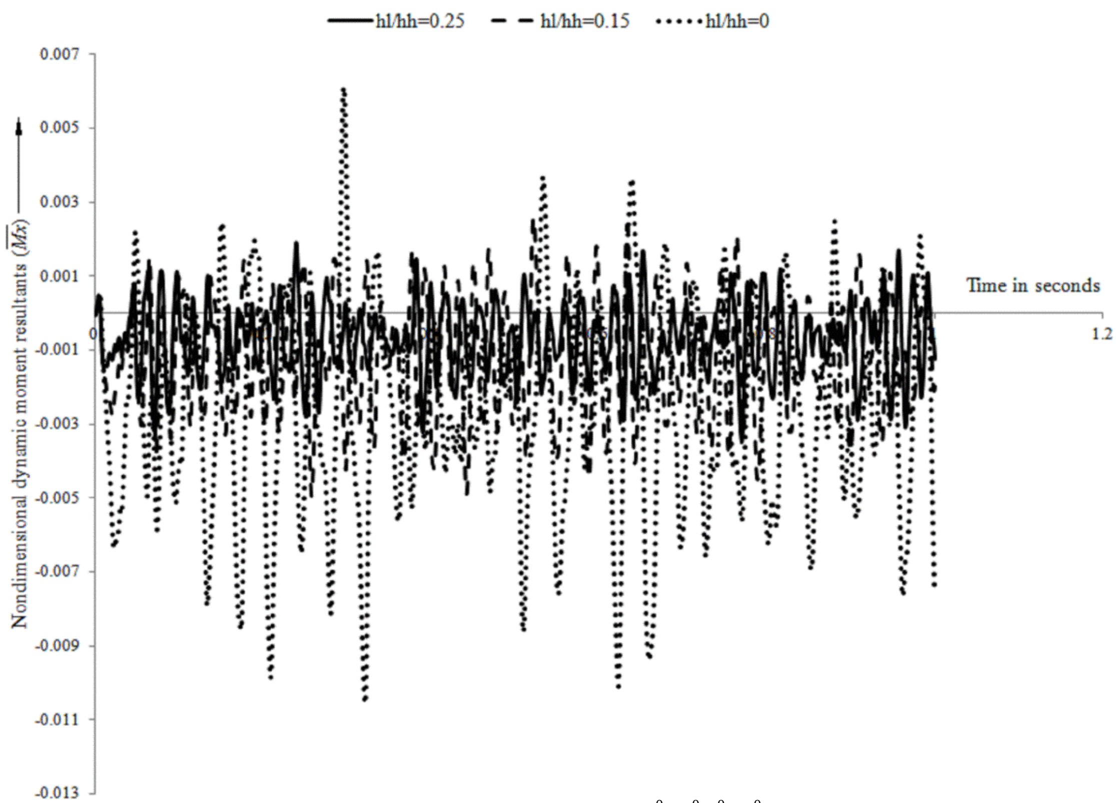


Fig. 10 Nondimensional dynamic moment resultant ($\overline{M}_x$) of $0^0/90^0/0^0/90^0$ shell for different *hl*/*hh* ratio

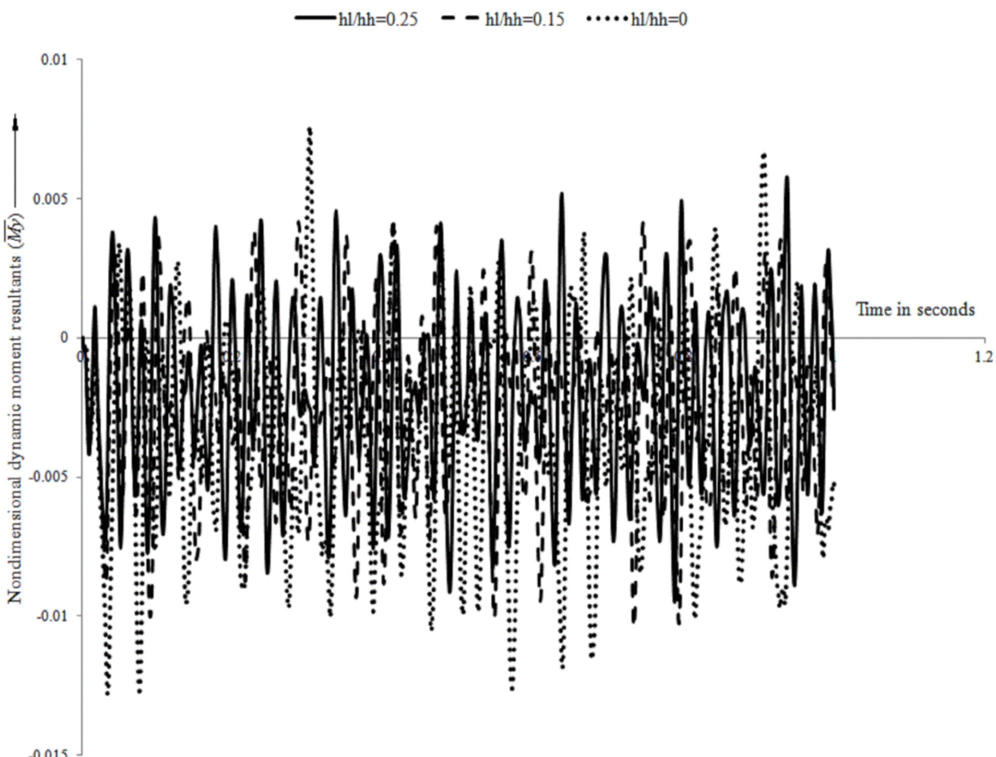


Fig. 11 Nondimensional dynamic moment resultant ($\overline{M}_y$) of $0^0/90^0/0^0/90^0$ shell for different *hl*/*hh* ratio

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Notations

a,b	Length and width of shell in plan.
D	Flexural rigidity matrix of the shell.
$\{d\}$	Global displacement of the shell.
$\{\ddot{d}\}$	Global acceleration of the shell.
hh, hl	Higher and lower heights of the conoidal shell respectively.
h	Shell thickness.
M_x, M_y, M_{xy}	Moment resultants per unit length.
$\overline{M}_x, \overline{M}_y$	Nondimensional moment resultants. [=(M_x or M_y)/qa^2]
$\overline{M}_{xy}$	Nondimensional torsion resultant. [= M_{xy} /qab]
N_x, N_y, N_{xy}	Force resultants per unit length.
$\overline{N}_x, \overline{N}_y$	Nondimensional inplane force resultants. [=(N_x or N_y)/qa]
$\overline{N}_{xy}$	Nondimensional inplane shear resultant. [= N_{xy} /qa]
Q_x, Q_y	Transverse shear resultants per unit length.
q_0	Peak value of the transient loads.
R_{xy}	Radius of cross curvature of conoidal shell.
R_{yy}	Radius of curvature of conoidal shell along y axis.

R	Radius of the spherical shell.
u, v, w	Translational degree of freedoms along x, y and z directions respectively.
$\overline{w}$	Nondimensional transverse displacement $\left[= wE_{22}h^3/\left(qa^4\right)\right]$
x,y, z	Global coordinates axes.
α,β	Rotations of the shell about y and x axes respectively.
γ_{xy} , γ_{xz}, γ_{yz}	Shear strains.
ε_x, ε_y	Inplane strains along x and y axes respectively.
θ	Angle of lamination with respect to x axis of the conoidal shell.
ν_{ij}	Poisson's ratio.
ξ, η	Natural coordinates of isoparametric elements.
ρ	Mass density.
σ_x, σ_y	Inplane stresses along x and y axes respectively.
τ_{xy}, τ_{xz}, τ_{yz}	Shear stresses.
k_x, k_y, k_{xy}	Curvatures of conoidal shell due to load.
ω	Fundamental frequency in radian/sec.
$\overline{\omega}$	Nondimensional fundamental frequency $\left[= \omega a^2\left(\rho / E_{22}h^2\right)^{1/2}\right]$

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