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Shape optimization of arch dams with frequency constraints by enhanced charged system search algorithm and neural network

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Abstract

The main aim of this paper is to find the optimum shape of arch dams subjected to multiple natural frequency constraints by using an efficient methodology. The optimization is carried out by charged system search algorithm and its enhanced version. Computing the natural frequencies by Finite Element Analysis (FEA) during the optimization process is time consuming. In order to reduce the computational burden, Back Propagation (BP) neural network is trained and utilized to predict the arch dam natural frequencies. It is demonstrated that the optimum design obtained by the Enhanced Charged System Search using the BP network is the best compared with the results of other algorithms. The numerical results show the computational advantageous of the proposed methodology.

Keywords: *Arch dam, Finite element, Frequency constraint, Enhanced charged system search, Optimum shape design, Neural network.*

1. Introduction

It is obvious that an appropriate shape design has a great influence on the economy and safety of an arch dam. Also, the natural frequencies are important parameters affecting the dynamic behavior of the arch dams. By imposing some constraints on the natural frequencies, the dynamic behavior of arch dam may be improved and the eventual resonance phenomenon during earthquake can be also eliminated. On the other hand, arch dams are supposed to be designer as economical as possible. Thus a frequency constraint cost optimization process should be performed to obtain these two aims simultaneously.

Traditionally, shape design of an arch dam is based on the experience of the designer, the model tests and trial and error procedures. To obtain a better shape, the designer should select several alternative schemes with various patterns and modify them to obtain a number of feasible shapes. The best shape satisfying the demands of design specifications is selected as the final shape.

The shape of the dam obtained in this way is feasible but not necessarily optimum or even good. Moreover, the time required for design is rather long.

To overcome these difficulties, the process can easily and reliably be performed by employing optimization techniques [1].

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In the recent decades, some progress has been made in optimum design of arch dams. Almost all of them have used conventional methods for analysis approximation and optimization [2-6]. The main disadvantage of these methods is the need for calculating the function derivatives and may also trap into local optima. Meta-heuristic algorithms are computationally efficient in comparison to the gradient based methods.

As a newly developed meta-heuristic algorithm, the charged system search (CSS) is introduced by Kaveh and Talatahari for design of structural problems [7]. Here, this method utilizes the governing laws of Coulomb and Gauss from electrostatics and the Newtonian laws of mechanics. Inspired by these laws, a model is created to formulate the structural optimization method. This algorithm and its enhanced form [8] are utilized to find optimal shape of arch dams with constrained natural frequencies [9-11]. The cost of the arch dam including concrete volume and the casting areas is considered as the objective function. The design variables are geometric parameters of the arch dam. In order to implement a practical design optimization, many constraints such as stress, displacement, stability requirement, and frequency constraints are considered. In the present study, in order to simplify the optimization operation, only frequency and some geometrical constraints are considered.

The stochastic nature of the meta-heuristic search techniques makes the convergence of the process slow. Therefore, complete optimization of arch dams for frequency constraints requires considerable computational effort. In order to accelerate the optimization process and reduce the computational time, the natural frequencies of

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arch dams are predicted using properly trained Back Propagation (BP) neural network instead of direct evaluation. Many successful applications of neural networks are reported in literature [12-14]. The numerical results demonstrate the computational merits of the proposed methodology for optimum shape design of arch dams.

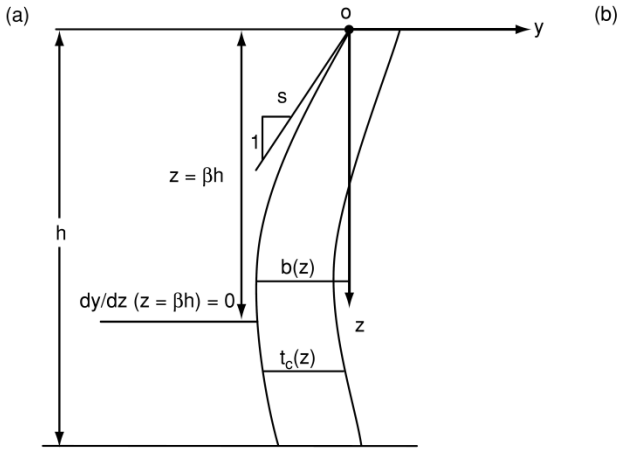
2. Geometrical Model of a Double-Curvature Arch Dam

The first stage of shape optimization of arch dams is to select an appropriate geometrical model. The model should be consistent with the stress state of arch dams and simple to facilitate construction. It is also important to construct the model on design variables with physical meaning. In this study, a geometric model for double-curvature arch dams that is based on practical requirements is presented. To define the geometrical model of an arch dam, the shape of a central vertical section is determined first, and then the shape of horizontal sections at various elevations is specified.

2.1. Shape of the central vertical section

For the central vertical section of a double-curvature arch dam, one polynomial of nth order can be used to determine the curve of the upstream boundary and another polynomial can be utilized to determine the thickness. In this study, for the curve of the upstream face a polynomial of second order is considered as [2, 3]:

$$y(z) = b(z) = -sz + sz^2 / (2\beta h) \tag{1}$$



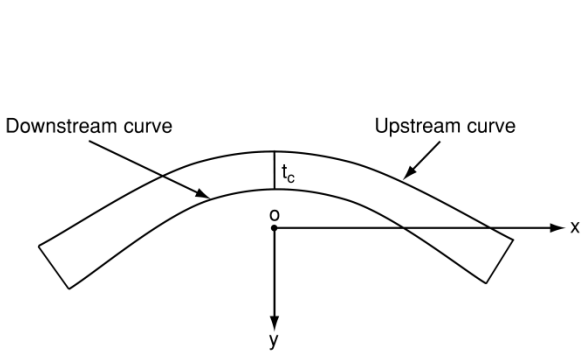


Fig. 1 (a) Central vertical section of an arch dam (b) Horizontal section of a parabolic arch dam

As shown in Fig. 1(a), h and s are the height of the dam and the slope at crest, respectively. The point where the slope of the upstream face equals to zero is $z = \beta h$ in which $0 < \beta \leq 1$ is a constant.

By dividing the height of the dam into n segments containing n+1 levels, the thickness of the central vertical section can be expressed as:

$$t_c(z) = \sum_{i=1}^{n+1} L_i(z) t_{c_i} \tag{2}$$

in which t_{ci} is the thickness of the central vertical section at the ith level. Also, in the above relation $L_i(z)$ is a Lagrange interpolation function associated with the ith level and can be defined as:

$$L_i(z) = \frac{\prod_{k=1}^{n+1}(z - z_k)}{\prod_{k=1}^{n+1}(z_i - z_k)} \qquad k \neq i \tag{3}$$

where z_i denotes the z coordinate of the ith level in the central vertical section.

2.2. Shape of the horizontal section

As shown in Fig. 1(b), for a symmetrical canyon and arch thickening from crown to abutment, the shape of the horizontal section of a parabolic arch dam is determined by the following two parabolic surfaces [3]:

At the upstream face of dam:

$$y_u(x,z) = \frac{1}{2r_u(z)} x^2 + b(z) \tag{4}$$

At the downstream face of dam:

$$y_d(x,z) = \frac{1}{2r_d(z)} x^2 + b(z) + t_c(z) \tag{5}$$

where r_u and r_d are radii of curvature corresponding to upstream and downstream curves, respectively, and functions of nth order with respect to z can be used for these radii:

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$$r_u(z) = \sum_{i=1}^{n+1} L_i(z) r_{ui} \quad (6)$$

$$r_d(z) = \sum_{i=1}^{n+1} L_i(z) r_{di} \quad (7)$$

where r_{ui} and r_{di} are the values of r_u and r_d at the ith level, respectively.

3. Finite Element Model of Arch Dam

In the present study, a finite element model based on modal analysis of double-curvature arch dam is presented. The arch dam is treated as a three dimensional linear structure. A twenty-node isoparametric solid element is used to mesh of the arch dam body. It is assumed that the reservoir is empty and dam foundation is rigid to avoid the extra complexities that would otherwise arise. These assumptions do not affect the proposed methodology for optimal design of arch dams. The material properties of the dam are given in Table 1 and the finite element model of a parabolic arch dam is depicted in Fig. 2.

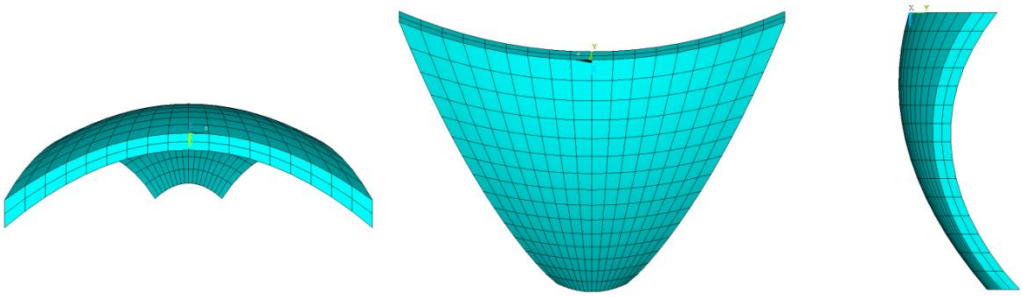

Fig. 2 The finite element model of the parabolic arch dam

Table 1 The material properties of dam

Characteristic	Material property
Elasticity modulus of concrete	27580 MPa
Poison's ratio of concrete	0.2
Mass density of concrete	2483 kg/m^3

The Block Lanczos eigenvalue solver is given in ANSYS [15] which uses the Lanczos algorithm. This method is useful for large-scale structures and typically, is applicable to the type of problems solved using the subspace eigenvalue method and is as accurate as the subspace method, but has a faster convergence rate.

3.1. Verification of the finite element model

In order to validate the finite element model of the arch dam, Morrow Point arch dam (263km southwest of Denver, Colorado) is investigated (Fig. 3). The dam structure is 141.7m high, with a crest length of 221m. The thin arch structure ranges in thickness from 3.7m at the crest to 15.9m at the base.

The material properties of the dam are given in Table 1. Other properties of the dam can be found in Ref. [16]. The first three natural frequencies of the mode of Morrow Point dam is determined from modal analysis of arch dam and the results is compared to those reported in the literature [12, 17]. It should be noted that the natural frequencies related to half of the dam are reported in the literature [12]. In order to perform an exact analysis, it is necessary to consider the complete dam. The natural frequencies from the literature and the finite element model are given in Table 2. It can be observed that a good conformity has been achieved between the result of present work and that reported in the literature.

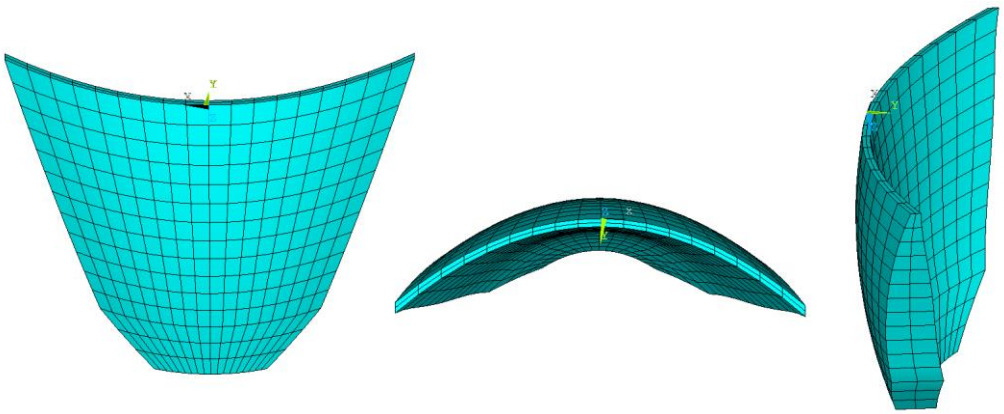

Fig. 3 Finite element model of the Morrow Point dam

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Table 2 Comparison of the natural frequencies (Hz) from the literature with the finite-element model

	Empty reservoir		
mode	Tan and Chopra [17]	Salajegheh et al. [12]	Present work
1	4.27	4.29	4.30
2	—	—	4.58
3	—	6.71	6.67

4. Arch Dam Optimization

4.1. Mathematical model and optimization variables

In the optimization process, the goal is to minimize the cost of arch dam while satisfying multiple constraints on natural frequencies. The optimization problem can be stated mathematically as follows:

$$\begin{aligned} &\text{Find}: X=[x_1,x_2,\ldots,x_n] \\ &\text{To minimize}: w(X) \\ &\text{Subject to}: g_i(X)\le 0\ ,\ i=1,\ldots,m \\ &X^l \le X \le X^u \end{aligned} \tag{8}$$

where X is the vector containing the design variables with n unknowns, $w(X)$ represents the objective function that should be minimized, and g_j is the jth constraint from m inequality constraints. Also, in the above equation X^l and X^u denote the lower and upper bounds of the design variable vector, respectively.

4.2. Design variables

The most effective parameters for creating the arch-dam geometry were mentioned in Section 2. These parameters can be adopted as design variables:

$$X^T=\{s\ \ \beta\ \ t_{c1}\ldots t_{cn+1}\ \ r_{u1}\ldots r_{un+1}\ \ r_{d1}\ldots r_{dn+1}\} \tag{9}$$

in which the X^T vector may have $3n+5$ components involving shape parameters of the arch dam.

4.3. Design constraints

Design constraints are divided into some groups including the behavior, geometric and stability constraints. The behavior constraints are limits on natural frequencies that may be defined as follows:

$$fr_k^l \le fr_k \le fr_k^u \Rightarrow \begin{cases} g_{bk}^l(X)=1-\dfrac{fr_k}{fr_k^l}\le 0 \\ g_{bk}^u(X)=\dfrac{fr_k}{fr_k^u}-1\le 0 \end{cases},\quad k=1,2,\ldots,n_{fr} \tag{10}$$

where fr_k, fr_k^l and fr_k^u are the kth natural frequency, lower and upper bounds on kth natural frequency, respectively. Also, n_{fr} is the number of natural frequencies that may be considered. The most important geometric constrains are those that prevent from intersection of upstream and downstream face as:

$$r_{di}\le r_{ui} \Rightarrow g_{gi}(X)=\frac{r_{di}}{r_{ui}}-1\le 0\ \ ,\ i=1,\ldots,n+1 \tag{11}$$

where r_{di} and r_{ui} are radii of curvatures at the down and upstream faces of the dam in ith position in z direction. The geometric constraint that is applied for facile construction is defined as:

$$s\le s_{all} \Rightarrow g_c(X)=\frac{s}{s_{all}}-1\le 0 \tag{12}$$

where s is the slope of overhang at the upstream face of dam and s_{all} is its allowable value. Also s_{all} is taken as 0.3 [2].

The constraints ensuring the sliding stability of the dam may be expressed as:

$$\varphi^l\le\varphi_i\le\varphi^u \Rightarrow \begin{cases} g_{si}^u(X)=\dfrac{\varphi_i}{\varphi^u}-1\le 0 \\ g_{si}^l(X)=1-\dfrac{\varphi_i}{\varphi^l}\le 0 \end{cases},\quad i=1,\ldots,n+1 \tag{13}$$

where φ_i is the central angle of arch dam and usually $90\le\varphi_i\le 130$ [18].

4.4. Objective function

In the present work, the cost of arch dam body is considered as objective function that may be expressed as:

$$w(X)=p_v v(X)+p_a a(X) \tag{14}$$

in which $v(X)$ and $a(X)$ are the volume of concrete and the areas of casting in dam. The unit price of concrete and casting are chosen as p_v =\$33.34 and p_a =\$6.67, respectively.

the volume of concrete can be determined by integrating of dam surfaces as:

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$$v(X)=\iint_{Area}\left|y_d(x,z)-y_u(x,z)\right|dx\,dz \tag{15}$$

in which $Area$ is an area produced by projecting of dam on xz plane.

The areas of casting can be approximately calculated by summing of the areas of upstream and downstream faces as follows [19]:

$$a(X)=a_u(X)+a_d(X)=\iint_{Area}\sqrt{1+\left(\frac{dy_u}{dx}\right)^2+\left(\frac{dy_u}{dz}\right)^2}\,dx\,dz+\iint_{Area}\sqrt{1+\left(\frac{dy_d}{dx}\right)^2+\left(\frac{dy_d}{dz}\right)^2}\,dx\,dz \tag{16}$$

where a_u and a_d are the casting areas of upstream and downstream faces, respectively. In order to evaluate the mentioned parameters a computer program is coded using MATLAB [20] based on numerical integration.

Actually minimizing $w(X)$ will lead to minimizing $\mathrm{Mer}(X)$, the merit function. Thus, an exterior penalty function, $w_{\mathrm{penalty}}(X)$, is employed to form the merit function, as follows:

$$Mer(X)=w(X)\times w_{penalty}(X) \tag{17}$$

where $w_{\mathrm{penalty}}(X)$ is the penalty function resulting from the violations of the constraints corresponding to the response of the structure. By considering some natural frequency constraints, the penalty function is defined as [9]:

$$\begin{aligned} &fr_j \le fr_j^* \quad \text{for some natural frequencies j}\\ &fr_k \ge fr_k^* \quad \text{for some natural frequencies k}\\ &w_{penalty}(X)=(1+\varepsilon_1.v)^{\varepsilon_2} \quad , \quad v=\sum_{i=1}^{q}v_i \end{aligned} \tag{18}$$

fr_j is the *j*th natural frequency of the structure and fr_j^* is its upper bound. fr_k is the *k*th natural frequency of the structure and fr_k^* is its lower bound. Also, q is the number of frequency constraints. If the *i*th constraint is satisfied v_i will be taken as zero, if not it will be taken as:

$$v_i=\left|1-\left(\frac{fr_i}{fr_i^*}\right)\right| \tag{19}$$

The parameters ε_1 and ε_2 are selected considering the exploration and the exploitation rate of the search space.

5. Enhanced Charged System Search

5.1. Review of the Charged System Search algorithm

Charged System Search (CSS) algorithm, proposed recently by Kaveh and Talatahari [7], is based on electrostatics and Newtonian mechanics laws. The CSS algorithm contains a number of agents which are called charged particles (CPs). In the shape optimization of arch dams, X_j is the position of the *j*th CP containing a number of design variables (i.e. $X_j=\{x_{i,j}\}$) as mentioned in section 4. Each CP is considered as a charged sphere which exerts an electric force on other CPs according to the Coulomb and Gauss laws. The resultant forces and the laws of motion determine the new location of the CPs [21]. The pseudo-code for the CSS algorithm can be summarized as follows:

Step 1. Initialization. Initialize CSS algorithm parameters. Initialize an array of charged particles (CPs) with random positions, as follows:

$$x_{i,j}^{(0)}=x_{i,\min}+\mathrm{rand}.(x_{i,\max}-x_{i,\min}) \quad , \quad i=1,2,...,n \tag{20}$$

where $x_{i,j}^{(0)}$ determines the initial value of the *i*th variable for the *j*th CP; $x_{i,\min}$ and $x_{i,\max}$ are the minimum and the maximum allowable values for the *i*th variable; rand is a random number in the interval [0,1]; and *n* is the number of variables. The initial velocities of the CPs are taken as zero,

$$v_{i,j}^{(0)}=0 \quad , \quad i=1,2,...,n \tag{21}$$

It seems that a CP with good results must exert a stronger force than the bad ones, so the amount of the charge will be defined considering the objective function value, *fit(i)*. Each CP has a charge of magnitude (*q*) defined considering the quality of its solution as:

$$q_i=\frac{fit(i)-fitworst}{fitbest-fitworst} \quad i=1,2,...,N \tag{22}$$

where *fitbest* and *fitworst* are the best and the worst fitness of all the particles; *fit(i)* represents the fitness of agent *i;* and *N* is the total number of CPs. The separation distance r_{ij} between two charged particles is defined as:

$$r_{ij}=\frac{\left\|X_i-X_j\right\|}{\left\|(X_i+X_j)/2-X_{best}\right\|+\varepsilon} \tag{23}$$

where X_i and X_j are the positions of the *i*th and *j*th CPs, respectively; X_{best} is the position of the best current CP, and ε is a small positive number to avoid singularities.

Step 2. CM creation. A number of the best CPs and the values of their corresponding fitness functions are saved in the Charged Memory (CM).

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Step 3. The probability of moving determination. Determine the probability of moving each CP toward the others considering the following probability function:

$$p_{ij}=\begin{cases}1 & \dfrac{fit(i)-fitbest}{fit(j)-fit(i)}>rand \vee fit(j)>fit(i)\\ 0 & \text{otherwise}\end{cases} \quad (24)$$

Step 4. Determination of the forces on CPs. The force vector is calculated for each CP as

$$F_j=q_j\sum_{i,i\neq j}\left(\frac{q_i}{a^3}r_{ij}.i_1+\frac{q_i}{r_{ij}^2}.i_2\right)ar_{ij}p_{ij}(X_i-X_j) \quad \begin{cases}j=1,2,...,N\\ i_1=1,i_2=0\Leftrightarrow r_{ij}<a\\ i_1=0,i_2=1\Leftrightarrow r_{ij}\geq a\end{cases} \quad (25)$$

where F_j is the resultant force affecting the jth CP and ar_{ij} indicates the kind of force and is defined as

$$ar_{ij}=\begin{cases}1 & rand<0.8\\ -1 & else\end{cases} \quad (26)$$

where *rand* represents a random number

Step 5. Solution construction. The new position and velocity of each CP will be obtained by using the following equations:

$$X_{j,new}=rand_{j1}.k_a.\frac{F_j}{m_j}.\Delta t^2+rand_{j2}.k_v.V_{j,old}.\Delta t+X_{j,old} \quad (27)$$

$$V_{j,new}=\frac{X_{j,new}-X_{j,old}}{\Delta t} \quad (28)$$

where $rand_{j1}$ and $rand_{j2}$ are two random numbers uniformly distributed in the range (1,0); m_j is the mass of the CPs, which is equal to q_j in this paper. The mass concept may be useful for developing a multi-objective CSS. Δt is the time step, and it is set to 1. k_a is the acceleration coefficient; k_v is the velocity coefficient to control the influence of the previous velocity. In this paper k_v and k_a are taken as:

$$k_v=0.5(1-\text{iter}/\text{iter}_{\max}) \quad , \quad k_a=0.5(1+\text{iter}/\text{iter}_{\max}) \quad (29)$$

where iter is the iteration number and $\text{iter}_{\max}$ is the maximum number of iterations.

Step 6. CP position correction. If each CP exits from the allowable search space, correct its position using the HS-based handling as described in Refs. [7, 22].

Step 7. CM updating. If the fitness function of some new CP vectors are better than the worst ones in the CM, include the better vectors in the CM and exclude the worst ones from the CM.

Step 8. Controlling the terminating criterion. Steps 3-7 are repeated until a terminating criterion is satisfied.

5.2. An enhanced charged system search algorithm

In the standard CSS algorithm, when the calculations of the amount of forces are completed for all CPs, the new locations of agents are determined. Also CM updating is fulfilled after moving all CPs to their new locations. All these conform to discrete time concept. In the optimization problems, this is known as iteration. On the contrary, in the enhanced CSS, time changes continuously and after creating just one solution, all updating processes are performed. Using this enhanced CSS, the new position of each agent can affect the moving process of the subsequent CPs while in the standard CSS unless an iteration is completed, the new positions are not utilized. All other aspects of the enhanced CSS are similar to the original one [9]. Fig. 4 shows the flowchart of the Enhanced CSS algorithm.

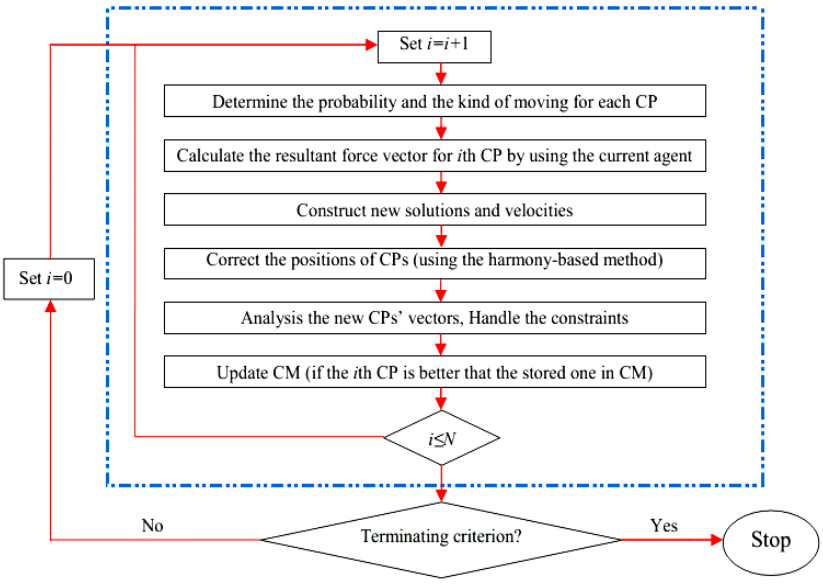

Fig. 4 The flowcharfor the Enhanced CSS algorithm

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6. Neural Network

Optimum design of arch dams for constrained frequencies is a time consuming process because a huge number of eigenproblems should be solved. In order to reduce the computational effort, neural network is used to predict the required natural frequencies during the optimization process.

Neural network training is the first stage of the presented methodology. In this case, a number of arch dams based on their geometric parameters are randomly selected and their natural frequencies are evaluated by conventional finite element analysis using ANSYS. Considering the geometric parameters of the generated dams as the inputs and their corresponding natural frequencies as the targets, the neural network is trained using MATLAB. In the second stage, the optimization task is achieved by CSS and Enhanced CSS algorithms. In these cases, the necessary natural frequencies of the dams are predicted by the trained neural networks and during the optimization processes the modal analysis of the dams is not required. This makes the methodology more efficient.

6.1. BP neural network

Standard BP [23] is a gradient descent optimization algorithm, which adjusts the weights in the steepest descent direction, according to the following equation:

$$W_{k+1}^{BP} = W_{k}^{BP} - \eta_k G_k \tag{30}$$

where W_{k+1}^{BP} and G_k are the weight and the current gradient matrices, respectively, and η_k is the learning rate. Levenberg–Marquardt (LM) algorithm was designed to approach second-order training speed without having to compute the Hessian matrix [24]. In the LM algorithm, the weights updating is achieved as follows:

$$\mathrm{W}_{k+1}^{BP} = \mathrm{W}_{k}^{BP} - \left[\mathrm{J}^{\mathrm{T}}\mathrm{J} + \mu \mathrm{I}\right]^{-1} \mathrm{J}^{\mathrm{T}} E \tag{31}$$

where J is the Jacobian matrix that contains first derivatives of the network errors with respect to the weights, and E is a vector of network errors. Also, μ is a correction factor. The value of μ is decreased after each successful step and is increased only when a tentative step would increase the performance function [24].

7. Implementation of Arch Dam Optimization

In the process of optimization, natural frequencies of generated dams are evaluated by ANSYS or trained BP network. Thus, with the mentioned conditions, optimization is implemented using four different methods listed in the following:

(a) CSS using accurate analysis employing a finite element analysis.

(b) Enhanced CSS utilizing accurate analysis using a finite element analysis.

(c) CSS using approximate analysis utilizing the trained BP network.

(d) Enhanced CSS using approximate analysis employing the trained BP network.

8. Numerical Results

In order to assess the effectiveness of the proposed methodology, the shape optimization of an arch dam with a height of 180m is examined. The widths of the valley in its bottom and top levels are 40m and 220m, respectively, as shown in Fig. 5. The dam is modeled by 11 shape design variables as:

$$X^{T} = \{s \quad \beta \quad t_{c1} \quad t_{c2} \quad t_{c3} \quad r_{u1} \quad r_{u2} \quad r_{u3} \quad r_{d1} \quad r_{d2} \quad r_{d3}\} \tag{32}$$

For this example, t_{c1}, t_{c2}, t_{c3} and r_{u1}, r_{u2}, r_{u3} and r_{d1}, r_{d2}, r_{d3} are values of t_c , r_u and r_d at z_1=0, z_2=0.55h and z_3=h, respectively. The lower and upper bounds of design variables can be determined using empirical design methods as:

$$\begin{array}{llll} 0 \le s \le 0.3 & 4m \le t_{c1} \le 12m & 50m \le r_{u1} \le 180m & 50m \le r_{d1} \le 180m \\ 0 < \beta \le 1.0 & 8m \le t_{c2} \le 30m & 40m \le r_{u2} \le 120m & 40m \le r_{d2} \le 120m \\ & 12m \le t_{c3} \le 40m & 10m \le r_{u3} \le 50m & 10m \le r_{d3} \le 50m \end{array} \tag{33}$$

Also, multiple natural frequency constraints are considered as:

$$fr_1 \ge 3\mathrm{Hz} \quad fr_3 \ge 6\mathrm{Hz} \quad fr_4 \ge 7\mathrm{Hz} \tag{34}$$

In this example, the main steps for the optimization of arch dams under frequency constraints by CSS algorithm can be summarized as follows:

Step 1: *Initialization.* Initialize CSS algorithm parameters. The number of CPs is set to 20. Using Eq. (33), initialize all CPs with random positions. With these initialized positions, calculate the separation distances between each two CPs. The initial velocities of the CPs are considered as zero.

Step 2: *CM creation.* Evaluate the values of the fitness function for the CPs, compare with each other and sort increasingly. Store 5 numbers of the first CPs and their values of corresponding fitness functions in the CM.

Step 3: *The probability of moving determination.* Determine the probability of moving each CP toward the others.

Step 4: *Determination of the forces on CPs.* Calculate the resultant force vector for each CP. In this paper, magnitude of CPs' radius is set to unity.

Step 5: *Solution construction.* Move each CP to the new position and find the new velocities. The maximum number of iterations is set to 100.

Step 6: *CP position correction.* If each CP exits from the allowable search space, correct its position using the HS-based handling as described in Ref. [7, 22]. The value of PAR is taken as 0.1.

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Step 7: *CM updating.* Evaluate and compare the values of the objective function for the new CPs, and sort them increasingly. If some new CP vectors are better than the worst ones in the CM, include the better vectors in the CM and exclude the worst ones from the CM.

Step 8: *Controlling the terminating criterion.* Stop repeating steps (3) to (7) after 100 iterations.

In the above steps, the natural frequencies of dams can be predicted by the trained BP network instead of evaluating by ANSYS. Therefore for training BP network the following steps are required:

(a) Some arch dams considering their geometric parameters are selected randomly.

(b) Natural frequencies of the selected dams are evaluated by ANSYS.

(c) By using the geometric parameters of the generated dams as the inputs and the corresponding natural frequencies as the targets, the BP network is trained.

For BP network testing, the errors between the exact and approximate frequencies are calculated using the following equation:

$$error = \frac{\left| fr_{ap} - fr_{ex} \right|}{fr_{ex}} \times 100 \tag{35}$$

where fr_{ap} and fr_{ex} represent the approximate and exact frequencies, respectively.

The optimization process is performed by a Core™ 2 Duo 2.66 GHz computer and the time of all computations is evaluated in clock time.

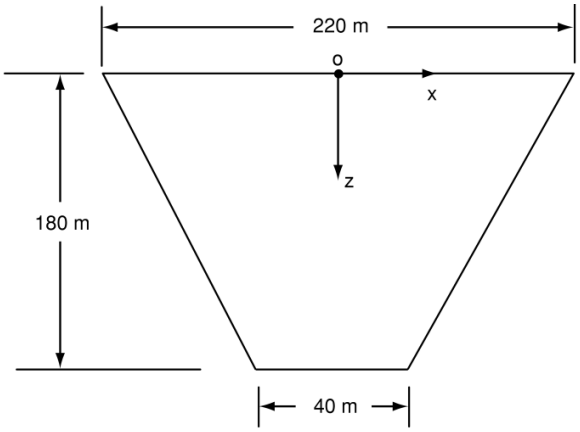

Fig. 5 The valley dimensions of the arch dam

8.1. Neural network training and testing

For neural network training, inputs and outputs are the design variables of the arch dams and their corresponding natural frequencies, respectively. Here, 350 training pairs are randomly generated and 250 and 100 samples are used for training and testing the network, respectively. The total time spent to data generation phase is equal to 33 min. Using the mentioned data, BP neural network is trained. The time spent to train the BP is 0.2 min, and the numbers of hidden layer neurons in BP network is considered as 10. It should be noted that the number of BP neurons is determined by trial and error. In this case, the number of BP neurons is changed and the testing errors are monitored. The best results are observed in the case of 10 hidden layer neurons. So, the size of network is 11-10-3. Training results are provided in Table 3. It is observed that the BP network possesses appropriate generalization and can be employed in the optimization process.

Table 3 Maximum and mean errors of BP network in testing mode

	Network	Errors (%)			Ave.
		fr_1	fr_3	fr_4	
Mean errors (%)	BP	0.708	0.797	0.601	0.702
Maximum errors (%)	BP	5.466	5.731	3.275	4.824

8.2. Optimization results

Optimum solutions obtained by the various methods are provided in Table 4. As can be observed from this table, the solutions found by the Enhanced CSS are more economical, and the best solution is attained by Enhanced CSS using BP network. It is observed that the overall time of the optimization by neural network is 0.3 times of optimization by EA. This demonstrates that the overall time of optimization can be significantly reduced using neural network.

Table 4 Optimum designs of the arch dam obtained by the various methods

Design variables	Optimum design			
	CSS		Enhanced CSS	
	Exact	BP	Exact	BP
s (m/m)	0.217	0.297	0.211	0.296
β (m/m)	1.000	0.997	1.000	0.979
t_{c1} (m)	8.321	7.611	7.143	11.899
t_{c2} (m)	8.000	8.000	8.000	8.000
t_{c3} (m)	12.001	12.000	12.000	12.000

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r_{u1} (m)	140.746	151.126	149.648	150.809
r_{u2} (m)	61.153	62.730	61.153	61.104
r_{u3} (m)	22.879	20.377	22.491	21.825
r_{d1} (m)	54.835	50.299	54.835	60.508
r_{d2} (m)	40.005	44.383	40.005	42.298
r_{d3} (m)	22.842	20.084	22.478	21.796
Cost ($\$10^6$)	**15.035**	**14.908**	**14.978**	**14.878**
Value of violated constraints (%)	0	0	0	0
Required iterations	100	100	100	100
Optimization time (min)	185	22.7	188	24
Data generation time (min)	—	33	—	33
Training time (min)	—	0.2	—	0.2
Overall computing time (min)	**185**	**55.9**	**188**	**57.2**

Table 5 represents the natural frequencies of optimum dams obtained by various methods mentioned in Table 4.

Table 5 Natural frequencies of optimum dams obtained by various methods in Table 4

Natural Frequency	CSS		Enhanced CSS	
	Exact	BP	Exact	BP
fr_1 (Hz)	5.02	5.13	5.04	4.81
fr_3 (Hz)	6.01	6.01	6.01	6.28
fr_4 (Hz)	7.00	7.00	7.00	7.00

Table 6 shows the central angles of optimum dams obtained by various methods mentioned in Table 4.

Table 6. The central angles of optimum dams obtained by various methods in Table 4

Central angle	CSS		Enhanced CSS	
	Exact	BP	Exact	BP
φ_1 (deg)	97.98	90.28	91.33	90.5
φ_2 (deg)	129.8	125.8	129.8	129.9
φ_3 (deg)	112.2	129.1	114	118.4

Fig. 6 shows the convergence curves obtained by 4 different methods.

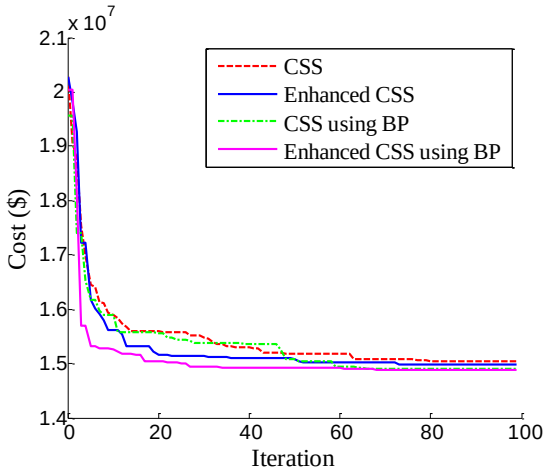

Fig. 6 The convergence curves obtained by the various methods

The errors of approximate frequencies of optimum designs predicted by BP network are compared to their corresponding accurate ones (obtained from finite elements analysis) in Table 7. It can be observed that the accuracy of approximate frequencies obtained by BP network is reasonably high.

Table 7 Error percentage of approximate frequencies of optimum dams

Frequency No.	CSS	Enhanced CSS
	BP	BP
1	0.98	1.05
3	0.83	0.48
4	1.16	2.94
Ave.	0.99	1.49

9. Concluding Remarks

Shape optimization of arch dams for constrained frequencies is performed in this paper. For this purpose, a finite element model based on modal analysis of arch dam is presented. The results of this model are compared to those of reported in literature and its performance is verified. In order to optimize the arch dam a combination of the new meta-heuristics algorithms and neural network is utilized. The meta-heuristics algorithms used in this investigation are CSS and Enhanced CSS. In order to reduce the computational cost of the optimization process the natural frequencies of the arch dams are evaluated using properly trained back propagation (BP) neural network instead of their exact modal analysis.

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It should be noted that the solutions found by EA is more accurate than those obtained by utilizing the neural networks, but the time spent in the case of using neural networks is about 0.3 of the time required for optimization using EA, while the computational errors appeared due to approximation, are negligible. It is demonstrated that the optimum design obtained by Enhanced Charged System Search using the BP network is the best compared to the other results. The numerical results demonstrate the efficiency and computational merits of the proposed methodologies for shape optimization of arch dams.

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