A Simple Strut-and-Tie Model for Prediction of Ultimate Shear Strength of RC Deep Beams

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Abstract: *In this paper a new method based on Strut-and-Tie Model (STM) is proposed to determine the shear capacity of simply supported RC deep beams and an efficiency factor for concrete with considering the effect of web reinforcements. It is assumed that, the total carried shear force by RC deep beam provided by two independent resistance, namely diagonal concrete strut due to strut-and-tie mechanism and the equivalent resisting force resulted by web reinforcements, web reinforcing reduces the concrete compression softening effect with preventing from the diagonal cracks opening or concrete splitting. The unknown function and parameters are determined from 324 experimental results obtained by other researchers. To validate the proposed method, the obtained results are compared with some of the existing methods and codes such as ACI 318-05 and CSA. The results indicate that the proposed method is capable to predict the shear strength of variety of deep beams with acceptable accuracy.*

Keywords: *Strut-and-tie, deep beams, shear strength, web reinforcement*

1. Introduction

As reinforced concrete deep beams have become an important structural element, their behavior and ultimate shear strength has been the subject of many researchers devoted to determine the influence of effective parameters. Several different modes of failure have identified from the experimental studies, due to the variability in the failure, the determination of their shear capacity and identification of failure mechanisms are very complicated.

The existing methods for analysis and design of deep beams consist of rational and semi-rational approaches as sectional approach or strut-and-tie Model (STM). Generally the conventional design formulas have been calibrated for normal strength concrete (NSC) and their reliability must be developed for high strength concrete (HSC)[1].

In STM compressive force are carried by a compressive field or concrete struts and tensile force by main longitudinal reinforcements, the concrete compression softening effect was usually applied to diagonal struts. STM was laid by Ritter (1899). Ritter's original goal was to explain that stirrups in RC members, Ritter's model was expanded later by Mörsch (1902), Mörsch proposed that the diagonal compressive stresses in the concrete need not be discrete zones, but could be continuous field in equilibrium with discrete stirrup forces [2].

In parallel to the sectional approach, the strut-and-tie method is gaining rapid popularity for deep beams.Other researchers proposed some approaches applicable in D-regions [3]. These approaches help design a complex structure maximally safe. STM has been adopted in some American codes such as the Canadian Standard Association (CSA) and ACI which most recently has included STM approach in 2008 edition of the Building Code Requirements for Structural Concrete (ACI 318)

There are many parameters affecting on the shear strength of RC deep beams, where the most important of them consist of concrete compressive strength, shear span-depth ratio and the amount and arrangement of vertical and horizontal web reinforcements.

The main aim of this paper is proposing a developed STM to determine the shear strength of deep beams, which is applicable for a wide

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range of concrete compressive strength and given a physical significance.

The present study involves a combination of two load carrying mechanisms, namely diagonal crushing of concrete strut due to STM and an equivalent resisting force perpendicular to strut due to web reinforcements.

To evaluate the applicability and accuracy of the proposed model, the results of 324 RC deep beam tests are used. All of the selected specimens are simple supported; top point loaded, had various compression strength of concrete and had been failed in shear modes. The obtained results closely tracked the experimental results and the results of some existing methods such as the simplified softened truss model of Mau-Hsu [4], the combined softened STM of Hwang-Lee [5], the combined softened STM of Matamoros et al [6], the formula proposed by Foster-Gilbert based on plastic truss model [7], the provided STM by ACI 318-05 [8] and CSA [9] codes.

Then it was concluded that the prediction by the proposed method are sufficiently conservative and accurate to be used reliably to compute the shear capacity of RC deep beams over a wide range of shear span-to-depth ratio and concrete strengths and various combinations and amount of web reinforcements.

2. Strut-and-Tie model basis

Fig.1 shows a typical deep beam and its STM, this beam is loaded on top face by two vertical point loads and supported at the opposite face. The longitudinal main reinforcements are located at a distance d from top. This beam is not detailed with any web reinforcement. Assuming that the flexure strength is sufficient, the failure of deep beam is governed by the compressive stress at the strut and its diagonal crushing. The shear strength is predicted by STM due to the diagonal struts and shear force flows along the strut from loaded point to the support.

The equilibrium of the applied forces leads to the following expressions (Fig.2).

$$T_s = C_c \cos\theta \tag{1}$$

$$V_c = C_c \sin\theta \tag{2}$$

where: C_c is the compression force in the diagonal strut, θ is the angle between strut and longitudinal reinforcements, T_s is the tension force on longitudinal reinforcements (or ties) and V_c is the applied load on top of the deep beam.

The inclined angle of the diagonal strut is given by

$$\theta = \tan^{-1}\left(\frac{jd}{a}\right) \tag{3}$$

where: ***a*** is the shear span measured center-to-center from load to support and jd is the distance of lever arm from the resultant compressive force to the center of the main tensile longitudinal reinforcements.

Using the assumption of Hwang et al [10], this term can be estimated as

$$jd = d - \frac{kd}{3} = \left(1 - \frac{k}{3}\right) d \tag{4}$$

where: kd is the depth of the compression zone or horizontal prismatic strut, this coefficient by neglecting the compressive longitudinal reinforcement is defined as; [10]

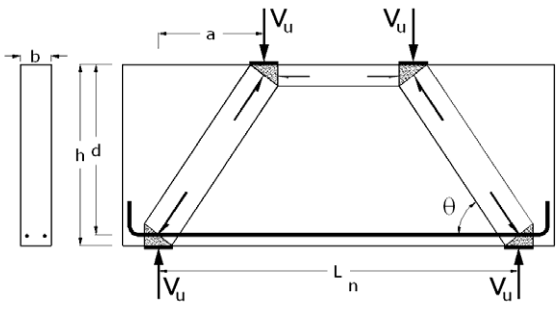

Fig.1. Geometry of concrete deep beam

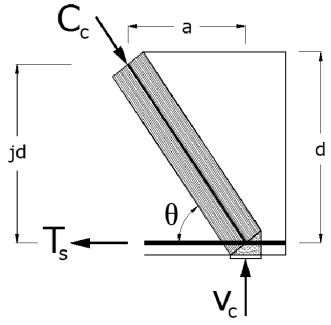

Fig.2. Equilibrium of strut in absence of web reinforcement

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$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \quad (5)$$

where: n is the modular ratio of elasticity, $(n = \frac{E_s}{E_c})$, E_s, E_c are the steel and concrete elasticity module, consequently ρ is the longitudinal reinforcements ratio, $(\rho = \frac{A_s}{bd})$, A_s, is the area of main longitudinal reinforcements and $\boldsymbol{b}$ is the width of beam.

Although the diagonal strut is formed in bottle-shape, in the current study it is assumed that the strut has a prismatic form with a uniform width. The mean compressive stress in the strut can be computed as the force acting on the strut dividing by its cross-sectional area by following expression;

$$C = f_{ce}^{'} A_{str} \quad (6)$$

where: f'_{ce} is the maximum strength of the softened concrete strut and A_{str} is the cross-sectional area of strut which can be calculated as

$$A_{str} = b\, a_s \quad (7)$$

where: b is the width of beam and a_s is the uniform width of strut which can be estimated as

$$a_s = \min\left(l_b \sin\theta + d_a \cos\theta, l_p \sin\theta + w_t \cos\theta\right) \quad (8)$$

where: l_p is depth of the bottom node, taken as twice the cover to the main reinforcements, w_t is width of the support bearing plate, l_b is the depth of the top node and d_a is the width of the loaded point bearing plate.

2.1. Compression strength of softened concrete

Cracked reinforced concrete can be treated as an orthotropic material with its principal axes corresponding to the directions of the principal average tensile and compressive strains. Cracked concrete subjected to high tensile strain in the direction normal to the compression is observed to be softer than concrete in a standard cylinder test [11]. This phenomenon of strength and stiffness reduction is commonly referred to as compression softening. Applying this softening effect to the STM, it is recognized that the tensile straining perpendicular to the strut will reduce the capacity of the concrete strut to resist compressive stresses. Hence according to Fig.4, $f_{ce}^{'}$ or softened concrete strength can be determined by

$$f_{ce}^{'} = \upsilon f_c^{'} \quad (9)$$

where: f'_c is specified compressive strength of concrete based on cylinder tests, and υ is the efficiency factor of concrete.

3. Equivalent web reinforcement force

A typical arrangement of web reinforcement is shown in Fig. 5; experimental tests prove that the effective resistance of web reinforcements is

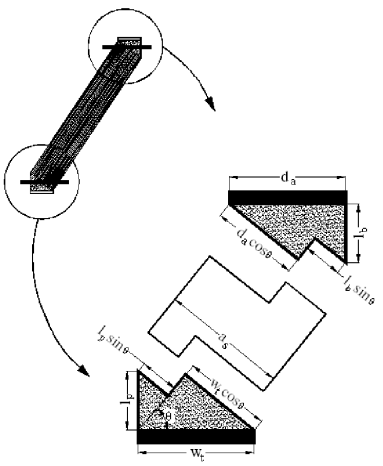

Fig.3. Prismatic strut geometry

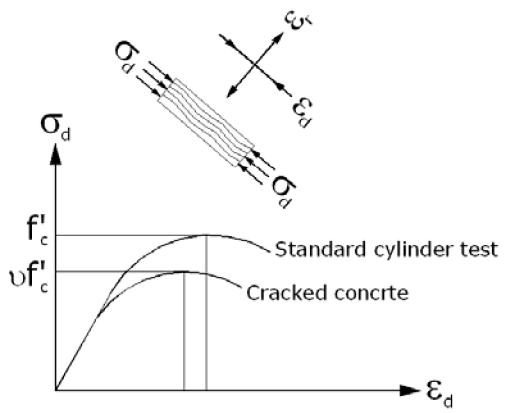

Fig.4. Softening of compressive stress-strain curve due to transverse tensile strain [10]

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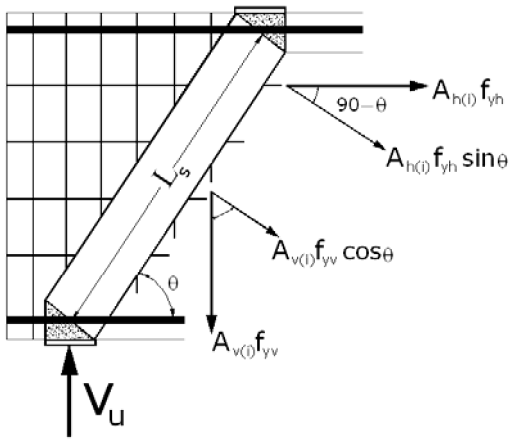

Fig.5. Resisting force perpendicular splitting crack [12]

always after extensive cracking of web. If the splitting crack is assumed to open without shear slip along the crack, the force in the reinforcement bars crossing the cracks must be calculated in uniaxial state. Otherwise, it can be assume that the shear slip the state of stress will be biaxial due to tension stress and shear stress applied in the reinforcement [12].

In this paper, the shear slip along the splitting crack was not considered. In STM it is assumed that the diagonal cracks occur in fully state and there is no force due to aggregate interlocking [12]. The maximum equivalent resisting force perpendicular to splitting crack $F_{p(\max)}$ can be written as [13]

$$F_{p(\max)} = A_v f_{yv} \cos\theta + A_h f_{yh} \sin\theta = \tag{10}$$

$$= \frac{A_v f_{yv}}{n_v s_v} L_s cos^2\theta + \frac{A_h f_{yh}}{n_h s_h} L_s sin^2\theta$$

where: n_v , n_h are the numbers of vertical and horizontal reinforcing bars crossing the crack, f_{yv} , f_{yh} are the tensile yield stress in the vertical and horizontal bars, respectively, s_v , s_h are the vertical and horizontal spacing between web reinforcements, $A_{h(i)}$ and $A_{v(i)}$ are the area of each of the horizontal and vertical web reinforcement crossing the crack, respectively, A_h and A_v are the total area of horizontal and vertical web reinforcements crossing the crack, respectively and L_s is the length of strut [13].

By substituting the $\rho_v = \frac{A_v}{b\, n_v s_v}$, $\rho_h = \frac{A_h}{b\, n_h s_h}$ and $f_{yv} = f_{yh} = f_y$ in Eq. 10, it becomes

$$F_{p(\max)} = \rho_v f_y\, bL_s cos^2\theta + \rho_h f_y\, bL_s sin^2\theta \tag{11}$$

The maximum tensile contribution on the perpendicular direction on the crack $f_{p(\max)}$ can be computed as [13]

$$f_{p(\max)} = \rho_v f_y\, cos^2\theta + \rho_h f_y\, sin^2\theta \tag{12}$$

$$= (\rho_v\, cos^2\theta + \rho_h\, sin^2\theta) f_y = \rho_p f_y$$

The new term ρ_p is defined as "equivalent reinforcement ratio perpendicular to splitting crack" and can be computed as

$$\rho_p = (\rho_v\, cos^2\theta + \rho_h\, sin^2\theta) \tag{13}$$

ρ_p , is an equivalent ratio that be calculated in tangential-section area of strut. $(A_{str(t)} = bL_s)$. where $A_{str(t)}$ is the area of tangential-section of strut.

In ACI 318-05 [8] according to Eq. (A-4) in appendix A, the value of ρ_p was defined as

$$\rho_p = (\rho_v\, cos\theta + \rho_h\, sin\theta) \tag{14}$$

3.1. Equilibrium of applied forces on strut

Fig. 6 shows the resulted internal forces in the STM due to applied loads, composed of two independent forces, namely C_c and F_p the total shear strength of beam is given by vertical equilibrium of C_c and F_p

$$V_u = C_c \sin\theta + F_p\, \cos\theta = V_c + V_w = \tag{15}$$

$$= C_c \sin\theta + \beta\, \rho_p\, bL_s \cos\theta$$

where: $C_c\, sin\theta$ or V_c is the shear strength provided by the STM due to the diagonal concrete compression strut and $F_p\, cos\theta$ or V_w is the shear strength resulted by resisting mechanism of web reinforcements against concrete splitting.

In Eq.11 it is assumed that the web reinforcements attain their yield stress, but in

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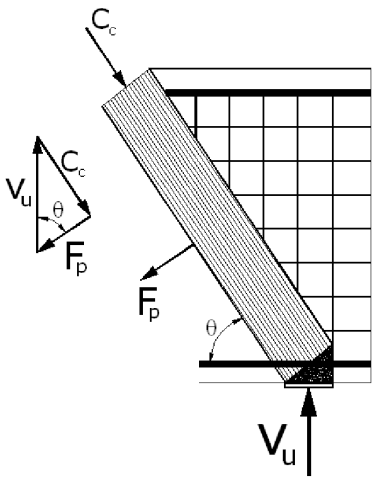

Fig.6. Equilibrium of strut reinforced by web reinforcement

practice, they are subjected to various stresses. The reinforcing bars located in the central region of strut have higher strain in comparison with the reinforcements near the supports or loaded point. Therefore the term f_y is substituted by mean stress in web bars equals to βf_y. According to experimental results β is a coefficient that depends on the equivalent perpendicular reinforcement ratio (ρ_p) and must be less than 1.0

Substituting Eq.15 equal to Eq. 2 gives

$$V_u = v_{cw} f_c^{'} A_{str} \sin\theta \tag{16}$$

where:

$$v_{cw} = v_c + v_w = v_c + \beta \rho_p \frac{a}{a_s} \left(\frac{f_y}{f_c^{'}}\right) \frac{1}{\sin\theta} \tag{17}$$

According to Eq.17, it is proved that after concrete diagonal cracking the reduction effect of concrete softening is reduced and the efficiency factor of concrete in the presence of web reinforcing can be substituted by v_{cw} the difference between v_{cw} and v_c just equals the v_w provided by the web reinforcements. Therefore it can be assumed that the shear strength of deep beam is governed only by the diagonal compression strut, but to determine the strut force in Eq. 2, the efficiency factor must be computed with considering the improvement effect of web reinforcements by Eq.17

4. Solution procedure

In this study, the investigation of the experimental results indicates that:

a. As the shear span-to-depth ratio $(\frac{a}{d})$ increases, the measured efficiency factor for the strut concrete decreases, a result of the increasing tensile straining.

b. By increasing concrete strength, concrete becomes brittle and the efficiency of strut decreases. Generally high-strength concrete deep beams exhibited abrupt shear failure without any warning.

c. There is a critical amount of web reinforcement depending on concrete strength, span-depth ratio and ratio of tensile main reinforcemen. If the reinforcement is less than critical amount, the width of cracks cannot be sufficiently controlled and the failure of deep beam is resulted by excessive cracks, therefore the shear strength of beam is reduced significantly by concrete softening.

But when the web reinforcement is sufficient or higher than the critical amount, controlling the width of cracks is done perfectly, although the additional reinforcements can strengthen the concrete, but its efficiency is less than efficiency of additional reinforcing in deep beams with light amount of web reinforcements. In other word, relative increment of shear capacity of deep beam with specified concrete strength and shear span-depth ratio depends on the web reinforcement amount. In other word, in empirical modeling of shear strength of deep beams, the first derivation of shear strength versus perpendicular equivalent reinforcement ratio must be a descending function.

According to above notes, the nonlinear model for computing the v_{cw} (Eq.17) can be obtained from Eq.18

$$v_{cw} = \frac{f_c^{'x}}{a + b\left(\frac{a}{d}\right)^2} + \beta \rho_p \left(\frac{f_y}{f_c^{'}}\right)\left(\frac{a}{a_s}\right) \frac{1}{\sin\theta} \tag{18}$$

The first term of Eq.18 presents the efficiency factor in the absence of web reinforcement, to obtain it, multiple compression softening models were used, after comparing the results the general expression for computing v_c was adopted from the simplified form of the proposed model by

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Table 1 Details of selected deep beam specimens

Reference	No. of tested beams	$(\frac{a}{d})$	$f'c$ (MPa)	Loading	
				Double	Single
Arabzadeh [15]	8	1.00-2.50	31.1-47.6		×
Clark [16]	34	0.35-1.18	18.6-27.6	×	×
De paiva an Siess [17]	10	0.67-1.33	21-42	×	
Foster-Gilbert [1]	16	0.70-1.90	50-120	×	×
Kong et al [18]	35	0.23-0.70	21-28	×	
Oh and Shin [19]	53	0.50-2.0	23.7-73.6		×
Ramakrishnan and Ananthanarayana [20]	13	0.20-0.90	14-28	×	×
Rogowsky et al [21]	45	0.80-1.60	26.1-46.8	×	×
Shin et al [22]	24	1.50-2.00	52-73	×	
Smith and vantsiotis [23]	52	1.00-2.08	16.1-22.7	×	
Tan et al [24]	18	0.27-2.15	41.1-58.8	×	
Yang et al [25]	16	0.75-2.29	31.4-78.5	×	
total	324				

Vecchio-Collins (1986) [14] based on MCFT (Modified Compression Field Theory)

There are three unknown parameters (a, b, x) ,an unknown function (β) in Eq.18 which are determined on the base of experimental results. For this purpose 324 deep beams have been selected from following references (Table 1). The specimens in the assembled data base

i) Had strut inclination angle θ greater than 25° and clear length to overall height ratio less than 4 (according to Appendix A-ACI 318-05)

ii) All beams are simply supported with one or two point load, the width of bearing plates in supports and loaded points are adopted from relevant papers. The governed failure mode of beams was shear (shear, shear-compression or shear-flexure) and anyone of the selected specimens had not been failed on flexural mode or due to bearing of loaded point or supports.

The unknown factors in Eq.18 are determined by a nonlinear regression and minimizing the residual errors. Finally

$$a = 0.5\,,\ b = 0.1\,,\ x = -0.3\,,\ \beta = 0.09\,\rho_p^{-0.35}$$

Therefore, Eq.18 becomes

$$v_{cw} = \frac{f_c'^{-0.3}}{0.5+0.1\left(\frac{a}{d}\right)^2} + 0.09\rho_p^{0.65}\left(\frac{f_y}{f_c'}\right)\left(\frac{a}{a_s}\right)\frac{1}{\sin\theta} \qquad (19)$$

Substituting Eq. 19 in Eq. 16 gives;

$$V_u = \frac{f_c'^{0.70}}{0.5+0.1\left(\frac{a}{d}\right)^2}\,A_{str}\,\sin\theta + 0.09\,\rho_p^{-0.35}A_{wp}\cos\theta \qquad (20)$$

where: A_{wp} is the equivalent area of perpendicular web reinforcements crossing strut and can be computed as;

$$A_{wp} = A_v\,cos\theta + A_h\,sin\theta = \rho_p\,A_{str(t)} \qquad (21)$$

where: A_v, A_h are the areas of vertical and horizontal reinforcement crossing strut respectively. According to Eq.19, v_w can be computed as;

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$$v_w = 0.09\rho_p^{0.65}\left(\frac{f_y}{f_c'}\right)\left(\frac{a}{a_s}\right)\frac{1}{\sin\theta}v_w = \quad (22)$$

$$= 0.09\rho_p^{-0.35}\frac{A_{wp}f_y}{A_{str}f_c'}\cot\theta = 0.09\rho_p^{-0.35}\ \frac{F_{p(\max)}}{C_{max}}\ \cot\theta$$

where: $F_{p(\max)}, C_{max}$ are the applicable perpendicular tensile force and the maximum applicable diagonal compressive force which can be applied in strut, respectively.

It indicates that, increasing the transverse tensile strength of the concrete strut versus the longitudinal compressive strength of it, can improve the softening effect on concrete strut.

Due to the term $0.09\rho_p^{-0.35}$ in Eq. 22, the first derivation of v_w versus ρ_p is a descending function, meaning that the additional web reinforcements in concrete deep beams with light amounts of web reinforcements is significantly more effective than the beams with a heavily reinforced web. Also for a high-strength concrete deep beam to exhibit a ductile shear failure, it requires more amount of web bars than normal-strength concrete beams.

In Eq. 22 the term $(0.09\rho_p^{-0.35}\ f_y)(A_{wp})$ can be substituted with mean effective equivalent resisting force perpendicular to diagonal crack, consequently the steel bars are subjected to lower stresses than f_y, equal to $0.09\rho_p^{-0.35}\ f_y$.

It is concluded that the mean stress of web bars depends on their amount.

5. Evaluation of proposed model reliability

The shear capacity (V_u)of the 324 considered specimens shown in Table 1, has been evaluated by means of the proposed model and some existing expression such as:

a. The plastic truss model revised by Foster-Gilbert [7]

b. The simplified softened truss model provided by Mau-Hsu [4]

c. The modified combined softened STM provided by Hwang-Lee [5]

d. The combined STM proposed by Matamoros et al [6]

e. The STM according to Appendix-A of ACI 318-05 [8]

f. The approach of CSA [9]

Table 2 summarizes the statistical results

Table 2 Summarizes statistical analysis of experimental-to-predicted shear strength ratio for existing method and proposed model, $\left(\frac{V_{exp}}{V_u}\right)$

The compared model	STD	VAR	Mean	COR
Foster-Gilbert	0.51	0.26	1.09	0.81
Mau-Hsu	0.85	0.73	1.84	0.76
Hwang-Lee	0.41	0.17	1.28	0.83
Matamoros et al	0.31	0.09	0.81	0.80
ACI 318-05 code	0.38	0.14	1.33	0.87
CSA code	0.55	0.3	1.60	0.89
Proposed model	0.20	0.04	1.02	0.93

STD: Standard diviation

VAR: The value of variation

Mean: The mean of $\left(\frac{V_{exp}}{V_u}\right)$ ***for specimens***

COR: Correlation between experimental and predicted results

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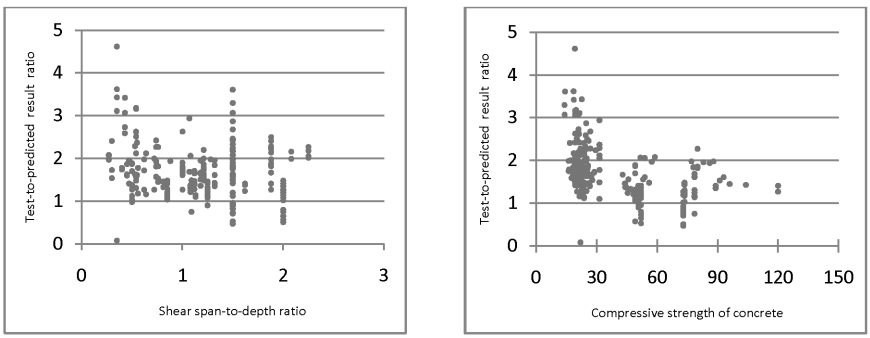
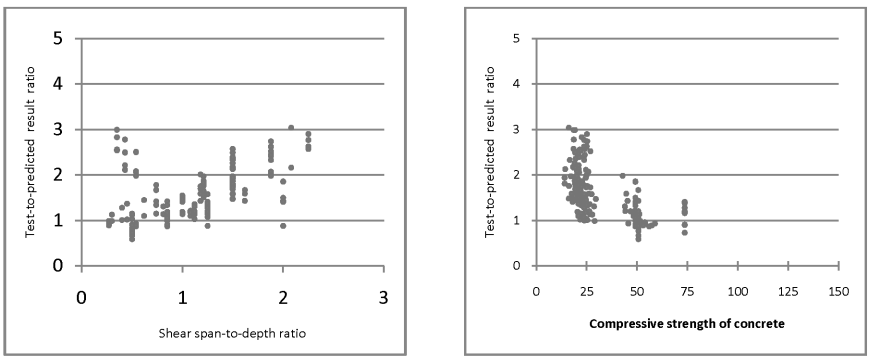
obtained from comparison.

The ultimate experimental shear strength versus predicted value is plotted for all selected models including proposed model in Figs.7, 8. According to Table 2 and Figs.7-8, it can be concluded that the proposed model is more reliable and accurate than other existing model, in particular:

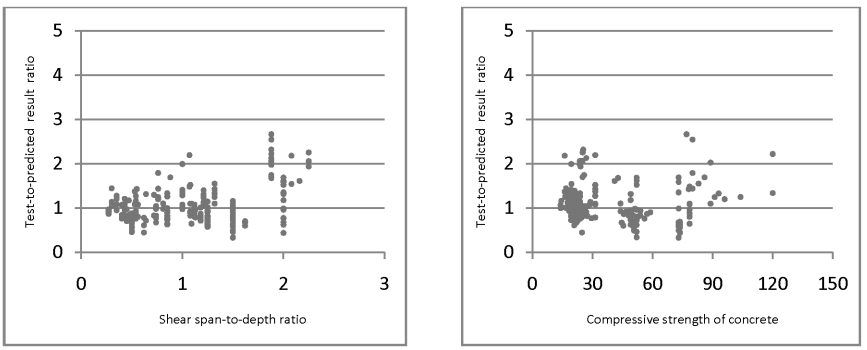
a. The formula of Mau-Hsu is the most conservative as the ratio $\left(\frac{V_{exp}}{V_u}\right)$ is scattered versus a



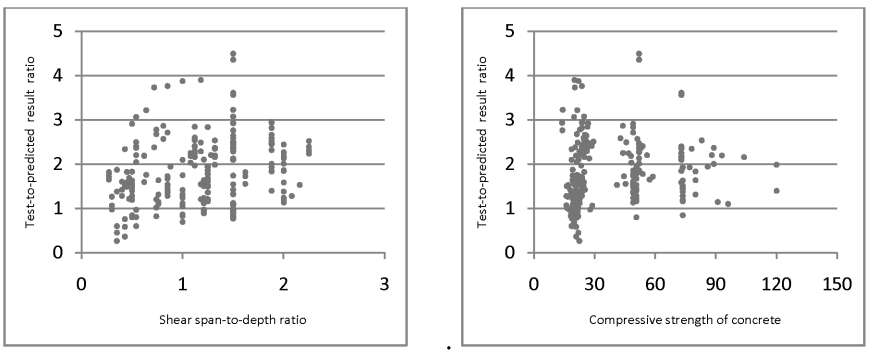
(a): Appendix.A ACI 318-05



(b): Approach of CSA



(c): Foster-Gilbert



(d): Mau-Hsu

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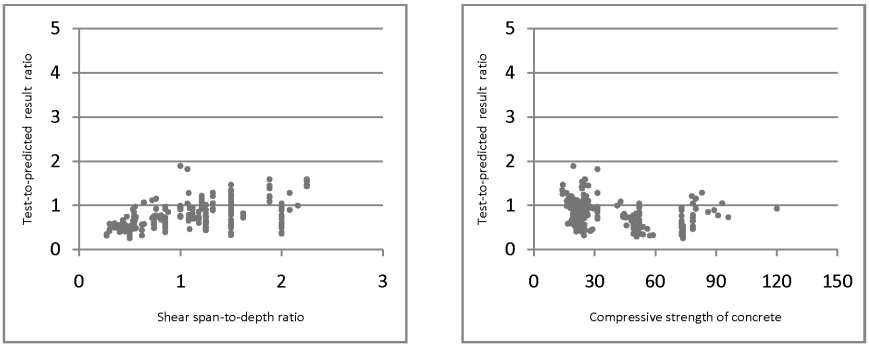
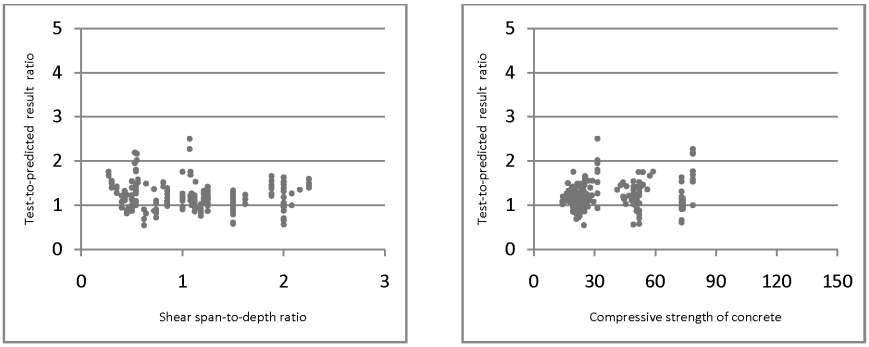
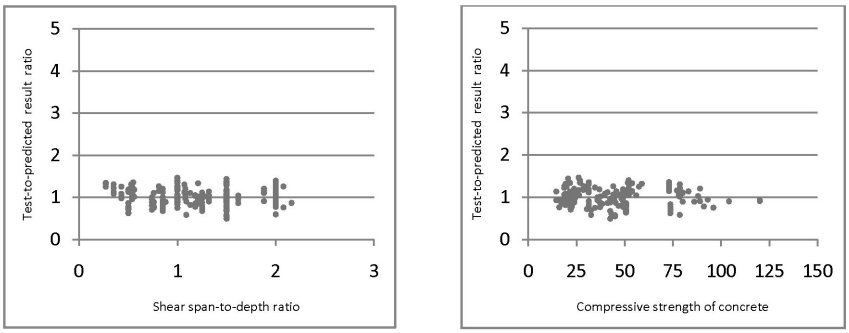


(e): Matamoros et al

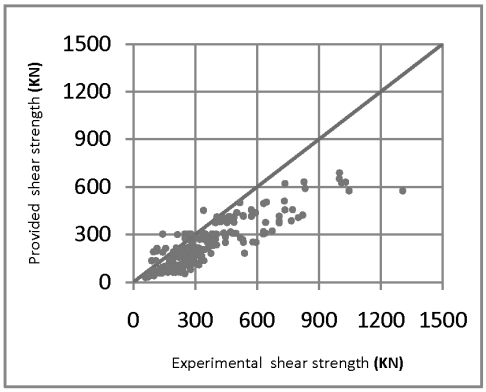


(f): Hwang-Lee

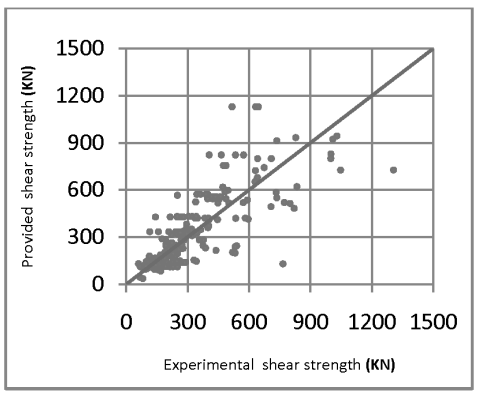


(g): Proposed model

Fig.7. Effect of shear span-to-depth ratio and Compressive strength of concrete on the ratio of experimental ?shear strength to predicted shear strength by selected model



(a): Appendix A ACI 318-05



(b): Foster-Gilbert

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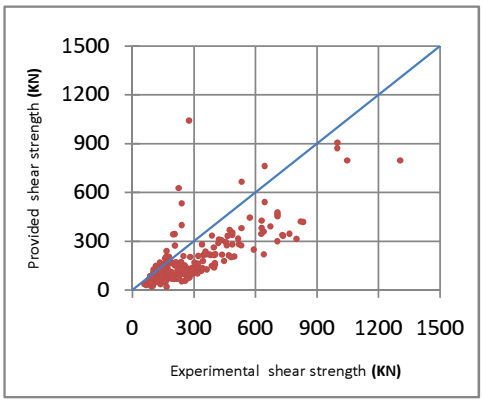
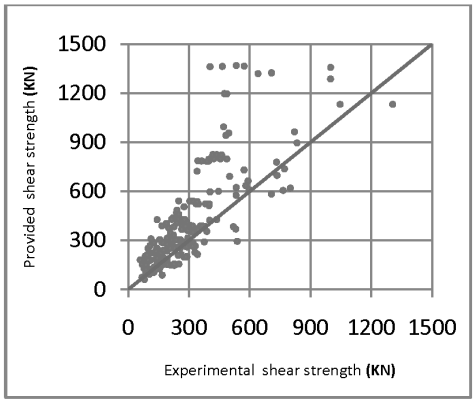
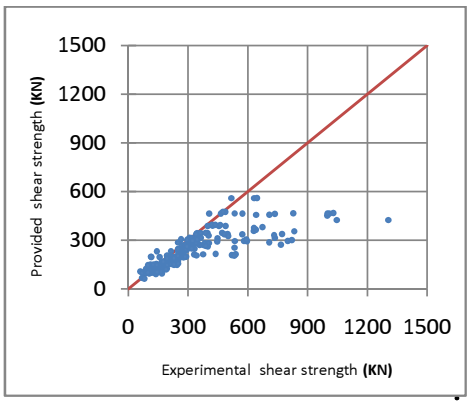


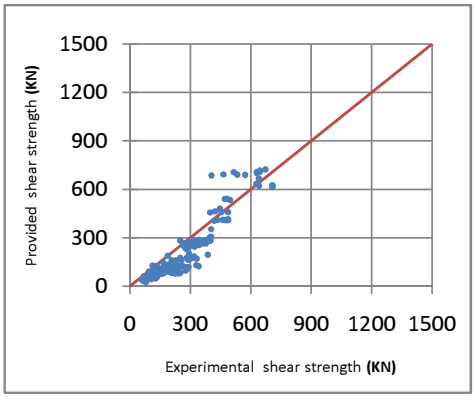
(c); Mau-Hsu



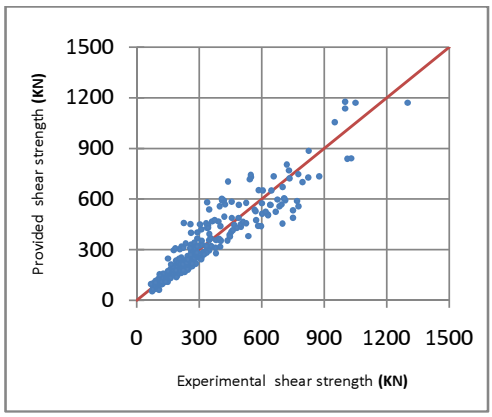
(d): Matamoros et al



(e): Hwang-Lee



(f): Approach of CSA



(g): Proposed model

Fig.8. Correlation between the experimental results and predicted strength by selected model?

wide range of $(\frac{a}{d})$ and f_c'.

b. The model of Matamoros et al has underestimated the shear strength of the beams by showing a poor correlation versus the experimental data of the beams with $(\frac{a}{d})$ less than 1

Although the results of the proposed models by Hwang-Lee and the method of Foster-Gilbert are scattered for high-strength concrete, but still, have given better correlation with experimental results in comparison with the formula of Mau-Hsu. Similarly the results provided by ACI and CSA haven't given a sufficient correlation with experimental results, but are less scattered.

6. Final design expression

In the proposed model, as the mean of the experimental-to-predicted shear strength ratio is

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equal to 1.0, it cannot be applied for the design and therefore must be modified. For this purpose, a design formula based on proposed model which statistically leaves only five percent of the selected specimens (16 specimens) is obtained.

This formula is a modified form of Eq.18 by multiplying the coefficient 0.72, hence the new predicted design expression becomes;

$$V_u = \frac{f_c^{'0.70}}{0.7+0.15\left(\frac{a}{d}\right)^2} A_{str} \sin\theta + 0.065\,\rho_p^{-0.35} A_{wp} \cos\theta \qquad (23)$$

where current parameters provided by Eq. 20, applying Eq. 23 to 324 selected RC specimens and investigation obtained results for the experimental-to-predicted shear strength leads to mean 1.42 with STD. and VAR. equal to 0.27 and 0.07 respectively.

7. Conclusions

By considering a simplified mechanical behavior of deep beams and proposing a applicable model to determine the shear strength, after comparison with 324 specimens available in the literature, the following conclusions can be drawn,

1. A consistent model to predict the shear capacity of RC deep beams is obtained by super-posing two independent factor in the shear resisting of the deep beams, namely diagonal concrete strut action due to strut-and-tie mechanism (STM) and resisting equivalent force perpendicular to diagonal cracks resulted by web reinforcements.
2. The STM approaches of ACI 318-05 and CSA codes generally leads to very conservative values, but with scattered prediction.
3. The proposed model for design and analysis of deep beams, yields to a simple formulas having a physical significance, in addition, over the large number of deep beams considered in this investigation.
4. In comparison with other models, the proposed model has the most accuracy, the lowest variant and standard deviation value and leads to $\left(\frac{V_{exp}}{V_u}\right)$ mean value of 1.0
5. The proposed formula is applicable to design deep beams with various parameters, such as concrete strength, arrangements and amount of web reinforcement and value of shear span-to-depth ratio.
6. The rational proposed expression to determine the efficiency factor of concrete can sufficiently present the improvement effect of web reinforcing in reduction the compression softening of concrete.
7. According to the proposed model and experimental observations due to decreasing the inclination angle of the strut or increasing the span-to-depth ratio of a deep beam, the efficiency of horizontal web reinforcements is reduced. In addition web reinforcement is the most efficient when is placed perpendicular to strut axis.

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Notations

The following symbols are used in this paper:

a = the shear span, ***mm***

A_h, A_v= the total area of horizontal and vertical web reinforcements, ***mm²***

a_s = the uniform width of strut, ***mm***

$A_{str(t)}$ = the area of tangential-section of strut, ***mm²***

A_{wp} = the equivalent area of perpendicular web reinforcements, ***mm²***

b = the width of beam, ***mm***

C_c = the compression force in the diagonal strut, ***N***

C_{max} = the maximum applicable compressive force applied in strut, ***N***

d_a = the width of the loaded point bearing plate, ***mm***

f'_c = the maximum strength of the concrete based on cylinder test, ***MPa***

f'_{ce} = the maximum strength of the softened concrete, ***MPa***

f_{yv}, f_{yh} = the tensile yield stress in the vertical and horizontal bars, ***MPa***

l_b = depth of the top node, ***mm***

l_p = depth of the bottom node, ***mm***

L_s = the length of strut, ***mm***

n_v , n_h = the numbers of vertical and horizontal reinforcing

s_v, s_h = the vertical and horizontal spacing

T_s = the tension force on longitudinal reinforcements (or ties) , ***N***

V_c = the shear strength provided by the STM, ***N***

V_w = the shear strength resulted by web reinforcements, ***N***

V_u = the total shear strength of beam, ***N***

w_t = width of the support bearing plate, ***mm***

θ = the angle between strut and longitudinal reinforcements

ρ_p = equivalent perpendicular web reinforcement ratio

ρ_h, ρ_v = horizontal and vertical web reinforcement ratio

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