International Journal of Civil Engineering

Geotechnique

Optimum calibration of DSC/HISS constitutive model parameters for rock fill materials

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Received: November 2013, Revised: January 2013, Accepted: June 2014

Abstract

In comparison with other geomaterials, constitutive modeling of rockfill materials and its validation is more complicated. This is principally due to the existence of more intricate phenomena such as particle crushing, as well as laboratory test limitations. These issues have necessitated developing more complex constitutive models including many parameters. In this study, a macroscopic approach called disturbed state concept (DSC) with modified hierarchical single yield surface (HISS) plasticity used to predict the rockfill material behaviour. An innovative method for optimum calibration of sophisticated constitutive model is introduced. Particle swarm optimisation (PSO) was implemented to optimise of rockfill material characteristics based on DSC-HISS constitutive model. Adaptive Neuro Fuzzy inference system (ANFIS) was conducted for completing the residual function in order to alleviate the poor trend of tri-axial test results in $\sigma_d - \varepsilon$ *and* $\sigma_d - \varepsilon_v$ *space. ANFIS was integrated with DSC/HISS result to exploit of substantial residuals. This study revealed that performance of proposed method is higher than empirical method. Two set of large tri-axial tests distinct from rockfill dams in Iran were used to delineate all significant factors affecting the rockfill material behaviour.*

Keywords: *Rockfill material, Disturbed state concept, Hierarchical single yield surface, Particle Swarm Optimization.*

1. Introduction

Crushed rocks are being used ever more extensively in embankment dams and other constructions, due to its significant features such as flexibility, capacity to absorb large seismic energy and adaptability to various foundation conditions, as well as relatively low production cost. However, the behaviour of this material has not been as well investigated as other finer particulate soils due to limitations of experimental apparatus. During the past decades great efforts have been made to study the shear behaviour of rockfill materials experimentally (Varadarajan et al. 2006; Varadarajan et al. 2003). Such investigations are usually carried out in large-scale tri-axial testing apparatus, which have revealed that rockfills exhibit a non-linear stress–strain relationship, stress-dependence of stiffness and a non-linear strength envelope, as well as intense shearing contraction or dilatancy.

An additional distinctive feature of such material which affects its mechanical characteristics is particle crushing during placement and/or shearing. Catering for this feature has separated the constitutive models of crushed rocks from other granular materials and thus the need for concepts such as Disturbed State (Desai et al. 1986). Nonetheless a number of constitutive models have been developed to predict the behaviour of rockfill material (Varadarajan et al. 2003; Xu and Song 2009).

A main objective of constitutive modeling revolved around the stress-strain relationships in order to predict the experimental test result. Constitutive models include a number of parameters that required determining from appropriate laboratory tests. From the point of experimental view, material variability in test specimens, mean pressure, initial density and stress paths were triggered by enhance of study on optimum model parameters. Das and Basudhar (2011) conducted the error-in-variables approach upon the probabilistic global search Lausanne and genetic algorithm to optimise the rock failure criterion parameters.

In view of the importance of the parameters in analysis and design, calibration of constitutive model has significant effect on finite element calculation for geotechnical issues which identifies suitable geologic material parameters to mimic the laboratory test result. In this perspective, the calibration of constitutive model becomes simple for linear elastic material behaviour. One the other hand, for the most engineering materials such as

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soil, manifested as nonlinear hardening-softening behaviour. This type of the behaviour is simulated by sophisticated constitutive models that often include number of considerable parameters. In such circumstances, the numerical modeling of geo-materials is mainly based on computational approximation and empirical knowledge; which inevitably may lead to some in accuracies. Thus, it is a challenging task to calibrate a potentially large number of model parameters while simultaneously satisfying the available experimental data. At this stage, an efficient calibration technique is necessary. The optimisation methods such as simplex reflection approach (Shah and Hoek 1992), gradient method (Li et al. 2000; Mahnken and Stein 1996), quasi-Newton method (Desai 2001; Desai and Chen 2006), genetic algorithm (GA) have been successfully applied for nonlinear stress-strain-time relationship of diatomaceous mudstone (Feng et al. 2002), and Rokonuzzaman and Sakai (2010) used Micro-genetic algorithms (mGAs) for calibration of five parameters hardening-softening constitutive model. In most cases, it consists of a better use of the available experimental data to extract a set of parameters that is the best compromise for the model's response along all the known loading paths. Conventional calibration of the nonlinear model encounter with number of experimental test demands and it is user dependent. The fitness functions used in the literature merely defined to optimise the material properties in $\boldsymbol{\sigma_d - \varepsilon}$ space. However, these methods are problem-dependent and thus may be quite elaborate if the model is highly nonlinear. Evolutionary algorithms form a subset of evolutionary computation inspired by biological evolution mechanisms such as reproduction, mutation, recombination, natural selection and survival of the fittest. In recent years evolutionary computational techniques have widely been applied in engineering problems (Cheng et al. 2007; Sadoghi Yazdi et al. 2012; Zhang et al. 2013; Zhao and Yin 2009) and its applied for optimization in calibration of model parameters; gradient methods, genetic algorithms and particle swarm optimization can very useful. Each method has its own merits and deficiencies. Deterministic algorithms like gradient-based techniques converge faster by using derivative information to identify a good search direction but get stuck in local minima. Also deterministic techniques perform poorly in minimizing functions for which the global minimum is surrounded by flat regions where the gradient is small. Stochastic optimization algorithms like genetic algorithms (GAs) and particle swarm optimization (PSO) algorithms achieve global optimization but require computational effort due to random searches. The PSO algorithm is easy to implement, has few parameters, and has been shown to converge faster than GA, for a wide variety of benchmark optimization problems (Angeline 1998). Even though there are more powerful algorithms, PSO performs better due to smaller numbers of variables to be identified and thus economizes on CPU time (Cheng et al. 2007). More discussion regarding the PSO performance will be discussed later. Disturbed state concept with the HISS model viable to capture rockfill material behaviour discussed follows:

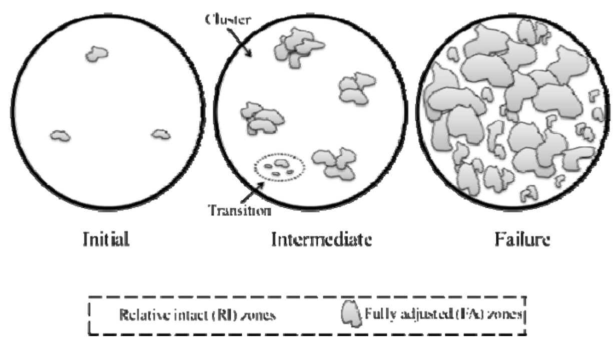
2. Disturbed State Concept with the HISS Model

The Hierarchical Single Surface (HISS) model based on the Disturbed State Concept (DSC) (Desai et al. 1986) has been adopted to characterize the behaviour of rockfill materials in the present study. In this concept, a deforming material element is assumed to be composed of two reference states: relatively intact (RI) and fully adjusted (FA) (Fig. 1). The material is assumed to transform continuously from the relatively intact (RI) state to the fully adjusted (FA) state (Fig. 1. *a*). The observed behaviour is expressed in terms of RI and FA states using the disturbance function (D), which acts as a coupling or interaction mechanism between RI and FA states (Fig. 1. *b*); as:

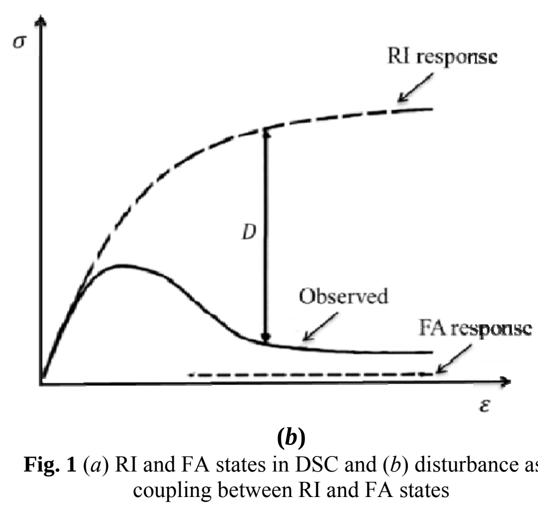
$$d\underline{\sigma}^a = (1-D)d\underline{\sigma}^i + Dd\underline{\sigma}^c + dD\left(\underline{\sigma}^c - \underline{\sigma}^i\right) \tag{1}$$

Where a, i, and c denote observed or actual, RI and FA responses, respectively, and σ is the tensor of stresses.

Brief descriptions of the models for the RI and FA states and the disturbance used in the DSC model are given below.



(***a***)



(***b***)

Fig. 1 (*a*) RI and FA states in DSC and (*b*) disturbance as a coupling between RI and FA states

2.1. Relatively intact (RI) state

The RI response can be represented by using such continuum theories as elasticity, plasticity, and viscoplasticity. Here, the hierarchical single surface (HISS) plasticity model is used, for which the yield function, F, is given by:

$$F = \left(\frac{J_{2D}}{P_a}\right) - \left[-\alpha\left(\frac{J_1 + 3R}{P_a}\right)^n + \gamma\left(\frac{J_1 + 3R}{P_a}\right)^2\right](1 - \beta S_r)^m = 0 \tag{2}$$

Where S_r is $\left(\sqrt{27}\, J_{3D}\right)/\left(2\, J_{2D}{}^{1.5}\right)$, J_{2D} and J_{3D} are the second and third invariants of the deviatoric stress tensor, respectively; J_1 is the first invariant of the total stress tensor; P_a is the atmospheric pressure; n is a phase change parameter used when the volume changes from compaction to dilation or vanishes; R is the reference stress which is proportional to the cohesive strength (that for rockfill materials study is zero); and both parameters γ and β are related to the ultimate condition (yield). The hardening or growth function for plastic yield can be expressed as:

$$\alpha = \frac{a_1}{\xi^{\eta_1}} \tag{3}$$

Where, a_1 and η_1 are hardening parameters, and $\xi = \int\left(d\varepsilon_{ij}^p d\varepsilon_{ij}^p\right)^{1/2}$ is the trajectory of the plastic strains (Desai 2001).

2.2. Fully adjusted (FA) state

The FA response can be characterized by assuming that the micro-cracked material parts, Fig. 2 *a*, can sustain (1) hydrostatic stress but no shear stress like a constrained liquid, or (2) continue to carry shear stress for given initial mean stress (pressure) and deform in shear without change in volume, like a constrained liquid-solid, which represents the critical state (Desai 2001). In this paper, the critical state concept is used to define the behaviour of the material in the FA state. For this purpose, the following two equations are used:

$$\sqrt{J_{2D}^c} = \bar{m} J_1^c \tag{4a}$$

Where $\bar{m}$ and λ are the slopes of the critical state line (CSL) in the $\sqrt{J_{2D}}$ vs. J_1 and e^c vs. $ln(J_1/3P_a)$ diagrams respectively and e_0^c is the value of critical void ratio corresponding to $J_1^c = 3P_a$.

2.3. Disturbance

The disturbance function (D) is expressed in terms of measured stresses, volume change, pore water pressure or nondestructive properties (velocity, attenuation), irreversible or plastic trajectory or dissipated energy (work). In terms of stress, it is defined as $(\sigma_i - \sigma_a)/(\sigma_i - \sigma_c)$ where σ_i, σ_a, and σ_c are the RI, observed, and FA stress values, respectively. To introduce D into the DSC model (Eq. (1)), a mathematical formulation in terms of basic variables, such as accumulated plastic strains or work, is necessary. Thus, in terms of the accumulated deviatoric plastic strains, D is expressed using the Weibull type function (Weibull 1951):

$$D = D_u\left(1 - exp(-A\xi_D^Z)\right) \tag{5}$$

Where D_u: ultimate disturbance, A and Z: disturbance parameters and ξ_D is deviatoric part of plastic strain trajectory (Desai 2001).

3. Particle Swarm Optimization and Adaptive Neuro-Fuzzy Inference System

Kennedy and Eberhart (1995) introduced PSO algorithm for the first time as a new population based optimization technique inspired by animal social behaviour. The initial intent of the particle swarm concept was to graphically simulate the graceful and unpredictable choreography of a bird flock, the aim of discovering patterns that govern the ability of such bird flock to fly synchronously and suddenly change direction with regrouping in an optimal formation. This method is employed in the wide variety of geotechnical issues such as: parameter identification for elasto-plastic modeling of unsaturated soils from pressure meter tests (Zhang et al. 2013), calibration of soil model parameters (Sadoghi Yazdi et al. 2012), simulation-based calibration of geotechnical parameters (Zhang et al. 2009), location of the critical, non-circular failure surface in two-dimensional slope stability analysis (Cheng et al. 2007). Performance of six heuristic global optimization methods in the location of critical slip surface investigated by Cheng et al. (2007). It is shown the PSO performance higher than other algorithms for various slope stability cases. Simulated annealing predominantly need to satisfy some requirements that make the SA converge very slow (Sadati et al. 2009). Owing to fairly performance of PSO, this algorithm was used in this study. The mathematical equation of PSO described in the appendix 1.

Adaptive Neuro-Fuzzy Inference System (ANFIS) model combined the neural network adaptive capabilities and the fuzzy logic qualitative approach, initially introduced by (Jang 1993). Neuro-fuzzy integrates the merits of both neural networks and fuzzy systems in a complementary way to overcome their disadvantage. Real-time processing of instantaneous system input and output data, offline adaptation instead of online system-error minimization, and fast learning time are some advantages of the ANFIS. The use of ANFIS extensively increased such as some applications of ANFIS in geotechnical engineering (Cabalar et al. 2012), reliability analysis of excavation damaged zone (Fattahi et al. 2013), prediction of soil liquefaction (Sadoghi Yazdi et al. 2012). More details of ANFIS architecture can be found in (Sadoghi Yazdi et al. 2012).

4. Methodology

In conventional soil mechanics, model parameters are calibrated based on experimental results appropriate for the type of boundary value problem under examination. However, sometimes the numbers of available experimental results, as well as the type of experiments, are not sufficient for the calibration of the model parameters. Therefore it's important for identification of complex constitutive model parameters. The overall procedure for evaluation of the DSC/HISS constitutive model parameters $\nu, \gamma, \beta, n, a_1, \eta_1, \bar{m}, A, Z, \lambda, e_c^0$, is presented in Fig. 2.

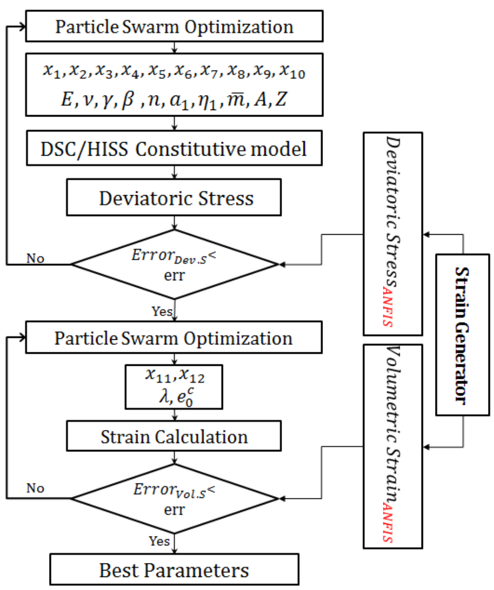


Fig. 2 The algorithm for determination of the DSC/ HISS constitutive model parameters

Initially the following logical limitations are imposed on the model parameters:

- Elasticity parameters: $\upsilon < 0.5$
- Ultimate parameters : $\gamma \geq \frac{J_{2D}}{J_1^2(1-\beta S_r)^{-0.5}}$ and $0 \leq \beta < 0.756$

Value of β is limited by $\beta \leq 0.76$ due to convexity of yield surface.

- Phases change parameters: $n > 2.0$

The value of n should be greater than 2.0 for a convex yield surface and may depend on such factors as initial density; however, as a simplification, an average value of n can be used. For dense granular materials like sands, n may be around 3.0, while for loose granular materials and other materials such as rock and concrete, it would be higher, often of the order of 6 to 10 (Desai 2001).

Also, the critical state parameters $(\bar{m}, \lambda, e_c^0)$ should theoretically be constants for samples of the same material with the same relative density under different confining pressures. However, imposition of such restriction on these parameters would drastically limit the prediction capability of the model. In all previous investigations on the calibration of the model parameters, the values of the critical state parameters are varied for sets of tests on the same material with constant relative density under different confining pressures (Desai and Chen 2006). This may indicate that these parameters are not constant indeed and vary with cell pressure. In other words the Critical State Line in $(\sqrt{J_{2D}} - J_1)$ and $(e - J_1/3P_a)$ spaces may actually be curves. However, this supposition would increase the variable parameters by at least two which would further complicate the calibration process. Therefore, in view of the limited available data, in this study the basic assumptions of the HISS model are reserved.

The "*particles*" for the calculation of deviatoric stress for each axial strain increment are generated randomly and Adaptive Neuro-Fuzzy Inference System (ANFIS) is used to arrive at a non-linear regression of the deviatoric stress per axial strain for each experimental result.

In the next step, the required parameters $(E, \nu, \gamma, \beta, n, a_1, \eta_1, \bar{m}, A, Z)$ to produce such results are randomly set and used in the simultaneous solution of the model equations. The solution procedure has been programmed using MATLAB and is set out in Table 1.

Table 1 The algorithm of DSC/HISS constitutive model

1. *Input data including* $E,\ \nu,\ \gamma,\ \beta,\ n,\ a_1,\ \eta_1,\ \bar{m},\ A,\ Z, \lambda, e_0^c, e_0, err, p_a$*. Here, err is assumed to be 0.0001 and* $p_a = 101.35\ Kpa$
2. *Replace the initial hardening parameter* α*, with the following equation when* $\sigma_x = \sigma_y = \sigma_z$

$$\alpha_0 = \gamma \left(\frac{J_1}{p_a}\right)^{2-n}$$

$$\xi_0 = \xi_{v0} = \left(\frac{a_1}{\alpha_0}\right)^{1/\eta_1}$$

$$\xi_{D0} = 0$$

3. *Input increment of strain* $d\varepsilon$.
4. *Calculate incremental stress* $(d\sigma)$ *based on elastic prediction*$(d\sigma = C^e d\varepsilon)$*. That* C^e *is the elastic material matrix.*
5. *Calculate stress vector* $\sigma^i = \sigma^{i-1} + d\sigma$.
6. *Calculate parameters* J_1^i *and* J_{2D}^i
7. *Determine the yield surface value according to*

$$F = \left(\frac{J_{2D}^i}{p_a^2}\right) - \left(-\alpha\left(\frac{J_1^i}{p_a}\right)^n + \gamma\left(\frac{J_1^i}{p_a}\right)^2\right)(1-\beta S_r)^{-0.5}$$

8. *IF* $(F \leq err)$*, Then go to 11;*

Else

9. *Calculate*

$$o^{i-1} = 1$$

$$d\xi = (o/10000)\sqrt{inv(C^e)^T . inv(C^e)}, \xi^i = \xi^{i-1} + d\xi$$

$$\sigma^i = \sigma^{i-1} + (o/10000), \alpha = \frac{a_1}{\xi^{\eta_1}}$$

$$o^i = o^{i-1} + 1$$

10. *Go to 6*

End IF

11. $D = D_u(1 - exp(-A\xi^Z))$
12. $dD = D_u A * Z * \xi^{Z-1} exp(-A\xi^Z) d\xi$

13. $\eta = J_1^i / \sqrt{J_{2D}^i}$
14. $d\sigma_{ij}^a = [D\overline{m}\eta + (1-D)]d\sigma_{ij}^i + (D - D\overline{m}\eta)\frac{dJ_1^i}{3}\delta_{ij} + [dD(\overline{m}\eta - 1) + D\overline{m}d\eta]S_{ij}^i$
15. $e^c = e_0^c - \lambda . ln(J_1^i / 3p_a)$
16. $d\varepsilon_v^c = -\frac{\lambda}{1+e_0}\frac{dJ_1^i}{J_1^i}$
17. $d\varepsilon_v^a = (1-D)d\varepsilon_v^i + Dd\varepsilon_v^c + dD\left(\frac{e^i - e^c}{1+e_0}\right)$

The solutions will produce errors (residuals) with respect the ANFIS model. Should the amount of the error exceed a pre-determined criterion (i.e. 0.1%) the parameters are re-evaluated using PSO and re-fed into the equations for a new solution. To have an objective comparison of the performance of the models against the experimental results, the fitness function Root Mean Square Error (RMSE) may be used:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{NO}\left(Dev.Stress_{DSC/HISS} - Dev.Stress_{ANFIS}\right)^2}{No}} \qquad (6)$$

That *No* is equal to number of incremental strain. Having obtained the optimized parameters, they are fed into calculation phase of volumetric strain. At this stage the other two parameters of the model λ, e_c^0 are optimized and the error function RMSE for volumetric strain is evaluated and optimized in the same manner as previously described.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{No}\left(Vol.Strain_{DSC/HISS} - Vol.Strain_{ANFIS}\right)^2}{No}} \qquad (7)$$

5. Validation

Two of the conventional compressive drained tri-axial test results used by Chen (1997) in his thesis on Leighton Buzzard sand under different confining pressures and initial void ratio have been used here to compare the performance of the proposed method with the optimized calibration procedure (Desai and Chen 2006) based least-square method. The two tests with given designation of "Group 2" and "Group 4" were carried out under confining pressure of 826.8 kPa and 89.6 kPa respectively. Relative densities and initial void ratios of both groups were %95 and 0.544 respectively (Chen 1997). The former test exhibited contractive behaviour while the latter had maximum dilative behaviour of approximately %5 volumetric strain. The model parameters evaluated using the proposed method are presented alongside the evaluations of the optimized calibration procedure (Desai and Chen 2006) using the least-square method in Table 2. The performances of the two methods are also shown in Fig.3. It is evident that both methods have performed well in predicting the variation of deviatoric stress with axial strain. However, the proposed method has clearly preformed much better in prediction of the variation of volumetric strain versus axial strain for "Group 4" with almost equal accuracy in "Group 2". In order to gain a quantitative assessment of the performances of the two methods, the RMSE and normalized RMSE (obtained by dividing RMSE value by the maximum value of each test) values of the two predictions are evaluated and presented in Table 3.

Table 2 Material parameters for Group 2 and 4

Material parameters		**Group 4, $\sigma_C = 89.6\ KPa$**		**Group 2, $\sigma_C = 826.8\ KPa$**	
		By least-squares	**By PSO**	**By least-squares**	**By PSO**
Elasticity	E	61.26	11.45	117.31	200.27
		0.35	0.42	0.43	0.33
Ultimate		0.08	0.09	0.08	0.072
		0.72	0.61	0.54	0.60
Phase change		2.68	3.31	2.40	2.56
Hardening		3 E -3	6 E -4	2 E -3	5 E -4
		0.31	0.47	0.31	0.41
Disturbance		17.89	13.85	1.27	10.35
		1.32	0.98	0.66	1.04
Critical state		0.29	0.16	0.11	0.03
		0.07	0.42	0.14	0.04
		0.12	0.48	0.40	0.84

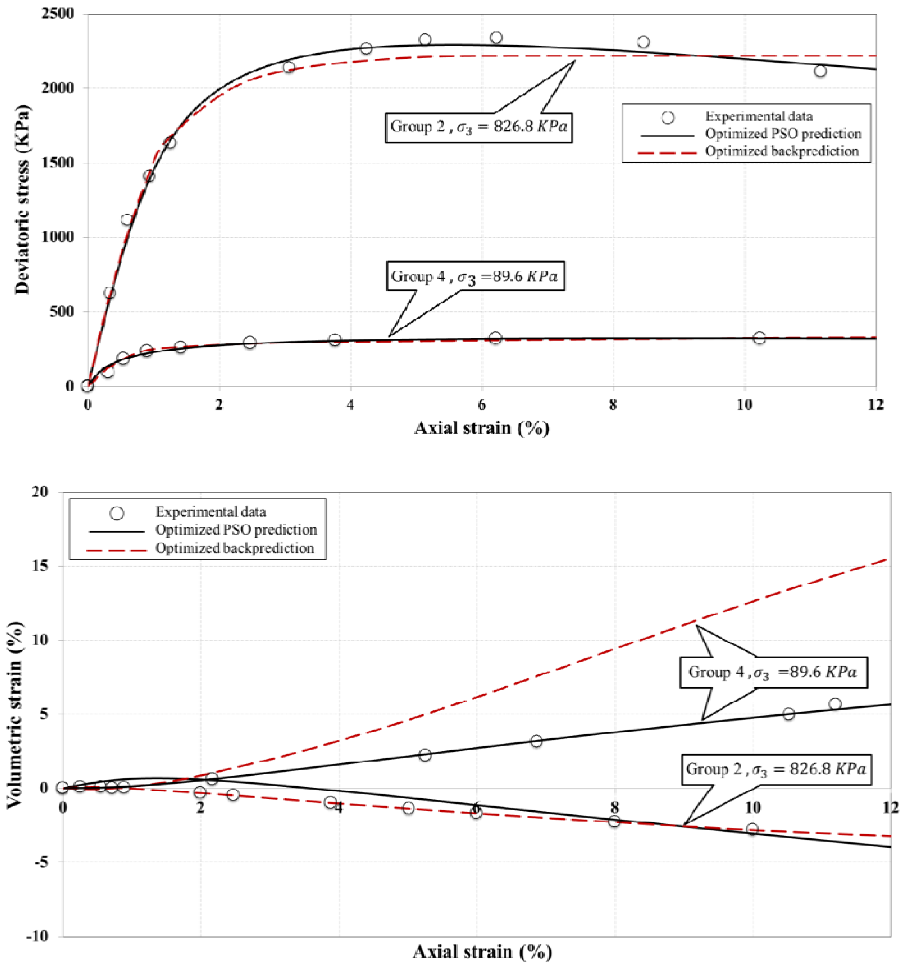


Fig. 3 Compression between least-squares and PSO prediction

Table 3 RMSE and Normalized RMSE value of two groups

Material name	Optimized by:	RMSE $_{\text{Dev. Stress}}$	Normalized RMSE $_{\text{Dev. Stress}}$	RMSE $_{\text{Vol. Strain}}$	Normalized RMSE $_{\text{Vol. Strain}}$
Group 4 $\sigma_C = 89.6\ KPa$	Least-squares	134.933	0.418	11.183	0.338
	PSO	14.966	0.046	0.151	0.0045
Group 2 $\sigma_C = 826.8\ KPa$	Least-squares	88.556	0.037	0.034	0.008
	PSO	40.851	0.017	0.644	0.155

5.1. Calibration for Rockfill Materials

Two large-scale tri-axial test results that were carried out for the Ministry of Energy in the Geotechnical Engineering Department of The Building and Housing Research Center (BHRC) (Rayhani et al. 2010) under different confining pressures are used here to calibrate HISS model. The materials used in the tests consisted of rounded and angular particles that were collected from quarries or borrow areas of rockfill dam projects in the Azarbaijan Province in the North West of Iran. Some of the rockfill material specifications are given in Tables 4 and more details may found in (Rayhani et al. 2010). Actual rockfill materials contained large size particles, modeled rockfill materials with maximum size of particles 50 mm, were prepared using parallel gradation technique for tri-axial testing. The grain size distribution curves of the modeled rockfill materials are shown in Fig. 4. Test Specimen with dimensions of 300 mm diameter by 813 mm long was used for testing.

Table 4 Properties of Rockfill Materials

Material from	Particle shape	Name of the rock	Los Angeles abrasion (%)	D_{50} (mm)	C_c	C_u
Aydoghmoosh dam	Angular	Andesite	19.0	7.1	2.9	25
Sahand dam	Rounded	Diorite and Dacite	46.2	2.4	0.4	14.3

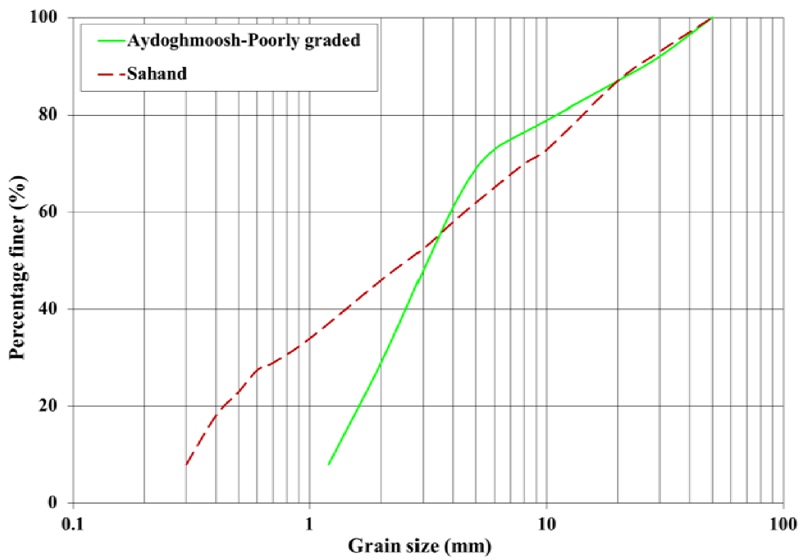
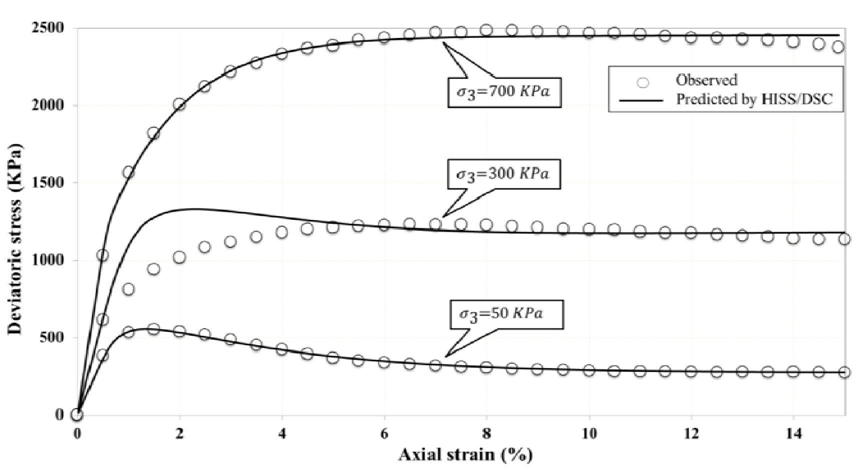


Fig. 4 Grain size distribution modeled rockfill materials

The tri-axial test results are also presented in the Fig 5 and 6 alongside the PSO predictions. The optimised parameters that produce the said predictions are presented in Table 5. In order to evaluate the overall trend of variations of the parameters, PSO was used only for predictions of two of the test results (minimum and maximum confining pressures) and the parameters of the other (mid confining pressure) test results were evaluated by normalized mean values of the parameters of the two predicted values. Obviously an independent PSO parameter evaluation would lead to a more accurate estimate, but by this approach a logical trend of the variation of the parameters with respect to confining pressures can be tracked. The more tangible parameters such as the modulus of elasticity follow an expected pattern of variation (i.e. increase with confining pressure) while some of the other parameters such as phase change or disturbance parameters do not vary much. The RMSE values of prediction is also presented in table as a quantitative measure of the accuracy of the procedure the achieved improvement of the proposed calibration technique the RMSE and normalized RMSE values are tabulated in Table 6.



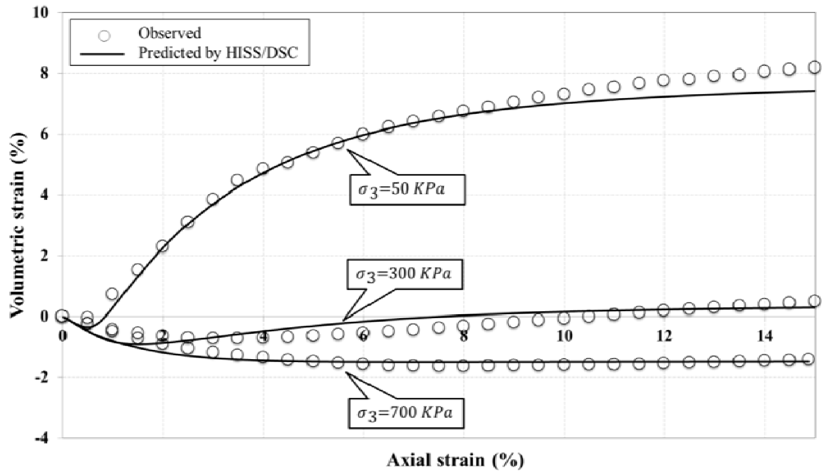


Fig. 5 Prediction of stress-strain-volume change response of Aydoghmoosh-Uniform rockfill materials

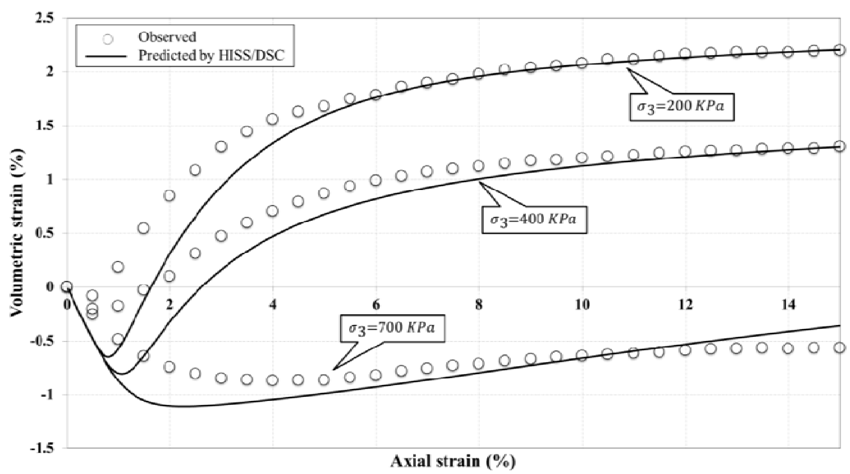


Fig. 6 Prediction of stress-strain-volume change response of Sahand rockfill materials

Table 5 Material parameters for the rockfill materials

Material Constants		Elasticity		Ultimate		Phase change	Hardening		Disturbance		Critical state		
Material name	$\sigma_{C\,(KPa)}$	$E_{(MPa)}$	ν	γ	β	n	a_1	η_1	A	Z	$\bar{m}$	λ	e_0^c
Aydoghmoosh Dam Materials	**50**	77.43	0.25	0.13	0.70	3.22	8 E-5	0.49	54.41	0.99	0.01	0.20	0.38
	300	131.61	0.28	0.09	0.63	3.07	6 E-5	0.47	130	1.20	0.21	0.16	0.39
	700	218.23	0.33	0.03	0.62	2.87	1 E-5	0.43	169.81	1.07	0.50	0.16	0.39
Sahand Dam Materials	**200**	136.83	0.32	0.09	0.75	3.19	4 E-5	0.37	125	1.08	0.12	0.24	0.52
	400	159.74	0.33	0.10	0.62	3.27	4 E-5	0.36	100	1.03	0.13	0.21	0.57
	700	194	0.38	0.10	0.73	3.45	3 E-5	0.30	100	1.10	0.14	0.17	0.55

Table 6 RMSE value of rockfill materials under different confining pressure

Material name	$\sigma_{C\,(KPa)}$	RMSE $_{Dev.\ Stress}$	Normalized RMSE $_{Dev.\ Stress}$	RMSE $_{Vol.\ Strain}$	Normalized RMSE $_{Vol.\ Strain}$
Aydoghmoosh Dam Materials	**50**	7.931	0.014	0.376	0.0459
	300	151.631	0.123	0.262	0.3727
	700	31.191	0.013	0.164	0.0998
Sahand Dam Materials	**200**	24.845	0.021	0.299	0.1362
	400	81.354	0.047	0.272	0.2088
	700	203.293	0.081	0.312	0.3606

It may be noted from both the figures and the RMSE values that the predictions are relatively accurate.

6. Discussion

The following points may be deduced from the above results:

- Despite complexities involved in the determination of parameters of highly non-linear models such as HISS, the proposed technique has proved to be quite fast and robust and insensitive to local minima in searching for globally optimized parameters.
- Comparison between the predictions of the least-squares method and the PSO technique shows marked improvement especially in the variation of the volumetric strain prediction.
- Previous attempts at calibration of the HISS model parameters either obtained pressure-independent values for the modulus of elasticity (Chen 1997) or used empirical correlations to determine the moduli of elasticity (Varadarajan et al. 2006; Varadarajan et al. 2003), whereas the proposed method has managed to determine the expected variation of the modulus of elasticity with confining pressure without any pre-determined supposition.
- Some of the HISS model parameters (including the critical state parameters) depend and vary with the confining pressure. It has been shown here that relatively accurate interpolated values for the model parameters may also be determined.

7. Conclusion

Calibration of constitutive models requires close examination of extensive experimental data. Even then, some of the more intricate and intangible parameters deployed in the more advanced and complicated models cannot simply be correlated to any experimentally measured properties and often simplifying assumptions have to be made. The goal of model calibration is to find appropriate parameters which yield the best model response in relation to the available experimental results. It has been shown here that a Neuro-Fuzzy model in conjunction with Particle Swarm Optimization (PSO) can effectively be used for calibration of the twelve parameters of Hierarchical Single Surface (HISS) constitutive model based on the Disturbed State Concept (DSC). Furthermore, the suppleness of the proposed technique provides the bases for more detailed examination of the behaviour of the model in response to parameter variations. The technique described in this paper has proven its capabilities as an identification procedure in many fields, including geomechanics. However, its versatility in calibration of model parameters as well as peripheral variables from the very basic and minimal experimental data can be viewed as a potent tool for developments of constitutive models. It is believed that the efficiency and the achieved improvement gained by this method in comparison with other methods of model calibration shall direct the future development in this field.

Appendix 1: particle swarm optimisation

The original PSO formulae defined for particle number i as a potential solution to a problem in M-dimensional space. Regarding the minimum problem, suppose $f(x)$ is the fitness function, the best position of particle i can be computed as follows:

$$P_i(t+1) = \begin{cases} P_i(t) & f\big(x_i(t+1)\big) \geq f\big(P_i(t)\big) \\ X_i(t+1) & f\big(x_i(t+1)\big) < f\big(P_i(t)\big) \end{cases}$$

where, $X_i = (x_{i1}, x_{i2}, \dots, x_{iM})$ is particle position; $P_i = (p_{i1}, p_{i2}, \dots, p_{iM})$ is memory of the best previous position; $V_i = (v_{i1}, v_{i2}, \dots, v_{iM})$ is velocity along each dimension. Then the following equation presents the evolutionary process:

$$v_i(t+1) = \omega v_i(t) + c_1 r_1(t)\big[P_{ij}(t) - x_i(t)\big] + c_2 r_2(t)\big[P_{gb}(t) - x_i(t)\big]$$
$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1)$$

where, P_{gb} represents the best position among all particles in the population; r_1 and r_2 are random numbers between (0, 1), and c_1 and c_2 are acceleration constants that control how far a particle moves in a single generation. w is the inertia weight calculated with $w = 0.5 + {rand}/{2}$ that controls the impact of previous velocity of particle on its current one (Yuhui and Eberhart 1998). $rand$ is a randomly generated number between zero and one.

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