STAGE DISCHARGE RELATIONSHIPS FOR TRIANGULAR WEIR

Masoud Ghodsian

Associate Professor of Hydraulic Engineering, Tarbiat Modarres University, Tehran, Iran
ghods@modares.ac.ir

Abstract: *The flow characteristics of triangular weir are studied under free and submerged flow condition. This paper reports the results of an investigation carried out to establish the stage-discharge relationship for sharp and broad crested triangular weir. The stage-discharge relationships are deduced by the application of the π -theorem of the dimensional analysis and the incomplete self-similarity theory coupled with experimental data.*

Keywords: *Triangular weir, sharp crested, broad crested, free flow, submerged flow, π-theorem, Incomplete self-similarity.*

1. INTRODUCTION

Weirs are flow-measuring devices that are used in hydraulic and water resources engineering. Triangular weirs are widely used in the field and laboratory channels for flow measurements. The sharp crested triangular weir is best described as a V-shaped notch symmetrically located in a thin plate, which is placed perpendicular to the sides and bottom of an open channel. A broad crested triangular weir is a structure with a triangular crest above which the fluid pressure may be considered hydrostatic. In the longitudinal section, it is simillar to the broad crested weir with horizontal crest [1]. In natural streams and irrigation canals where a wide range of discharges must be measured, the triangular weir has a number of advantages, which recommend its use. First, at large flows it provides a large breadth so that the backwater effect is not excessive. Second, at low flows, the width is reduced so that the sensitivity of the weir is acceptable [2]. Moreover it is easy to make and is inexpensive structure. The dimensions of triangular weirs depend on the maximum discharge to be measured and the available width of the approach channel.

Previous works on triangular weirs, as compare to rectangular weir, are less. Bos [3] related the discharge coefficient to notch angle for sharp crested triangular weir and to ratio of total head to weir thickness for broad crested triangular weir. Results of experiments on the variations of discharge coefficient for sharp crested triangular weirs are reported by Montes [4], Smith [5] Brater and King [6].

The flow charactristics over broad crested triangular weir with notch angle 90 degree was investigated by Smith and Liang [7] and Pitlo and Smith [8]. Their measurments included both free flow and submerged flow conditions. Exessive studies on broad crested triangular weir with notch angle 120 deggree were conducted at the agricultural university at Wageningen [9 and 10]. Boiten and Pitlo [11] obtained the discharge coefficient of broad crested triangular weir as a function of boundary layer thickness, notch angle and ratio of weir thickness to flow depth. Boiten [12] developed the graphical representation of discharge coefficient for broad crested triangular weir as a function of flow depth to weir thickness. Gill [13] obtained the discharge coefficient of broad crested triangular weir as a function of ratio of depth of flow and weir thickness. Smith [5] performed experiments for free and submerged flow over broad crested triangulare weirs.

In this paper results of experiments on sharp crested and broad crested triangular weir under free and submerged flow conditions are reported. The stage-discharge relationship is obtained by theoretical analysis based on the application of dimensional analysis and the application of incomplete self-similarity theory coupled with an experimental investigation carried out by using a laboratory flume.

2. THEORY

Figs. 1 and 2 show schematic view of the flow over a sharp crested and broad crested triangular weirs respectively. The flow over a sharp crested

and broad crested triangular weir is expressed by (French [2] and, Montes [4]):

- Sharp crested triangulare weir:

$$Q = \frac{8}{15} C_d \sqrt{2g} h^{2.5} \tan(\theta) \qquad (1\text{-a})$$

- Broad crested triangular weir:

$$Q = \frac{16}{25\sqrt{5}} C_d \sqrt{2g} h^{2.5} \tan(\theta) \qquad (1\text{-b})$$

in which Q = discharge over the weir; C_d = discharge coefficient; h = total head over the weir in the upstream section; g = acceleration due to gravity; and θ = half notch angle of the weir.

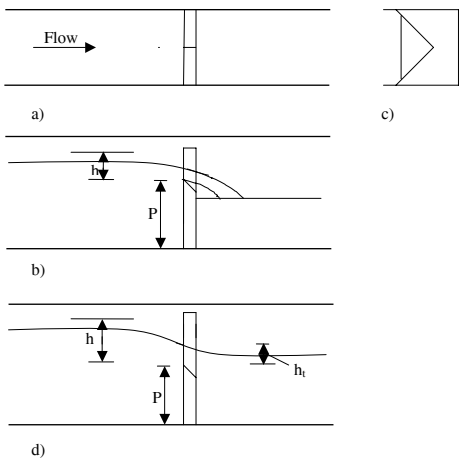

Fig. 1. Flow over sharp crested weir; a) Plan, b) Free flow condition; c) Front view; and d) Submerged flow condition

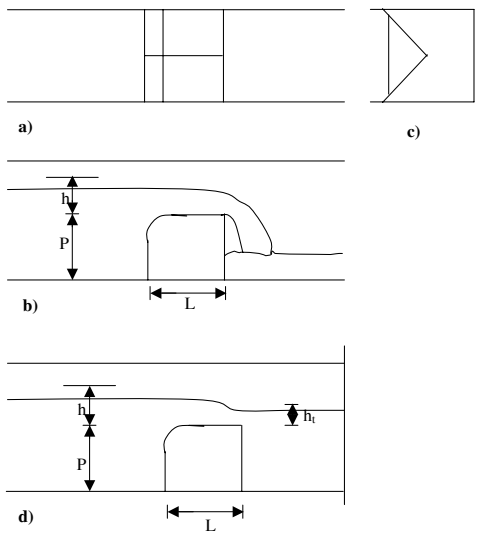

Fig. 2. Flow over broad crested weir; a) Plan, b) Free flow condition, c) Front view, and d) Submerged flow condition

The discharge coefficient includes the effects of viscous, turbulence, non-uniform velocity distribution of flow, type and shape of crest. The generall equations for sharp crested and broad crested triangular weirs are obtained in foregoing sections of paper.

The stage-discharge relationship, for a sharp crested triangular weir can be expressed by the following functional relationship:

$$f(Q, h, P, g, \mu, \theta) = 0 \qquad (2)$$

in which:

P = weir height; and μ = viscosity of water.

Eq. (2), according to the Buckingham π-theorem and after neglecting the effect of μ, can be expressed as a non dimensional form as follows:

$$f(\pi_1, \pi_2, \pi_3) = 0 \qquad (3)$$

in which π_1, π_2 and π_3 = dimensionless groups given by:

$$\pi_1 = \frac{Q}{g^{0.5} P^{2.5}} \qquad (3\text{-a})$$

$$\pi_2 = \frac{h}{P} \qquad (3\text{-b})$$

$$\pi_3 = \theta \qquad (3\text{-c})$$

Therefore, Eq. (3), after combining π_1 and π_3, for sharp crested triangular weir can be written as:

$$\frac{K}{P} = f\left(\frac{h}{P}\right) \qquad (4)$$

in which:

$K = Q^{2/5} g^{-1/5} (\tan\theta)^{-2/5}$.

It is obvious that the condition of incomplete self-similarity, as explained by Ferro [14] prevail for the above relationship. Following the incomplete self-similarity approach one obtain the following stage-discharge relationship for sharp crested triangular weir:

$$\frac{K}{P} = a\left(\frac{h}{P}\right)^b \qquad (5)$$

in which: *a* and *b* are numerical constant to be determined by using experimental data.

For broad crested triangular weir, the stage-discharge relationship can be expressed as:

$$f(Q, h, L, g, \mu, \theta)=0 \qquad (6)$$

in which:

L = thickness of weir, i.e. length of weir in the direction of flow.

Using the Buckingham π-theorem and after neglecting μ, Eq. (6) leads to the following

dimensionless group:

$$\frac{K}{L} = f\left(\frac{h}{L}\right) \tag{7}$$

in which $K = Q^{2/5} g^{-1/5}(\tan\theta)^{-2/5}$. The following stage-discharge relationship derives from incomplete self-similarity condition for broad crested triangular weir:

$$\frac{K}{L} = c\left(\frac{h}{L}\right)^{d} \tag{8}$$

in which c and d are numerical constant to be determined by using experimental data.

3. EXPERIMENTS

Experiments were conducted in a re-circulating flume with a length of 12m, width of 0.3m and depth of 0.45m. The flume had a mild steel bed and the glass sides. A mild steel triangular weir of crest thickness L was installed at the mid length of flume. In order to obtain different tail water depth, for studying the effect of submergence, a tailgate was provided at the end of flume.

For the broad crested weir, the upstream corner was rounded with a radius equal to 0.1 of maximum water depth to avoid flow separation. A schematic view of the experimental setup is shown in Fig. 3. The discharge was measured by volumetric method.

In order to have sufficient accuracy, for each run, discharge measurement was repeated at least three times and the average of them was considered. Flow depths (i.e. upstream depth of flow over the weir and tail water depth over the weir) were measured at the centerline of channel with a point gauge having an accuracy of ±0.1mm. Four different notch angles were used (i.e. 30, 60, 90 and 120 degree). Experiments were carried out for various combinations of notch angler, weir height and upstream depth for free flow. For submerged flow condition, in addition to above variables, the tail water depth was varied to obtain various degree of submergence. Table 1 and Table 2 show the range of various parameters covered in the present study.

4. ANALYSIS

In order to check the feasibility of Eq. (5) and Eq. (8) and to obtain the proper form of stage-discharge relationship for triangular weir, the present experimental data were used. Fig. 4 shows the graphical representation of Eq. (5) for sharp crested triangular weir. It is evident that the assumed stage-discharge relationship for sharp crested triangular weir (Eq. 4) is acceptable. The values of constant a and b in Eq. (5) were obtained as 0.7907 and 1.01 respectively.

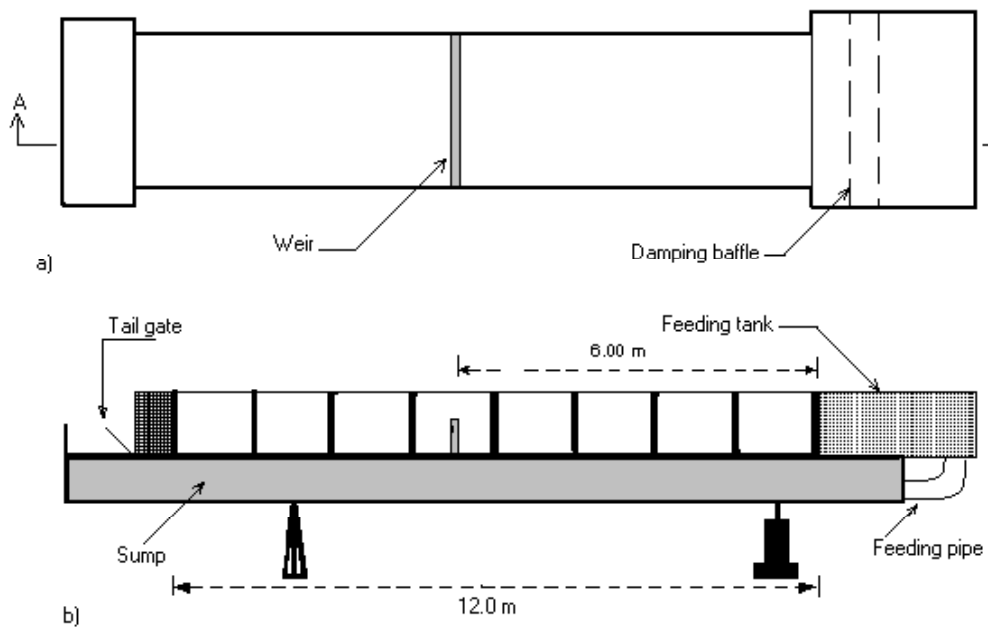

Fig. 3. Schematic view of experimental setup; a) Plan and b) Longitudinal section

Therefore, the proposed stage-discharge relationship for sharp crested triangular weir under free flow condition is:

$$\frac{K}{P} = 0.7907\left(\frac{h}{P}\right)^{1.01} \tag{9}$$

Introducing submergence coefficient K_s, Eq. (9) for submerged flow condition can be written as:

$$\frac{K}{P} = 0.7907 K_s\left(\frac{h}{P}\right)^{1.01} \tag{10}$$

in which K_s is the ratio of discharge under free flow and submerged flow condition and can be assumed to be a function of ratio of tail water head to upstream depth of flow (i.e. h_t/h). Fig. 5 shows the variations of K_s with h_t/h for sharp crested triangular weir with different notch angle under submerged flow condition. It is evident that up to about $h_t/h \leq 0.4$, the value of K_s is unity, and further increasing h_t/h, the value of K_s decreases.

The line drawn through the experimental data in Fig. 5 is an average line fitted to the data with the equation as:

$$K_s = \left[1-\left(\frac{h_t}{h}\right)^{2.5}\right]^{0.3737} \tag{11}$$

Therefore, the general form of stage-discharge relationship for sharp crested triangular weir is obtained from Eqs. (10 and 11) as:

$$\frac{Q}{g^{0.5}P^{2.5}\tan\theta} = 0.5559\left(\frac{h}{P}\right)^{2.525}\left[1-\left(\frac{h_t}{h}\right)^{2.5}\right]^{0.9337} \tag{12}$$

Comparing Eq. (12) with Eq. (1-a), one obtain the discharge coefficient C_d as:

$$C_d = 0.73708\left(\frac{h}{P}\right)^{0.025}\left[1-\left(\frac{h_t}{h}\right)^{2.5}\right]^{0.9337} \tag{13}$$

For the free flow condition, last terms in Eq. (12) and Eq. (13) drops out and results in to the stage-discharge relationship and discharge coefficient for this condition respectively.

Fig. 6 shows the graphical representation of Eq. (8) for broad crested triangular weir with different notch angles. The values of constant c and d in Eq. (8) were obtained as 0.692 and 1.01 respectively.

Table 1. Range of parameters studied for sharp crested weir

Notch angle (deg)	Weir height P (cm)	h/P	ht/h	Q (L/s)	Type of flow	No. of data
30	2-4 2&4	0.07-4.98 2.1-6.1	- 0.15-0.84	0.075-2.75 0.46-2.3	Free Submerged	27 14
60	2-20 2&19.5	0.08-4.91 0.1-5.0	- 0.15-0.9	0.05-4.4 0.097-2.8	Free Submerged	28 18
90	2-26 16&25	0.04-4.53 0.1-0.46	- 0.13-0.7	0.03-6.3 0.5-3.2	Free Submerged	29 10
120	1.5-13 7.8&16.8	0.09-4.8 0.24-0.9	- 0.29-0.83	0.02-7.5 0.75-4.0	Free Submerged	32 13

Table 2. Range of parameters studied for broad crested weir

Notch angle (deg)	Weir height P (cm)	Weir thickness L (cm)	h/L	ht/h	Discharge (l/s)	Type of flow	No. of data
30	19 19	50 50	0.06-0.41 0.30-0.40	- 0.8-0.9	0.74-5.9 3.0-4.4	Free Submerged	16 14
60	20 20	50 50	0.04-0.40 0.26-0.40	- 0.84-0.97	0.054-11.8 4.6-7.82	Free Submerged	20 22
90	25.7 25.7	30.5 30.5	0.05-0.50 0.27-0.50	- 0.8-0.97	0.05-10.25 2.5-8.4	Free Submerged	24 18
120	22 22	20 20	0.07-0.55 0.28-0.40	- 0.7-0.99	0.057-6.82 1.44-2.55	Free Submerged	12 20

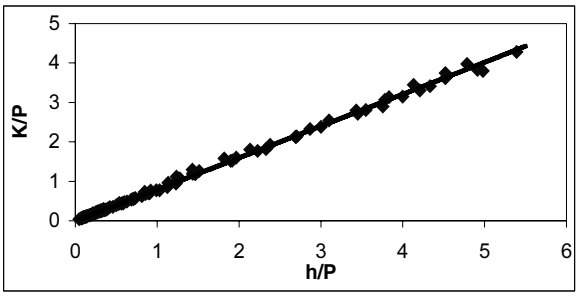

Fig. 4. Variations of K/P with h/P for sharp crested triangular weir with different notch angle

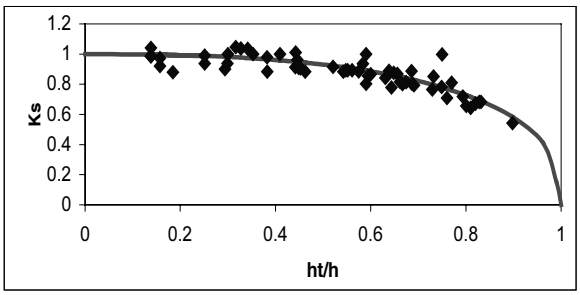

Fig. 5. Submergence coefficient for sharp crested triangular weir with different notch angle

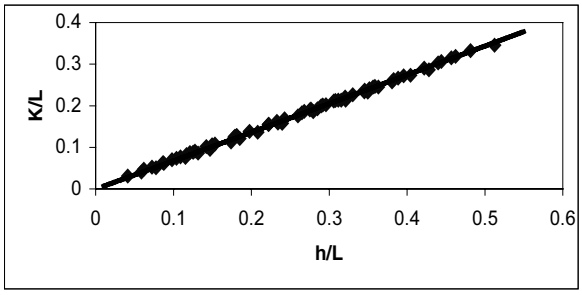

Fig. 6. Variations of K/L with h/L for broad crested triangular weir with different notch angle

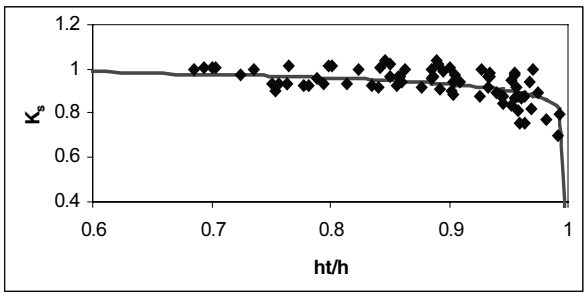

Fig. 7. Submergence coefficient for broad crested triangular weir with different notch angl**e**

Therefore, the proposed stage - discharge relationship for broad crested triangular weir under free flow condition is:

$$\frac{K}{L} = 0.692\left(\frac{h}{L}\right)^{1.01} \qquad (14)$$

Introducing submergence coefficient K_s, one can obtain the following stage-discharge relationship for broad crested triangular weir under submerged flow condition:

$$\frac{K}{L} = 0.692 K_s\left(\frac{h}{L}\right)^{1.01} \qquad (15)$$

Fig. 7 shows the variations of K_s with h_t/h for broad crested triangular weir for different notch angles. This Figure shows that $K_s = 1$ for $h_t/h \leq 0.6$. The trend of variations of h_t/h is almost similar for broad crested and sharp crested weirs.

The line drawn through data points in Fig. 7 is an average line fitted to the data and can be expressed by the following equation:

$$K_s = \left[1-\left(\frac{h_t}{h}\right)^{2.5}\right]^{0.0488} \qquad (16)$$

Therefore, the general form of stage-discharge relationship for broad crested triangular weir is obtained from Eqs. (15 and 16) as:

$$\frac{Q}{g^{0.5}L^{2.5}\tan\theta} = 0.39835\left(\frac{h}{L}\right)^{2.525}\left[1-\left(\frac{ht}{h}\right)^{2.5}\right]^{0.122} \qquad (17)$$

Comparing Eq. (17) with Eq. (1-b), it can be shown that for broad crested triangular weir C_d is given by:

$$C_d = 0.98413\left(\frac{h}{L}\right)^{0.025}\left[1-\left(\frac{h_t}{h}\right)^{2.5}\right]^{0.122} \qquad (18)$$

It is obvious that for free flow condition the last terms in Eq. (17) and Eq. (18) drops out to yield the stage-discharge relationship and discharge coefficient for this condition respectively. In order to check the validity of Eq. (14, 15 and 17) the data collected by Boiten [15] on triangular broad crested weir for free and submerge flow were used. The range of Boitens data is given in Table 3.

Fig. 8 shows the comparision of K/L versus h/L for the data of Boiten[15] and thoes obtained by Eq. (14). It is evident that Eq. (14) give satisfactory results for the data of Boiten [15]

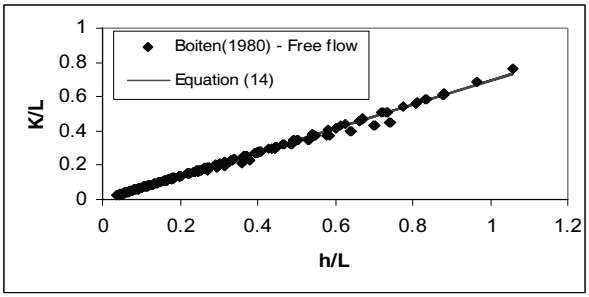

Fig. 8. Validation of equation (14) with Boitens (1980) data for free flow condition

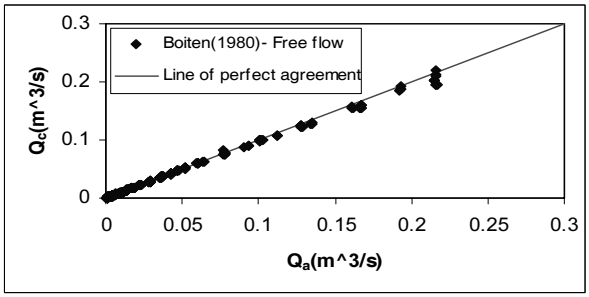

Fig. 9. Comparison of computed discharge using Eq. (14) and actual discharge for Boitens(1980) data under free flow condition

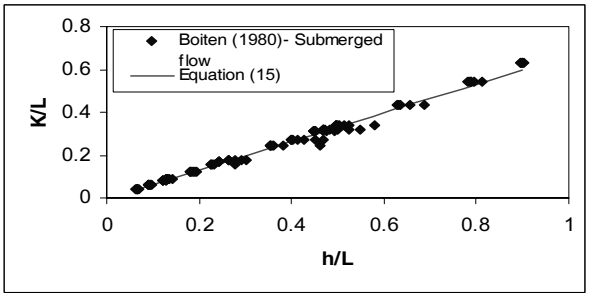

Fig. 10. Validation of equation (15) with Boitens (1980) data for submerged flow condition

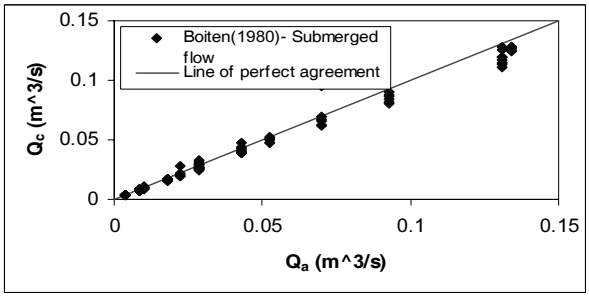

Fig. 11. Comparison of computed discharge using Eq. (15) and actual discharge for Boitens (1980) data under free flow condition

Table 3. Range of parameters studied by Boiten(1980)

Notch angle (deg)	Weir height P (cm)	Weir thicknes L (cm)	h/L	ht/h	Discharge (l/s)	Type of flow	No. of data
90	25 25	45 , 90 45, 90	0.052-1.058 0.22-0.91	- 0.35-0.98	0.44-216.28 22.3-134	Free Submerged	147 44
120	30.5	45, 90	0.049-0.878	- 0.84-0.97	0.87-216.2 10-42.8	Free Submerged	172 22
150	0.36 0.30	0.45-0.9 0.45,0.9	0.03-0.75 0.036-0.25	- 0.75-0.94	0.92-216.2 3.7-18.1	Free Submerged	95 36

under free flow condition even for noch angle 150 degree. Fig. 9 shows the comparision of computed values of discharhe Q_c [using Eq. (14)] with actual values of discharge Q_a for the data of Boiten[15] under free flow condition. It is obvious that Eq. (14) is capable of predicting the discharge over a braod crested triangular weir for free flow accurately.

Figs. 10 and 11 show the validation of Eqs. (15 and 17) respectively for the data of Boiten [15] under submerged flow condition. Fig. 10 shows the comparision of K/L versus h/L for the data of Boiten[15] and thoes obtained by Eq. (15). It is evident that Eq. (15) give satisfactory results for the the data of Boiten [15] under submerged flow condition even for noch angle 150 degree. Fig. 11 shows the comparision of computed values of discharhe Q_c using Eq. (17) with actual values of discharge Q_a for the data of Boiten[15] under submerged flow. It is obvious that Eq. (17) is also capable of predicting the discharge over a braod crested triangular weir

for submerged flow accurately. The verification of developed equations for sharp crested triangular weir is possible when data on this type of weir is available.

5. CONCLUSION

In this paper the results of an experimental investigation on sharp crested and broad crested triangular weir, under free and submerged flow condition, are reported. The analysis shows that the stage-discharge relationship for sharp crested and broad crested triangular weirs can be theoretically obtained by applying the Buckingham theorem of dimensional analysis and the incomplete self-similarity theory. New equations for stage discharge relationship and discharge coefficient are developed.

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