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Spatial development of urban road network traffic gridlock

H.S. Qi[1], Y.Y[1], Dian Hai Wang[1,]*, Y.M. Bie[2]

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Abstract

Abstract: Gridlock is an extreme traffic state where vehicle cannot move at all. This research studies the development of gridlock by theoretical and numerical analysis. It is shown that the development of gridlock can be divided into several stages. The core of the development is the evolution of congestion loop. A congestion loop is comprised of a number of consecutively connected spillover links. The evolution of a congestion loop always tends to be stable, i.e. the state of all related links tends to be identical.. Under the stable condition, traffic states of all links are identical. A novel concept, "virtual signal" is proposed to describe the queue propagation and spillover during the stabilization. Simulation results show that congestion propagates in an accelerated way. The prevention of the first congestion loop is crucial. The achieved results have potential use for future network traffic control design and field applications.

Keywords: *Virtual signal; Signal cooperation; Traffic congestion; Traffic control.*

1. Introduction

Gridlock is an extreme traffic state under. Under gridlock, no vehicle can move forward. In urban networks, gridlock is often characterized by some consecutively connected congestion links. In most cases it consists of some loops. The mitigation of the congestion of each link within the gridlock depends on the downstream link within the same gridlock. Hence at last traffic congestion of any link cannot be cleared quickly.

The formation of gridlock is influenced by various factors, including network topology, signal parameters, flow structure etc. The development of a gridlock involves the evolution of the traffic state of each link within the gridlock. Thus it is very important to examine the link traffic dynamics in the context of network traffic flow framework.

Wright and Roberg (1998) provided the first insight of this topic. Their research was simulation-based, with an analytical link model. In the link model, ahead flow and turning flow were constant. Propagation time of a downstream obstruction was the function of the flow rate. According to the relationship between through flow queue propagation time and turning flow queue propagation time, spatial structure of the jam was outlined. Their simulation method was followed by many researchers, with different objectives.

* *Corresponding author: wangdianhai@zju.edu.cn*

1 College of Civil Engineering, Zhejiang University, Hangzhou, China

2 School of Transportation Science and Engineering, Harbin Institute of Technology, Harbin, China

Roberg and Abbess (1998) applied a simulation model to investigate the diagnosis and treatment of traffic jams. Long (2008) explored the formation of traffic jam, focusing on the influence of stop line assignment and channelized section length. Roberg-Orenstein et al. (2007) developed several alternative strategies for protecting networks from gridlock and dissipating traffic jams once they had formed. The treatment focused on the installation of bans at specific network locations.

Different from above researches, Daganzo (2007) argued that the modeling and control of the gridlock should be established in an aggregate level, i.e. traffic dynamics should be analyzed through aggregate parameters. In his research, the relationship between aggregate vehicle number and system outflow was assumed to be unimodal. If the initial accumulation was on the decreasing part of this relationship, then accumulation increased at a rate that itself grew with accumulation, i.e. gridlock took place. However, the relationships between these aggregate level traffic parameters are not clear yet. Besides, the two assumptions, i.e. slowly evolving demand and evenly distributed traffic state also should be reexamined especially during peak hour.

Gridlock consists of a series of spillover links. The interaction between adjacent link queues plays an important role in gridlock development. Kerner (2011) studied the gridlock formation in a signal controlled link. He showed that even under small inflow, when no traffic gridlock should be expected, spontaneous traffic breakdown with subsequent city gridlock took place with some probability. Different from Kerner's approach, we adopt a deterministic way which describes the queue formation and propagation based on traffic wave theory. We focus our research on the interaction between links,

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rather than within the link. We hope that future empirical studies can solve the problem: which of the two approaches is more associated with traffic gridlock in city network.

There may exist many isolated gridlocks simultaneously and the mutual effect of such gridlocks also contributes to the dynamics of network traffic jam. None of these literatures above provide useful information about this topic. One reason maybe the difficulty in spillover modeling (Chen, Shi et al. 2009). The spillover event is the key for the development of gridlock. It consumes the capacity of upstream intersection (Kim and Cassidy 2012). The imbalance between the demand and supply of upstream links is then enhanced. The resulting queue propagation is accelerated. This paper realizes the spillover effect modeling with the concept of "speed of virtual signal", which is determined by traffic demand (upstream arrival) and supply (link capacity, which depends on signal split).

Another reason may be the jam evolution complexity. The jam evolution is influenced by various parameters (Jin, Huang et al. 2011). Pure analytical results can hardly be obtained. Current researches on one hand simplify the modeled system (Corthout, Flötteröd et al. 2012); on the other hand, simulation approach is applied. By the combination of analytical and simulation approaches, some interesting results about gridlock development are found. All of these are realized through the concept of "virtual signal". The core idea was that queue evolution can be interpreted as the propagation of virtual red signal. When the queue occupies a specific location, the virtual signal of this location shows red, otherwise green signal lasts. Spillover can be modeled by the same way. By applying the idea to the network level, we show that the mechanism of the network traffic jam evolution presents a more complex nature.

Section 2 lists all related notations. Section 3 provides the link traffic model, which is based on traffic wave theory. Spillover modeling is also presented. Definition and description of relationships between congestion loops are given in section 4. Section 5 derives the stability condition both for single congestion loop and congestion loop groups. Based on the analysis of congestion loop, section 6 outlines the steps of the formation of network traffic jam. Considering the analytical difficulties, Section 7 explores the gridlock development when link inflow is variable based on numerical simulation. Section 8 concludes with some remarks.

2. Notations

q_m - capacity of each lane, m is short for "maximal";

k_j - jam density;

w - backward wave speed;

v_f - free flow speed;

$l_{\max}$ - maximal queue length within a cycle;

$G=(N,A)$ - the whole network, N is node set and A is link set;

G_{CL} - sub-graph comprised by links that belong to a congestion loop;

$\bar{G}_{CL}$ - complementary set of G_{CL};

G_{FC} - sub-graph comprised by spillover links;

$\bar{G}_{FC}$ - complementary set of G_{FC};

$CL_k=\{I^1_{CL_k}, I^2_{CL_k}, I^3_{CL_k}, ...\}$ - congestion loop k expressed as a series of consecutive intersections;

$I^m_{CL_k}$ - the *m*-th node at congestion loop *k*;

$L^m_{CL_k}$ - the *m*-th link at congestion loop *k*;

$\lambda_{L^i_{CL_k}}$ - split of the *i*-th link at congestion loop *k*;

$I^{m+1}_{CL_k}$ - upstream node of node $I^m_{CL_k}$;

$L^{m+1}_{CL_k}$ - upstream link of link $L^m_{CL_k}$;

$CL_k=\{L^1_{CL_k}, L^2_{CL_k}, L^3_{CL_k}, ...\}$ - congestion loop k expressed as a series of links;

$R_{I^i_{CL_k} I^j_{CL_m}}=\{R^t_{I^i_{CL_k} I^j_{CL_m}}\}$ - all routes from node $I^i_{CL_m}$ to node $I^j_{CL_m}$;

λ_{CL_k} - stable split of congestion loop *k*;

$N^I_{CL_k}$ - intersections number in CL_k;

$N^L_{CL_k}$ - links number in CL_k, $N^L_{CL_k}=N^I_{CL_k}$;

s_a - length of link a;

$s_{L^3_{CL_k}}$ - length of link $L^3_{CL_k}$;

$s^t_{I^i_{CL_k} I^j_{CL_m}}$ - route length for *t*-th route $R^t_{I^i_{CL_k} I^j_{CL_m}}$ from node $I^i_{CL_m}$ to node $I^j_{CL_m}$;

C - signal cycle length;

3. Link Traffic Model

3.1 Critical condition for oversaturation queue

Consider a one-lane link controlled by signal without endogenous flow. The length is L. Inflow rate is q_i and outflow rate is q_o. Red time and green time are r and g respectively. Assume that the fundamental diagram is of triangular shape. Arriving flow is q and the corresponding density is k. Fig. 1**Error! Reference source not found.**(a) presents the formation and dispersion of vehicle queue behind the stop line under signal control. Green time is g and red time is r. At first (the beginning of red time), stopping wave (line OB in Fig. 1**Error! Reference source not found.**(a), which represents the queue tail) propagates upstream with the velocity u_0 and a queue develops. When the signal switches to green, a starting wave (line

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AB in Fig. 1**Error! Reference source not found.**(a)) appears and propagates upstream either with a greater speed (denoted as u_1). After time t', the starting wave catches up with the stopping wave and the queue has dispersed. A new wave (denoted by its speed, u_2) comes into being. It takes the time t'' for the wave u_2 to propagate over the stop line. If the effective green time g is equal to or greater than $t'+t''$, traffic state will reproduce cycle by cycle. Otherwise, the wave propagation profile will vary from cycle to cycle (see Fig. 1(b)). During the second cycle, stopping wave firstly spreads with speed u_1 which makes the queue tail of this cycle further from stop line than the former cycle. As time elapses, the queue tail becomes further and further. Link traffic state is unstable.

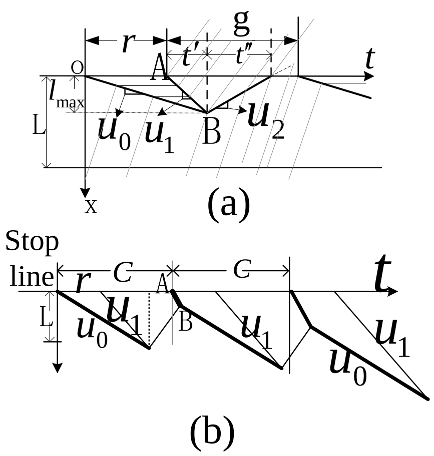

Fig. 1 Traffic wave propagation (a) unsaturated condition; (b) oversaturated condition

Based on the analysis above, some formulas can be formulated:

$$u_1=\frac{q_m}{k_j-k_m}=w;u_0=\frac{q}{k_j-k};u_2=\frac{q_m-q}{k_m-k}=v_f \tag{1}$$

$$u_0(r+t')=u_1t'\Rightarrow t'=\frac{u_0r}{u_1-u_0} \tag{2}$$

$$l_{\max}=u_1t'=\frac{u_1u_0r}{u_1-u_0} \tag{3}$$

$$t''=\frac{L_{\max}}{u_2} \tag{4}$$

$g=t'+t''$ is the critical condition for oversaturation. This equation determines a specific signal split λ, which is defined as the division of cycle length by green time. The decrease of λ will result in oversaturation.

3.2. Oversaturation and spillover modeling

When critical condition is not satisfied, vehicle queue will grow as shown in Fig. 2(a). Each green signal shortens the queue length while each red signal prolongs queue length. The length shortened or prolonged is proportional to the time duration, because wave speeds are fixed.

Suppose h_{g_i} and h_{r_i} are queue length increment generated by green signal and red signal. According to the geometric relationship in Fig. 2(b) and (c), they can be derived as follows:

$$h_{r_i}=\frac{r_iu_0u_1}{u_1-u_0} \tag{5}$$

$$h_{g_i}=\frac{g_iu_2u_1}{u_1+u_2} \tag{6}$$

Stopping wave speed u_0 is the function of upstream arrival. Under unstable condition, queue evolution is determined by link signal parameters and the inflow.

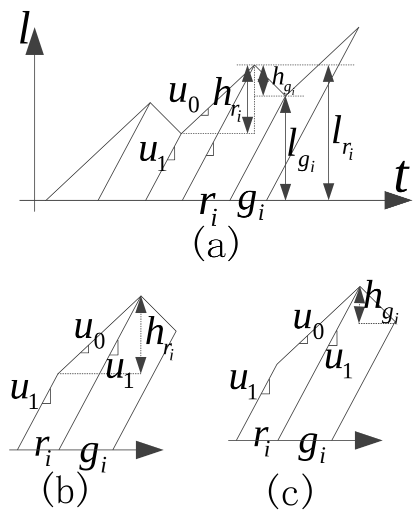

Fig. 2 Stopping wave and starting wave with insufficient split

Vehicle queue evolution directly reflects the traffic jam development. A novel concept, “virtual signal” is proposed to describe the queue dynamics. The core assumption suggests that there are (virtual) traffic signals everywhere within the link. For a specific position, if it has not been occupied by vehicle queue, this virtual signal shows green, otherwise red signal governs the position. The propagation of vehicle queue is equivalent to the changes of virtual signal both in spatial and temporal dimension. Fig. 3 presents the idea in a link without initial queue. Two locations are chosen for description. During the first cycle, queue tail already spreads over location 2, thus the virtual signal attached at location 2 shows red. But virtual signal of location 1 always displays green in the first cycle. At the second cycle, the red time of the virtual signal at location 2 increases, and the virtual red signal develops at location 1 either. Thus the variation of virtual signal relates to the queue propagation, which depends on real

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signal parameters and arriving flow.

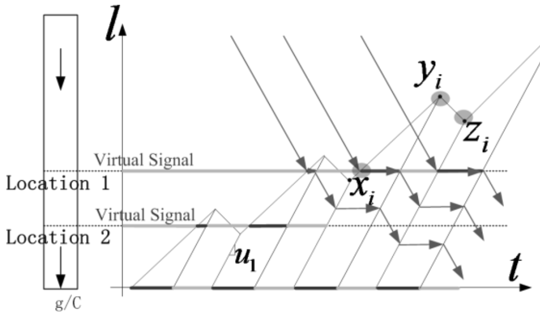

Fig. 3 Method of virtual signal

Fig. 4 presents the wave propagation within a typical cycle i. Three points of the queue tail, namely x_i, y_i, z_i are labeled. For any location within x_i, virtual signal parameters are identical to that at stop line; for any location l within the interval [x_i, y_i], virtual red time is computed by:

$$r_v = r_i \frac{y_i - l}{y_i - x_i} \tag{7}$$

For any location beyond y_i, virtual red time is zero. Hence the three points represent the traffic state along the link. They are defined as **characteristic points**, and can be recorded iteratively:

$$\begin{cases} x_i = z_{i-1} \\ y_i = x_i + h_{r_i} \\ z_i = y_i - h_{g_{i-1}} \end{cases} \tag{8}$$

The velocity of queue tail propagation is called Speed of Virtual Signal (SVS). SVS takes both stopping wave and starting wave into account. During a cycle, the queue length increment:

$$\Delta l_i = h_{r_i} - h_{g_{i-1}} \tag{9}$$

Therefore:

$$SVS = \frac{\Delta l_i}{C} = \frac{\frac{r_i u_0 u_1}{u_1 - u_0} - \frac{g_i u_2 u_1}{u_1 + u_2}}{C} = \frac{(1-\lambda)u_0 u_1}{u_1 - u_0} - \frac{\lambda u_2 u_1}{u_1 + u_2} \tag{10}$$

$$SVS = \frac{u_0 u_1}{u_1 - u_0} - \lambda[\frac{u_2 u_1}{u_1 + u_2} + \frac{u_0 u_1}{u_1 - u_0}] = f(q, \lambda) \tag{11}$$

It can be seen that SVS is a linear function of signal split λ. Queue tail equation of Eq. (8) is:

$$\begin{cases} x_i = x_{i-1} + SVS \times C \\ y_i = y_{i-1} + SVS \times C \\ z_i = z_{i-1} + SVS \times C \end{cases} \tag{12}$$

In reality, cycle inflow may vary, which leads to the generalized formula:

$$SVS(i) = \frac{u_0(i) u_1}{u_1 - u_0(i)} - \lambda[\frac{u_2(i) u_1}{u_1 + u_2(i)} + \frac{u_0(i) u_1}{u_1 - u_0(i)}] \tag{13}$$

where $u_0(i)$ is the stopping wave speed at cycle i. $SVS(i)$ is the speed of virtual signal at cycle i.

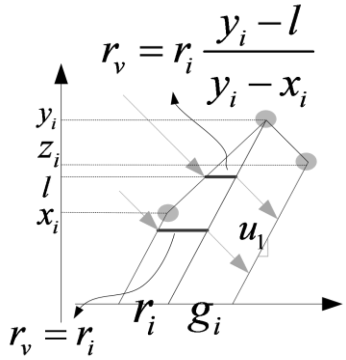

Fig. 4 Virtual Split Calculation

When the queue tail reaches upstream intersection, spillover event takes place. Even if a movement gets RoW (right–of-way), it still cannot traverse the intersection. Three types of signals at the intersection: virtual signal, displayed signal and operational signal. The ultimate flow dynamics is determined by the operational signal. The operational signal itself depends on the former two types of signals. If both signals show green, the operational signal displays green, otherwise red governs. In other words, operational signal is the result of the cooperation between virtual signal and displayed signal. Consider two traffic signals. g1, r1, o1 is the green time, red time and offset for signal 1. For simplicity we set o1 to zero. Similarly we have g2, r2 and o2. C is the cycle length. According to the relationship between o2, g1 and the relationship between o2+g2 and C, six conditions are enumerated, which are shown in Fig. 5.

The cooperation of multiple signals, i.e. multiple spillovers, can be derived by the same way.

Since the cooperation of two signals is not green when either of them is red, the overall split must be smaller than both original splits. Thus the cooperation always decrease virtual split. The decrease is called split lost. Different offset will result in different split lost, which then influence the dynamics of traffic jam.

Traffic dynamics of spillover link is very simple. The traffic wave speed always equals u_1. And the trajectory of each vehicle is similar to that of the front one, but with a temporal and spatial shift. Virtual signal of all locations are the same within the cycle, which means that traffic state of the link can be represented by a single split. This feature is the start point of following research.

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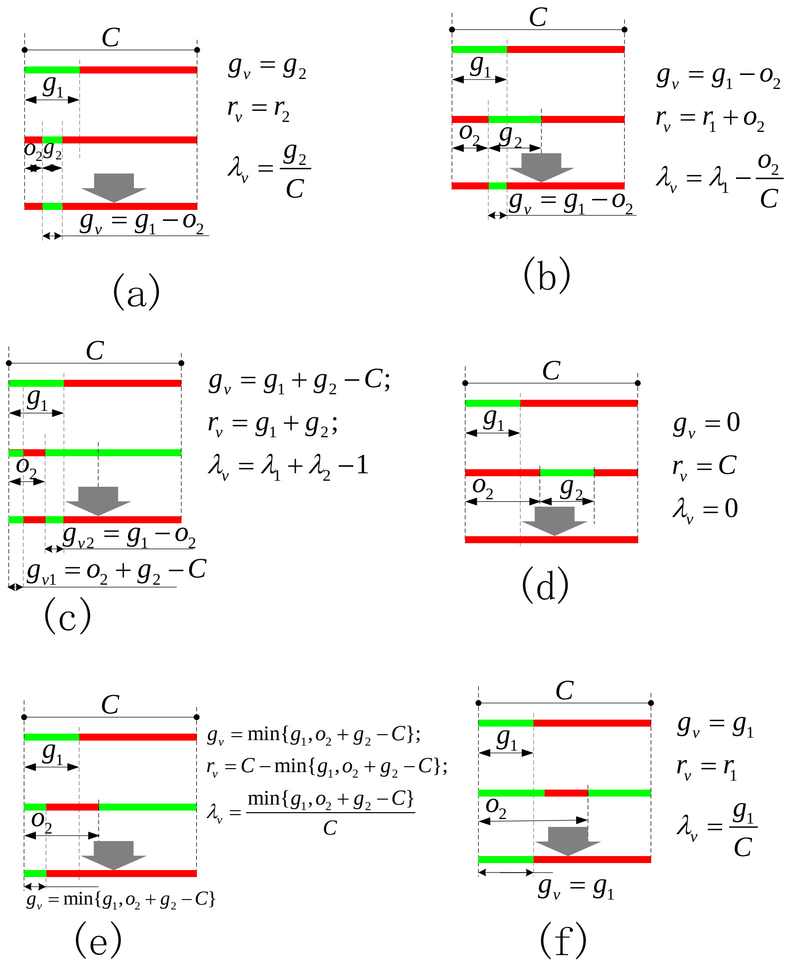

Fig. 5 Virtual signal cooperation

(a) Condition 1; (b) Condition 2; (c) Condition 3; (d) Condition 4;(e) Condition 5; (f) Condition 6;

4. Definition of Congestion Loop

During peak hours, vehicle queue will propagate around the network. Some spillover links connect consecutively, which makes up a loop. A congestion loop is defined as a series of interconnected links with the same direction, i.e., all are clockwise or all are counterclockwise. All links must be in spillover condition and hence perform as the virtual signal on upstream links.

4.1. Measuring congestion loop size

Two congestion loop measurements are established:

M1: Links number of a congestion loop, $M1 = N^L_{CL_k} = N^I_{CL_k}$. A greater M1 means there are more spillover links within a congestion loop. These links may perform as the seeds for new congestion loops.

M2: Average virtual split of all links within the congestion loop, $M2 = \frac{\sum_i \lambda_{L^i_{CL_k}}}{N^I_{CL_k}}$; from link traffic dynamics we know that virtual split can express the congestion degree of a spillover link. Similarly M2 also measures the congestion level of a loop. From the stability analysis below, we can see that virtual split of all links within the same congestion loop tend to be the same.

4.2. Relationship between congestion loops

Generally there are four relationships between congestion loops in time and space: link-sharing, node-sharing, mixed and reproductive.

Link-sharing congestion loops

A link may belong to many congestion loops. These congestion loops are of link-sharing relationship. Suppose congestion loop k and congestion loop m are link-sharing congestion loops. The shared links set is $CL^L_k \cap CL^L_m$. Shared links number is $N^L_{GL^L_k \cap GL^L_m}$. When all links are shared, these congestion loops are considered to be identical just as in Fig. 6(a).

Node-sharing congestion loops

When two congestion loops share some common nodes, the relationship is of node-sharing type. In fact, link-sharing congestion loops are node-sharing at the same time. They share the nodes that are directly connected with shared links. However, the opposite may not be true. Similar to the link-sharing congestion loops, we have shared nodes set $CL^I_k \bigcap CL^I_m$ and the number of shared nodes $N^I_{CL^I_k \bigcap CL^I_m}$.

Mixed type congestion loops

Mixed type congestion loops, just as its name implies, are congestion loops that share some nodes as well as some links. An example is shown in Fig. 6(c). Congestion loop 1 and congestion loop 2 have four nodes and two links in common.

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The formation of a congestion loop can trigger the development of other congestion loops. Congestion loop produced by existing congestion loop is called child congestion loop. The original one is called parent congestion loop. It can be seen that the relationship between the parent congestion loop and child congestion loop must be one of the three above relationships. Mixed type is an extension of the link-sharing and node-sharing relationship. Thus we only discuss the former two relationships.

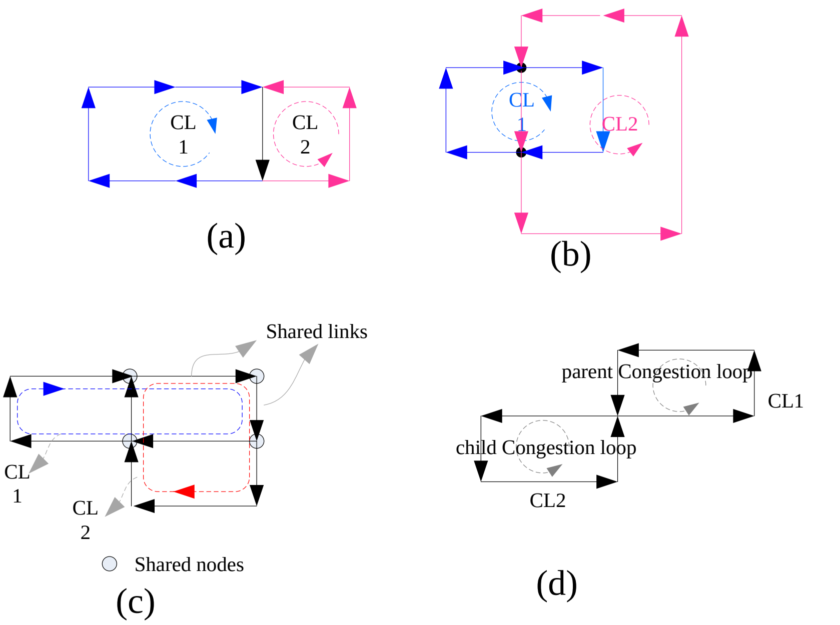

Fig. 6 Types of loops relationship, (a): link-sharing; (b): node-sharing(c):mixed type; (d): reproductive

5. Stability of Congestion Loops

5.1. Stability of a single congestion loop

Suppose a congestion loop is comprised by a series of links, $L^1_{CL_k}, L^2_{CL_k}, L^3_{CL_k}, \ldots\ldots$ Define $L^{i+1}_{CL_k}$ as the upstream link of link $L^i_{CL_k}$. According to the link traffic dynamics, virtual split of link $L^{i+1}_{CL_k}$, i.e. $\lambda_{L^{i+1}_{CL_k}}$, is the cooperation between virtual split $\lambda_{L^i_{CL_k}}$ and that of some other links. Thus $\lambda_{L^{i+1}_{CL_k}} \le \lambda_{L^i_{CL_k}}$. Furthermore, According to the loop shape, it can be concluded that:

$$\lambda_{L^{i+1}_{CL_k}} \le \lambda_{L^{i}_{CL_k}} \le \lambda_{L^{i-1}_{CL_k}} \le \lambda_{L^{i-2}_{CL_k}} \ldots\ldots \le \lambda_{L^{i-N^L_{CL_k}}_{CL_k}} \quad (14)$$

While $\lambda_{L^{i-N^L_{CL_k}}_{CL_k}} \equiv \lambda_{L^i_{CL_k}}$, hence the necessary condition for the truth of Eq.**Error! Reference source not found.** is:

$$\lambda_{L^{i+1}_{CL_k}} = \lambda_{L^{i}_{CL_k}} = \lambda_{L^{i-1}_{CL_k}} = \lambda_{L^{i-2}_{CL_k}} \ldots\ldots = \lambda_{L^{i-N^L_{CL_k}}_{CL_k}} \quad (15)$$

Eq. (15) is the stability principle of a single congestion loop. The physical meaning is that traffic states of all links tend to converge to the same split. Virtual split of all links may not be the same at the beginning. The congestion loop will evolve gradually. Since each link is influenced directly by its downstream link, once there is split difference between any pair of adjacent links, i.e. traffic sate is different, virtual split will propagate upstream. Since the wave speed under spillover state, u_1 is constant, it will take the time s_A/u_1 for the virtual split of link B to propagate over link A. s_A is the length of link A. The propagation speed of virtual split within a congestion loop is independent on the split itself but only determined by the physical length of the congestion loop $\sum_i s_{L^i_{CL_k}}$. The stability time then is $\sum_i s_{L^i_{CL_k}} \Big/ u_1$ if the virtual split remains unchanged during the propagation. If the virtual split becomes smaller during the propagation, the process will

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restart. The propagation will end until the virtual splits within the congestion loop are the same. If there is an inner or outer disturbance of virtual split such as an incident, a new stabilization process will begin and change the traffic state of all links in a cyclical way. The stable virtual split is always smaller than or equal to the original minimal split within the congestion loop. Under the worst condition where the virtual split is zero, vehicles within the same congestion loop cannot move at all. Gridlock ultimately forms.

5.2. Stability of a congestion loop group

Congestion loop group is defined as a series of interconnected congestion loops. Each congestion loop must be in one of the three types of relationships referred to above with other congestion loops. With respect to link-sharing congestion loops, since the stabilization process of a congestion loop involves all the links, the stable virtual split of all related congestion loops must be the same, i.e. under stable condition, traffic states of all links within a link-sharing congestion loop group are identical.

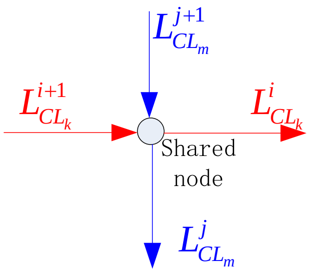

Fig. 7 Node-sharing congestion loops link and shared node

As for node-sharing congestion loops, first consider the common node as shown in Fig. 7. Two congestion loops share the same node. The links in congestion loop k is $L^i_{CL_k}$ and $L^{i+1}_{CL_k}$. Similarly we have $L^j_{CL_m}$ and $L^{j+1}_{CL_m}$ in congestion loop m. Due to the spatial relationship, we have Eq. (16) and Eq. (17):

$$\lambda_{L^{i+1}_{CL_k}} \le \lambda_{L^{j}_{CL_m}} \tag{16}$$

$$\lambda_{L^{j+1}_{CL_m}} \le \lambda_{L^{i}_{CL_k}} \tag{17}$$

According to the stability condition of a single congestion loop, the virtual split of all links within a congestion loop are equal, i.e.:

$$\begin{cases} \lambda_{L^{i}_{CL_k}} = \lambda_{L^{i+1}_{CL_k}} & (a) \\ \lambda_{L^{j}_{CL_m}} = \lambda_{L^{j+1}_{CL_m}} & (b) \end{cases} \tag{18}$$

Substitute the left side in Eq. (16) using Eq. (18)(a):

$$\lambda_{L^{i}_{CL_k}} \le \lambda_{L^{j}_{CL_m}} \tag{19}$$

Substitute the left side in Eq. (17) using Eq. (18)(b):

$$\lambda_{L^{j}_{CL_m}} \le \lambda_{L^{i}_{CL_k}} \tag{20}$$

The necessary condition that guarantees the truth of both Eq. (19) and Eq. (20) is:

$$\lambda_{L^{j}_{CL_m}} = \lambda_{L^{i}_{CL_k}} \tag{21}$$

Similarly:

$$\lambda_{L^{j+1}_{CL_m}} = \lambda_{L^{i+1}_{CL_k}} \tag{22}$$

Hence the stable virtual split of both congestion loops must be the same:

$$\lambda_{CL_k} = \lambda_{CL_m} \tag{23}$$

Similar results can be derived about the mixed type. In sum, single congestion loop group tend to be stable ultimately. The stable traffic states for all congestion loops within the same group are identical.

At the beginning of the congestion loop group formation, the virtual split of all links may be different. The group then will evolve gradually. As long as there is split difference between adjacent links, the stabilization process will last. Suppose there are two congestion loops, loop k and loop m in the same group. The virtual split of link $L^i_{CL_k}$ (connected with node $I^i_{CL_k}$) is minimal among the group while other virtual splits are assumed to be the same. Link $L^j_{CL_m}$ (connected with node $I^j_{CL_m}$) will then converge to $\lambda_{L^i_{CL_k}}$. The arising question is how much time at least it will take for link $L^j_{CL_m}$ to reach the stable condition. To answer this, we construct a sub-graph that consists of the congestion loop group $G_{\{CL_n\}}, n=k, \mathrm{m}$. Since the virtual split always propagate upstream, the spread route of virtual split must be in one of the routes from node $I^j_{CL_m}$ to node $I^i_{CL_k}$ in sub-graph $G_{\{CL_n\}}$. The spreading speed of virtual signal is u_1 which is fixed. Hence the time it will take for split $\lambda_{L^j_{CL_m}}$ to decrease to $\lambda_{L^i_{CL_k}}$ is $\dfrac{\min\{s^t_{I^i_{CL_k} I^j_{CL_m}}\}}{u_1}$ if minimal virtual split remains unchanged, where $\min\{s^t_{I^i_{CL_k} I^j_{CL_m}}\}$ denotes the shortest path length from node $I^j_{CL_m}$ to node $I^i_{CL_k}$. Actual stable time for node $I^j_{CL_m}$ is greater than or equal to $\dfrac{\min\{s^t_{I^i_{CL_k} I^j_{CL_m}}\}}{u_1}$ due to the fact that minimal split may diminish. Therefore for the whole group, the stable time is greater than or equal to the physical length of maximal shortest path from all

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nodes to propagation source node $I_{CL_k}^i$ divided by u_1.

5.3. Interaction between congestion loop groups

There may simultaneously exist more than one congestion loop group in the network. When they are separated, i.e. there is no spillover links that connect them, the stabilization processes of these congestion loop groups are isolated. When some spillover links appear between these congestion loop groups and connect them, the smaller virtual split will spread via these links from one group to another. At last these congestion loop groups constitute a larger, new group. The stable virtual split of this bigger group is smaller than any one of the original splits. If the virtual split changes (must be smaller) during the propagation, the stabilization process would return back to propagation source with the new split, hence displays an oscillatory feature. If the ultimate split is zero, gridlock takes place, where no vehicle can move forward.

6. Stages Of Network Traffic Congestion Propagation

The urban traffic congestion can be divided into several stages below:

a) There is no oversaturated link in the whole network;
b) Some isolated links are oversaturated at first, and the vehicle queues are growing gradually;
c) Spillover takes place at some nodes, which leads to more spillovers. However, there is no congestion loop yet. At this stage, the propagation of virtual split is unidirectional;
d) Several spillover links comprise some congestion loops. At the same time, these congestion loops converge to the stable condition gradually, which is accompanied by the worsening of network traffic state;
e) Existing congestion loops soon trigger more loops to form, which results in loop groups. At this stage, traffic congestion may not be controlled easily.

When virtual split of some congestion loops or spillover links becomes zero, gridlock takes place. Large scale traffic congestion comes into being.

7. Gridlock Simulation

7.1. Simulation scenario

In reality traffic events such as spillover naturally decrease the output flow rate, which serves as the demand of downstream links or nodes. Nonetheless, incorporation of such issues into the deduction may introduce analytical difficulties. The numerical study is carried out to explore the dynamic nature of gridlock with the help of a dynamic revision of the link model in section 0.

The simulated network is shown in Fig. 8. It is a grid network with the size of 7*7. The links are bidirectional as shown in the figure. There are 45 nodes and 360 lanes in sum. The boundary nodes are OD nodes and inside nodes are signal controlled. The phase settings are shown either. There are four phases: south-north through phase, south-north left-turn phase, east-west through phase, east-west left-turn phase. The lost time for each phase is 3 seconds. Green time is assigned according to Webster's formula (Webster and Cobbe 1966).

Offset of the intersections are given as follows: there are two offset values, horizontal offset o_1 and vertical offset o_2. Offset of adjacent nodes that is in horizontal relationship adopt horizontal offset. Offset of adjacent nodes in vertical relationship adopt vertical offset. Because the influence of offset is not the major concern, they both are set to zero during the simulation. The given flows are routes flow. The routes are the static reasonable routes defined in Sheffi's literature (Sheffi 1985). There are 32 OD pairs overall that is listed in Table 1. 684 reasonable routes in sum are found out. The flow of a link is the sum of related routes flow. All route flows are generated randomly. At the beginning, link flows are loaded directly according to the routes flow. And then we choose a link, set the split to a minor one (=0.01 in the simulation). Hence oversaturation is introduced. In the simulation, the initial oversaturation position is set at left turn lane of the west approach at node 23. Then all links evolve according to the link model in the appendix. Here reroute effect of traffic jam is neglected since it relates to the drivers' evaluation of gridlock. We assume that their routes (reasonable route between OD pairs) are determined once they enter the network.

Table 1 OD pairs and reasonable routes Number

O points	D points	Number of reasonable routes	O points	D points	Number of reasonable routes
1	45	70	45	1	70
2	44	15	44	2	15
3	43	1	43	3	1
4	42	15	42	4	15
5	41	70	41	5	70
6	40	70	40	6	70
13	33	15	33	13	15
20	26	1	26	20	1
27	19	15	19	27	15
34	12	70	12	34	70

Other parameters are set as below: u_f : 54km/h; w : 21.6km/h; k_j :133veh/km; (q_m can be derived from the fundamental diagram, which is about 2000veh/h.).

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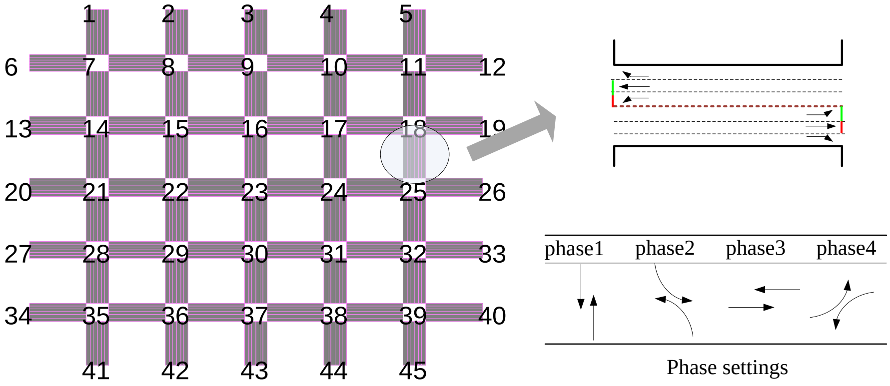

Fig. 8 Simulation scenario

7.2. Simulation results

The simulation results are described by the indexes below:

P1: The number of congestion loops, which varies with time;

P2: congestion loop size at a specific moment;

P3: congestion loop reproducing time;

P4: congestion loop reproducing rate, i.e. the number of congestion loops produced per unit time;

By definition, we have:

$$p_4 = \frac{dp_1(t)}{dt} \tag{24}$$

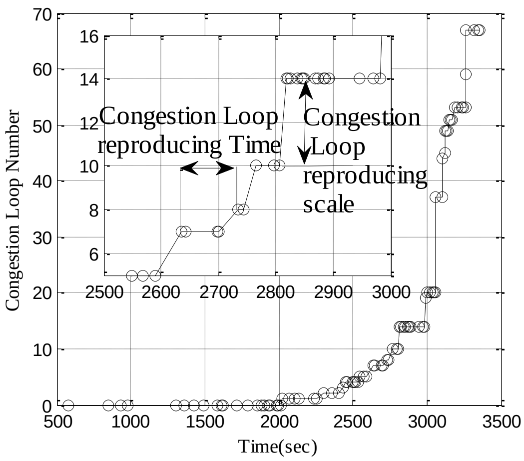

Fig. 9 Congestion loops number (P1) evolution

Fig. 9 presents the congestion loops number curve. Congestion loops number increases gradually with time. The curve becomes sharper, which implies that the congestion loop reproduce itself in an increasing speed. The horizontal difference between neighboring jumps of congestion loop number can be interpreted as congestion loop reproducing time while the vertical difference then can be seen as congestion loop reproduction scale as shown in Fig. 9. Fig. 10 shows the congestion loop reproducing time needed for existing congestion loop. It will take about 1500 seconds for the first congestion loop to come into being. Once it has formed, the reproduction time sharply decreases, and the reproducing process seems to accelerate to a relatively stable speed. It implies that the control of the formation of the first congestion loop is crucial. If the control or prevention fails, the congestion loop will quickly reproduce itself around the network.

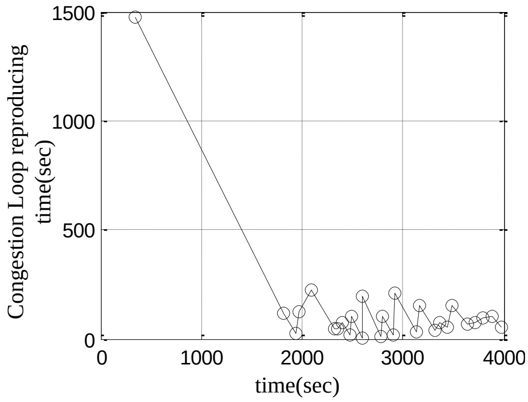

Fig. 10 Congestion loop reproducing time (P3)

Fig. 11 displays the reproduction indexes, namely congestion loop reproduction scale and congestion loop reproduction rate. The scale is measured by the increment of congestion loops number. Congestion loop reproduction rate is defined as the increment of loops number per unit time. Congestion loop reproduction scale presents an increasing trend. It is understandable because the jam propagation will naturally increase the reproducing scale of a congestion loop. Since the propagation of vehicle queue is determined by flow rate as well as signal parameters, reproduction rate which is depend on the same factors then varies in a chaotic profile which is shown in Fig. 11(b).

Fig. 12 gives the loop size curve during simulation. The three sub figures are maximal size, average size and minimal size respectively. The maximal loop size is increasing all the time. This is not true for average congestion loop size because smaller congestion loops come into being which is comprised by two lanes. Fig. 12(c) illustrates the minimal congestion loop. At about 4000sec, two-lane congestion loops appear. Despite that

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the formation of smaller congestion loop may decrease average congestion loop size, the overall trend of congestion loop size still rises which is the nature result of congestion propagation.

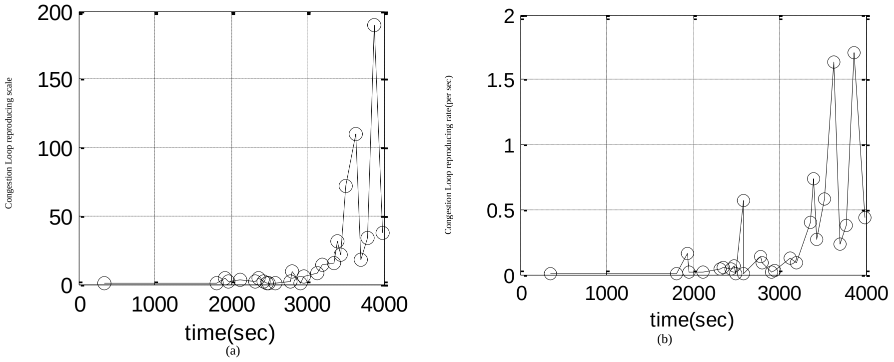

Fig. 11 Congestion loop reproduction index: (a) congestion loop reproduction scale ; (b) congestion loop reproduction rate (P4)

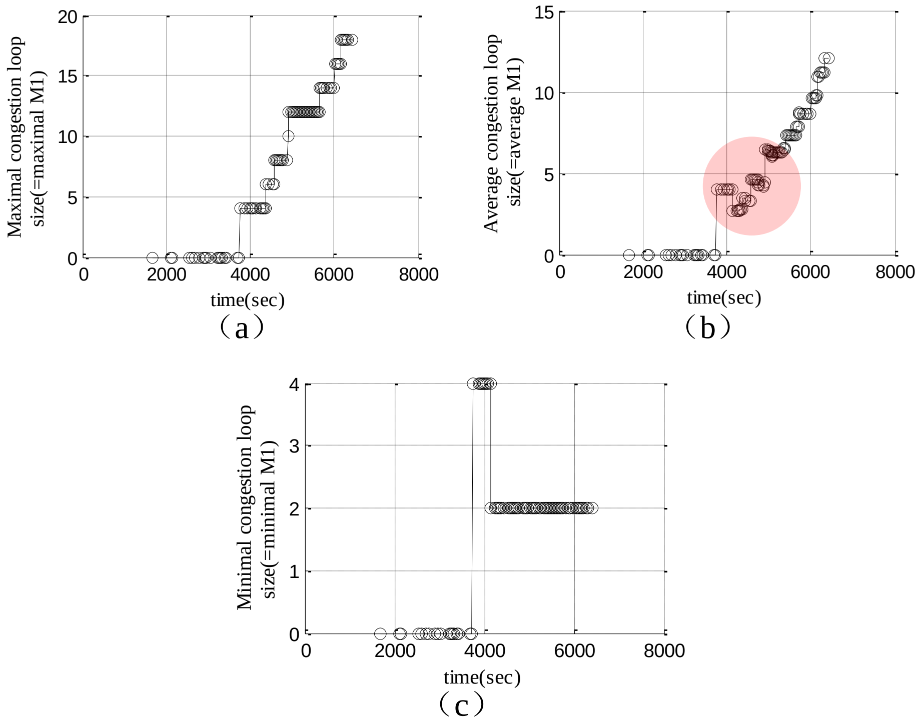

Fig. 12 Congestion loop size dynamics (P2); (a) maximal congestion loop size; (b) average congestion loop size; (c) minimal congestion loop size

Fig. 13 presents one of the biggest congestion loop that is measured in link number at about t=4081.9s. It consists of 18 links that are distributed around the network. Dark gray lanes denote the lanes that are in spillover condition. Blue ones mean congestion loop lanes. We can see that not all the spillover lanes are congestion looped. Since G_{CL} means the sub-graph comprised by congestion loop lanes and G_{FC} denotes that comprised by spillover lanes, we have $\bar{G}_{CL} \bigcap G_{FC}$, which represents the lanes that are in spillover condition but do not belong to any congestion loop.

Fig. 14(a) gives the lanes proportion variation of these two sub-graphs: congestion loop sub-graph and spillover lane sub-graph. We can see that there are some lanes which are in spillover condition but not congestion looped. However, these lanes may serve as the potential congestion loop seed. Fig. 14(b) presents the influence of proportion difference on congestion loop reproduction. x-axis, which is calculated by $N^L_{G_{FC}} - N^L_{G_{CL}} \big/ N_G$, denotes the difference between the two lane proportions. Congestion loop reproducing time reduces against the difference. It is understandable because these non-congestion-looped, spillover lanes will probably constitute a new congestion loop. The more they are, the quicker new congestion loop will be reproduced.

Fig. 15(a) gives an arbitrary chosen congestion loop. This congestion loop consists of 10 lanes. The stabilization is presented by the maximal, minimal and average virtual split of all the lanes in Fig. 15(b). At first one lane is oversaturated. The congestion loop comes into being at

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about t=2900 second. However, at that moment, virtual splits of the related lanes within this congestion loop are not identical. At about t=3800 second, the virtual splits of all lanes decrease to zero, which means that no car within this congestion loop system can move ahead. Thus the stabilization time is about 3800-2900=900 seconds.

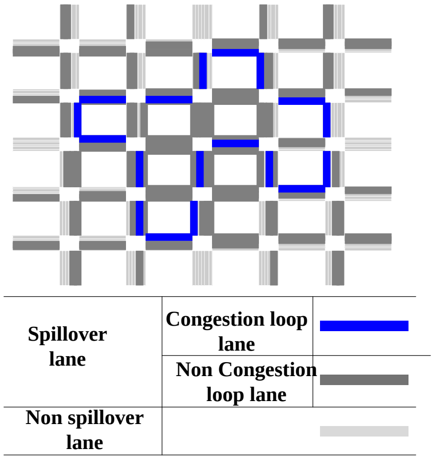

Fig. 13 The biggest congestion loop at t=4081.9s

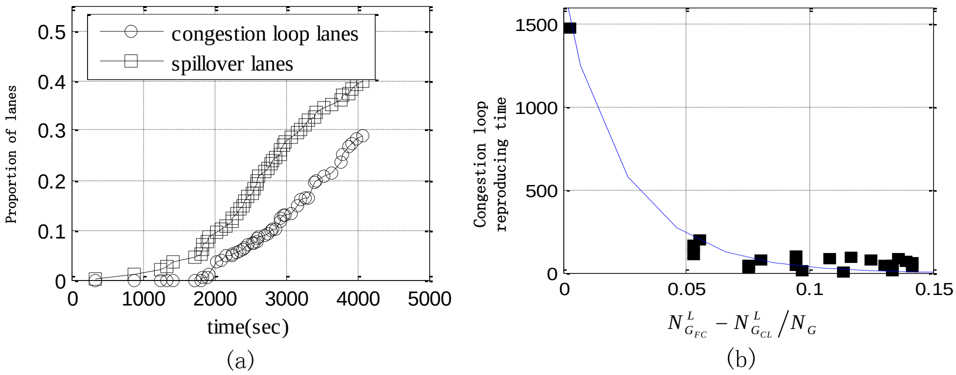

Fig. 14 Fraction of lanes and congestion loop index:(a)Fraction of lanes;(b) P3 versus difference of fraction

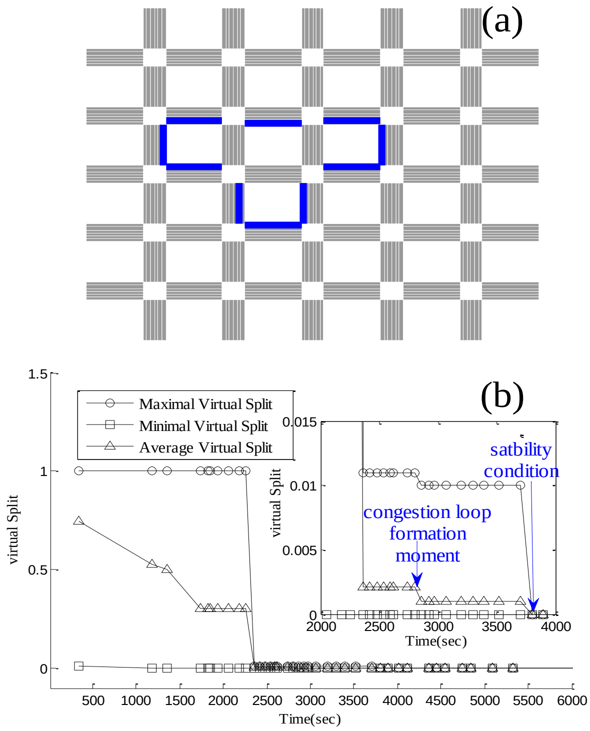

Fig. 15 A congestion loop and stabilization process

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8. Conclusion and Future Work

Traffic congestion of network scale is very complex compared with that within a link. It not only involves road traffic dynamics analysis, but also relates to the interaction between adjacent links. With the help of the concept of virtual signal, we define the congestion loop whose size is determined by the traffic state. From the stability analysis, it can be seen that with respect to a congestion loop group under stable inflow, there always exists a stable condition where virtual splits for all links are identical. Because the virtual split always diminishes, the stable process is accompanied by the deterioration of traffic state. The simulation results suggest that once the original congestion loop forms, the following reproducing rate soon increases sharply, which implies the importance of the prevention of the first congestion loop.

Since traffic jam is characterized by vehicle queue, and vehicle queue evolution itself is determined by arrival and intersection control measures as proposed in this paper, thus control method based on the evolution feature can be established. Because the congestion would reassign the route flow, which results in the variation of link flow, and finally changes the link queue evolution. Incorporation of these issues will be our next work, which serves as the theoretic basis for network jam control.

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