Numerical Investigation on Accuracy of Mass Spring Models for Cylindrical Tanks under Seismic Excitation

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Abstract: *The main objective of this article is evaluation of the simplified models which have been developed for analysis and design of liquid storage tanks. The empirical formulas of these models for predicting Maximum Sloshing Wave Height (MSWH) are obtained from Mass Spring Models (MSM). A Finite Element Modeling (FEM) tool is used for investigating the behavior the some selected liquid storage tanks under available earthquake excitations. First, the results of FEM tool are verified by analyzing a liquid storage tank for which theoretical solution and experimental measurements are readily available. Then, numerical investigations are performed on three vertical, cylindrical tanks with different ratios of Height to Radius (H/R=2.6, 1.0 and 0.3). The behaviors of the tanks are initially evaluated using modal under some available earthquake excitations with various vibration frequency characteristics. The FEM results of modal analysis, in terms of natural periods of sloshing and impulsive modes period, are compared with those obtained from the simplified MSM formulas. Using the time history of utilized earthquake excitations, the results of response-history FEM analysis (including base shear force, global overturning moment and maximum wave height) are compared with those calculated using simplified MSM formulations. For most of the cases, the MSWH results computed from the time history FEM analysis demonstrate good agreements with the simplified MSM. However, the simplified MSM doesn't always provide accurate results for conventionally constructed tanks. In some cases, up to 30%, 35% and 70% average differences between the results of FEM and corresponding MSM are calculated for the base shear force, overturning moment and MSWH, respectively.*

Keywords: *Liquid Storage Tank, Seismic Analysis, Finite Element Modeling, Mass Spring Model, Impulsive Mode, Convective Mode, Sloshing Wave hieght.*

1. Introduction

Liquid storage tanks have always been an important connection in systems of water distribution, chemical and refined petroleum products. Water storage tanks are located near populated areas to assure constant water supply over time, while oil and liquefied chemical tanks are constructed in refinery plants. Water supply is immediately essential, following to the destructive earthquakes, not only to cope with possible subsequent fires, but also to avoid outbreaks of disease. Therefore, due to the requirement of remaining functional after a major earthquake event, the seismic performance of liquid storage tanks has been a matter of special importance (beyond the economic value of the structure).

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The most common type of tank is the vertical cylindrical tank. Damages to liquid storage tanks in past earthquakes motivated several experimental and analytical investigations of the seismic response of vertical cylindrical tanks. Some simplified Mass Spring Model (MSM) have been proposed for predicting tank responses, such as base shear force, overturning moment and Maximum Sloshing Wave Height (MSWH). The most frequent failures of these tanks are related to shell buckling near the bottom of the tank wall, where large compressive membrane stresses are induced to resist the overturning moment from the earthquake shaking of the system. If the sloshing freeboard is not properly accounted in the design procedure, sloshing of the contained liquid can also cause yielding in the connection of the tank roof and its wall. Other types of damage, including failure of the piping system connected to the tank (due to its inability to follow the shell movement and rupture at the junction of the tank wall and the base plate) may appear due to excessive plastic yielding.

It should be noted that the seismic analysis of storage tanks is a complicated and challenging task due to complexity of the case. The evaluation of general safety of the structure against earthquake loading should be carried out by considering the local earthquake properties and the characteristics of the construction site. Moreover, it is shown that the earthquake responses of liquid storage tanks are significantly affected by the interaction between the contained liquid and flexible steel structure of the tanks. Therefore, fluid-structure interaction effects must be considered during the design procedure.

In general, investigations on the seismic response of liquid storage tanks have been conducted over the past 30 years. Housner (1954, 1957) proposed a simple MSM for computing the seismic response of liquid storage tanks which is still widely used with certain modifications for the analysis of rectangular and cylindrical tanks [1,2]. His simplified MSM is a two degree-of-freedom (DOF) system for a rigid tank; one DOF accounting for the motion of the tank-liquid system, in which a part of the contained fluid being rigidly attached to the tank wall (impulsive mode) and the other DOF for the motion of the sloshing fluid effect on the tank wall (convective mode).

In further studies, Housner's simplified MSM has been modified to account for the flexibility of the tank wall. Veletsos and Yang (1976) used one mass for the impulsive component and two convective mass in their simplified MSM [3]. Haroun and Housner (1981) divided the impulsive mass into two parts; one part rigidly connected to the ground and one part representing the mass participating in the relative movement due to the deformation of the tank shell [4]. Malhotra et al. (2000) modified the properties of the simplified MSM proposed by Veletsos and Yang (1976) using one convective mode [5].

Uplifting of unanchored tanks, as well as soil-structure interaction effects, has been also extensively studied by several researchers [6-12]. Some of above mentioned works have constituted the basis for the seismic design provisions for vertical cylindrical tanks in Euro code 8-part 4.3 [13] and American Petroleum Institute (API) [14].

In present work, ANSYS Finite Element Modeling (FEM) package is used to study the quality of the results produced by simplified MSM which was proposed by Malholtra (2000) for the analysis and design of liquid storage tanks. For this purpose, Malholtra's simplified MSM is introduced in section 1. In section 2, a tank-liquid system is introduced and the results of the results of the application utilized software is validated by comparing the FEM results with available experimental measurements as well as theoretical solution. The results of numerical analyses, including details of modal and time history analysis of considered tanks, are presented in section 3. Finally, section 4 summarizes the conclusions regarding the accuracy assessment of the MSM simplified formulations.

2. MASS SPRING MODEL (MSM)

The simplified Mass Spring Model MSM proposed by Malhotra (2000) is illustrated in Figure 1. The procedure was based on the work of Veletsos and his co-workers with certain modifications including; 1- Combining the higher impulsive modal mass with the first impulsive mode and combing the higher convective modal mass with the first convective mode. 2- Modifying modal heights of mass to account for the contribution of higher modes to the base overturning moment. 3- Generalizing the formula for the impulsive period so that it could be applied to tanks of various wall thicknesses.

The effects of liquid-structure interaction and

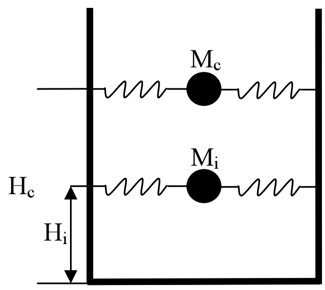


Fig. 1. Simplified Mass Spring Model (MSM) proposed by Malhotra et al.(2000)

an uncoupled manner by dividing the hydrodynamic pressure acting on the shell into two components; 1) The impulsive pressure caused by the portion of the liquid, M_i , which is rigidly attached to the shell wall, and 2) The convective pressure caused by the portion of the liquid, M_c, sloshing in the tank. These components were then modeled as single DOF oscillators. For very large values of fluid Height to tank Radius (*H/R*), the sloshing mass is only a small portion of the total mass. As *H /R* less than unity, more than half the total mass can participate in the convective mode. The values proposed by Malhotra for the parameters of the simplified model can be obtained from equations 1 and 2 as well as table 1.

$$T_{imp} = C_i \frac{H\sqrt{\rho_L}}{\sqrt{t_s / R}\sqrt{E}} \tag{1}$$

$$T_{con} = C_c \sqrt{R} \tag{2}$$

In the above equations, *M* is the total mass of the liquid and K_c is the stiffness assigned to M_c. H_i and H_c are the respective heights of the resultant force of the hydrodynamic pressure due to the motion of the impulsive M_i and the convective M_c masses, respectively. In equations 1 and 2, t_s is the wall thickness, *E* is the modulus of elasticity of the tank material, ρ_L , T_{imp} and T_{con} are the mass density of the liquid, the periods of the impulsive and convective modes, respectively.

3. FINITE ELEMENT MODEL (FEM)

3.1. FEM Strategy

The Finite Element Modeling (FEM) strategy is utilized to analyze the storage tank walls, as well as the contained liquid. ANSYS software which combines structural and fluid simulation capabilities is used to perform modal and nonlinear seismic analysis. Four-node 24 DOF quadrilateral elastic shell elements that have both membrane and bending capabilities are used to model the tank shell. In this study, modulus of elasticity for tank wall is considered as E_S*=2E+08 (KPa)*, its Poisson's ratio and density are considered as $\upsilon=0.3$ and $\gamma_s=78$ *(kN/m³)*, respectively. The fluid domain is modeled with three dimensional, eight-node, 24 DOF fluid elements. This element type has 3 DOF at each node (displacement based in three directions) and is used to model contained fluids (with zero net in/out flow rate).

The fluid structure interaction is also considered by properly coupling the nodes that lie in the common faces of the two domains. This means that the fluid is attached to the shell wall but can slip in the wall tangential directions , and therefore, only can exert normal pressures to the tank wall. The assumption that the fluid cannot separate from the shell wall corresponds to the simplified mechanical model of Housner (1957).

3.2. Verification of FEM Results

Prior to application of FEM for parametric study on storage tank, the accuracy of the introduced modeling strategy is investigated in this section. For this purpose, the results of free surface displacement obtained from numerical model are compared with experimental results reported by M.S.Chalhoub [12]. A set of measurements experimental scaled tank is used for the purpose of comparison was cylindrical steel tank, 1mm. thick, 60.96 cm in height and 121.92 cm diameter. Similar to the experimental test the El Centro earthquake record at 0.635 cm peak table displacement and the peak table acceleration 0.114 g was considered as an input base excitation for the FEM [15]. Taking

Table 1 Parameters of the simplified MSM (Malhotra 2000)

H / R	C_i	C_c(s/√ m)	M_i/M	M_c/M	H_i/H	H_c/H
0.3	9.28	2.09	0.176	0.824	0.400	0.521
0.5	7.74	1.74	0.300	0.700	0.400	0.543
0.7	6.97	1.60	0.414	0.586	0.401	0.571
1.0	6.36	1.52	0.548	0.452	0.419	0.616
1.5	6.06	1.48	0.686	0.314	0.439	0.690
2.0	6.21	1.48	0.763	0.237	0.448	0.751
2.5	6.56	1.48	0.810	0.190	0.452	0.794
3.0	7.03	1.48	0.842	0.158	0.453	0.825

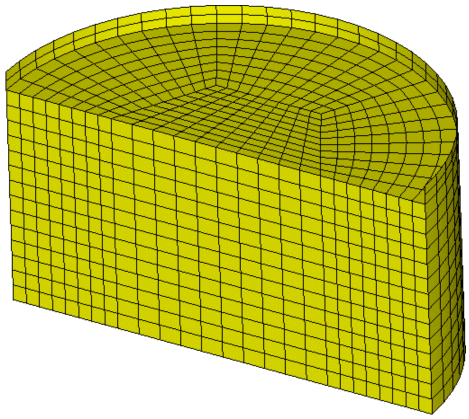


Fig. 2. Finite Element Mesh for FEM of a tank (using symmetry condition)

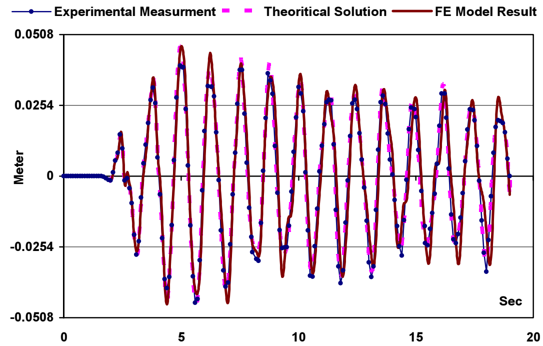


Fig. 3. Comparison between FEM results, experimental measurements and theoretical solution for the MSWH time history at the shell wall

base excitation for the FEM [15]. Taking advantage of the symmetry of a cylindrical tank,only half the storage tank is modeled considering uniaxial earthquake in shaking with direction parallel to the plane of symmetry (figure2).

In figure 3, the results of free surface displacements at the shell wall computed by FEM are compared with measured free surface elevation as well as linear theoretical solution given by M.S.Chalhoub [12]. The first three modes of sloshing are used to calculate free surface theoretical solution. As can be seen in figure 3, there are good agreement between FEM results, the theoretical solution (for more information about the theoretical solution see 12) and the experimental measurements [12]. The FEM results present maximum differences of 15% and 14%, respectively, with measured and analytical peak free surface elevation. These levels of discrepancy are unavoidable because of the errors in measurements and the assumptions made in the theoretical solution.

4. NUMERICAL INVESTIGATIONS

4.1. Specification of Utilized Tanks

In present study, three tanks of different aspect ratios including broad tank *(H/R=0.3)*, medium tank *(H/R=1.01)* and tall tank *(H/R= 2.6)* are utilized. Each tank was designed based on API code of practice [15]. Dimensions and other geometry characterizes of these tank are listed in table 2.

4.2. Modal Analysis of Three Tanks

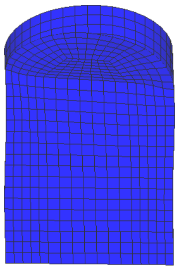
In order to examine the accuracy of simplified formula proposed by Malhotra (Eq.1 and Eq.2) for computing fundamental periods of the tanks, a modal analysis is performed in this section. The linear representation of the tank-fluid system allows the modal eigenvalue analysis. For the particular fluid elements, the lumped mass matrix is formulated and the “Reduced Method” is utilized for the modal analysis. This method uses a reduced number of DOF, which are called “master” DOF, to formulate the mass and stiffness matrices of the system. The first two mode shapes involve sloshing of the contained liquid without any participation of the shell walls. This shows that the eigenvalues and eigenvectors for convective modes to be independent of the stiffness of the walls. The most significant mode

Table 2 Geometry Dimensions of the tanks Radius

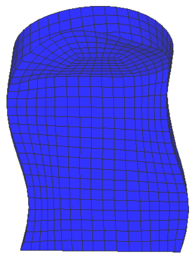
	Radius (m)	Tank Height (m)	Liquid height (m)	Lower shell thickness (m)	Upper shell thickness (m)	Liquid Density (Kg/m2)	Type of liquid	Bulk modulus of elasticity (N/m2)
Tank1	54.5	17.5	15.85	0.03	0.03	885	Crude oil	1.65E+09
Tank2	37	40.6	37.4	0.033	0.033	480	LNG	2.00E+09
Tank3	2.5	8	6.5	0.006	0.006	1000	Water	2.10E+09

Table 3 Natural periods of the considered Tanks (sec)

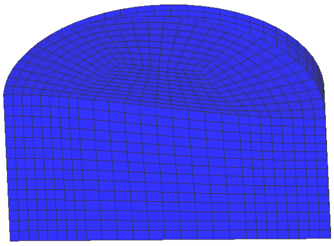
Tank	Result	Mode Shapes Number			
		1st sloshing	2nd sloshing	1st coupled	2nd coupled
Tank 1	**FEM result (Sec)**	15.87	6.85	0.466	0.435
	MSM result (Sec)	15.43	---	0.47	---
	Difference (%)	-2.77	---	0.85	----
Tank 2	**FEM result (Sec)**	9.37	5.47	0.46	0.44
	MSM result (Sec)	9.24	---	0.45	---
	Difference (%)	-1.38	---	-2.17	---
Tank 3	**FEM result (Sec)**	2.475	1.467	0.067	0.037
	MSM result (Sec)	2.34	----	0.062	---
	Difference (%)	-5.45	----	-7.46	---



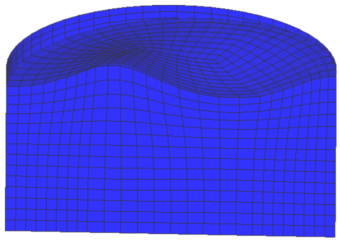
4-5: 1st coupled mode Tank3 (T = 0.067 sec)



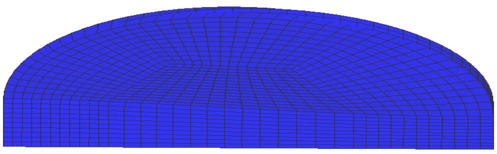
4-6: 2nd coupled mode Tank3 (T = 0.037 sec)



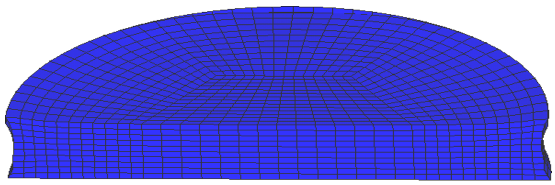
4-3: 1st sloshing mode Tank2 (T = 9.37 sec)



4-4: 2nd sloshing mode of Tank2(T = 5.47 sec)



4-1: 1st sloshing mode Tank1 (T = 15.87 sec)



4-2: 1st vertical (??) mode Tank1 (T = 0.052 sec)

Fig. 4. Computed sloshing coupled mode shapes for three utilized tanks

shapes of the tank-fluid system consist of the first mode shape of liquid sloshing and mode shape correspond to coupled motion of the tanks and fluids are plotted in figure 4.

Table 3 summarizes the natural periods computed by FEM and compares the sloshing and impulsive periods with those computed from the simplified formula provided by Malhotra et al. There are good agreements between the results of the FEM and the Malhotra's simplified formula results for Tank 1 and Tank 2. However, the predicted results of the simplified formula for Tank 3 are slightly higher than others (7.4%). It seems that the accuracy of natural periods predicted by simplified formula is degraded by decreasing the fundamental natural period of sloshing mode.

4.3. Time History Analysis of Three Tanks

4.3.1. Utilized earthquake excitations

Table 4 Earthquake specifications used for the time history analysis

Earthquake	Year	Peak Ground Acceleration (g)	Predominant Period (Sec)	Abbreviation
Elsentro	1940	4.1	0.5	EL
Imperial Vallay	1979	2.9	0.24	IV
Chichi	1999	3.03	1.14	CHI
Tabas	1978	8.3	0.2	TAB
Northriche	1994	5.9	0.54	NOR

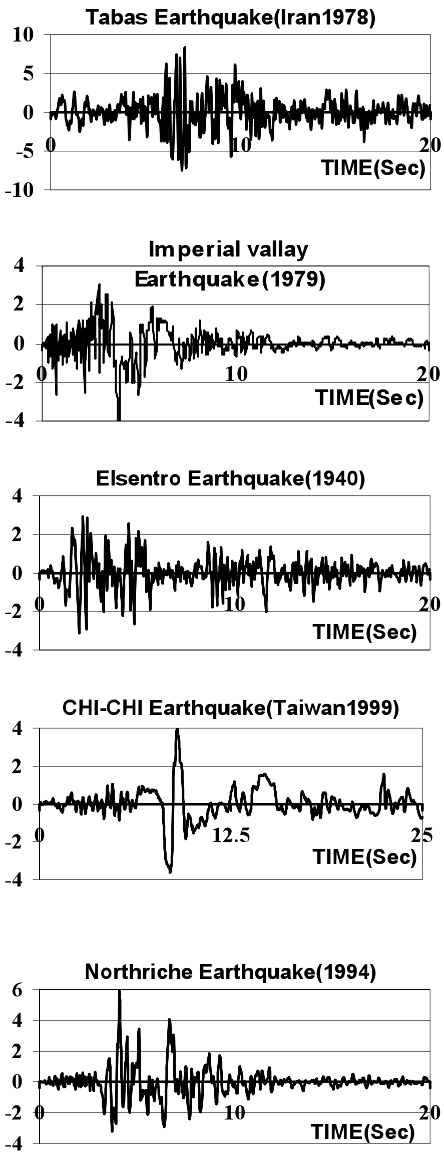


Fig. 5. Time history acceleration of earthquake excitations (The unit of vertical axis m / sec^2)

Primary objective of this article is to evaluate the accuracy of MSM formulations which are simplified for prediction of seismic response of storage tanks. For this purpose, the FEM results of three tanks introduced in section 2 under time history acceleration of various available earthquakes records are utilized. Five readily available earthquake records are utilized as time histories for base excitation of the system, in which, the peak ground accelerations are considered between 2.9 to 8.3 m/s2. Five earthquake specifications used for the time history analysis are listed in table 4 and acceleration time histories for the utilized earthquakes are plotted in figure 5. Among the applied earthquake excitations, the CHI earthquake record with predominant period at about 1.14 Sec presents a long period motion. Such low frequency excitations may give rise to large values of MSWH (Maximum Sloshing Wave Height) due to significantly long fundamental natural periods for contained liquid of the utilized tanks (which are typically in the range of 6 to 12 seconds for broad tanks).

4.3.2. Mass spring model (MSM)

In order to evaluate the time histories response of Malholtra's model under the utilized earthquake excitations, the corresponding MSM of considered tanks are also simulated using ANSYS 5.4 package. For this purpose, structural mass element option of the software is used. The simulated model consists of two independent DOF (impulsive and convective) that both contribute to the base shear and the overturning moment. Each mass is connected to the ground through a spring element that is fixed at the base ground. Considering the contributions of both DOFs, the base shear and global overturning moment of the tanks are considered as the total shear force and moment at the base of the system.

4.3.3. Finite element model (FEM)

The same FEM strategy, which is described in section 2 and verified by experimental measurements, is used to simulate the behavior of the three tanks (with *H/R=2.6, 1.0 and 0.3*) under the five seismic excitations introduced in previous section. Newmark's Method is used to simulate the nonlinear time history response of both FEM and MSM for each earthquake ground motion. A Rayleigh damping matrix is also defined in both computational models, related to the damping ratio desired in the two significant modes (the first sloshing mode and the first horizontal coupled or impulsive mode) (. For the first sloshing mode, damping ratio is considered to be 0.5% and for the first coupled mode is defined to be 2.0% (which is considered for contribution of steel wall of cylindrical tank, corresponding to performance in the linear elastic range).

4.3.4. Discussion on time history results

This section presents the results of the time history analyses of the three considered tanks performed by FEM and simplified MSM. The results are presented in terms of base shear and overturning moment in tables 5 and 6. "FEM" label refers to the numerical results extracted from ANSYS analyses. The overturning moment is computed with respect to the center of the cylindrical tank, neglecting the contribution of hydrodynamic pressures exerted on the base plate.

"SUM", "SRSS", "IMP" and "CONV" labels

Table 5 Maximum Base Shear force (kN)

			EL	IV	CHI	TAB	NOR
Tank1	*FEM results*		221551	179155	193340	532728	331382
	MSM results	*SUM*	301481	232385	184548	536808	380960
		SRSS	301243	237982	181552	551889	382038
		IMP	301222	237488	161460	551116	382020
		CONV	3563	15327	83018	29212	3787
Tank2	*FEM results*		415386	412824	348148	984647	684681
	MSM results	*SUM*	440256	388615	337056	1136640	655783
		SRSS	442663	400159	339798	1142549	657445
		IMP	442619	399901	329600	1142190	657437
		CONV	6310	14377	82624	28655	3425
Tank3	*FEM results*		515	635	467	1862	722
	MSM results	*SUM*	540	686	444	2097	714
		SRSS	532	676	474	2112	754
		IMP	530	667	429	2108	741
		CONV	54	115	202	142	144

Table 6 Maximum Overturning Moment (kN-m)

			EL	IV	CHI	TAB	NOR
Tank1	*FEM results*		1496730	1186920	1290000	3875070	2217540
	MSM results	*SUM*	1911880	1463540	1316360	3375930	2413250
		SRSS	1909974	1510983	1231971	3502390	2422208
		IMP	1909747	1505674	1023656	3494075	2422006
		CONV	29419	126555	685479	241203	31269
Tank2	*FEM results*		6571200	7021780	5731900	17179600	11158900
	MSM results	*SUM*	6881390	6006410	5336620	17770200	10264000
		SRSS	6937361	6275189	5504203	17910279	10302339
		IMP	6935840	6266449	5164832	17898117	10302038
		CONV	145319	331102	1902831	659924	78877
Tank3	*FEM results*		1439	1806	1282	5374	1774
	MSM results	*SUM*	1611	2056	1481	6137	2038
		SRSS	1582	2050	1638	6238	2302
		IMP	1557	1962	1260	6195	2178
		CONV	281	595	1047	733	748

Table7 Errors of MSM (%) on evaluation of maximum response of Base Shear force

		EL	IV	CHI	TAB	NOR
Tank1	*SUM*	36.08	29.71	-4.55	0.77	14.96
	SRSS	35.97	32.84	-6.10	3.60	15.29
	IMP	35.96	32.56	-16.5	3.45	15.28
Tank2	*SUM*	5.99	-5.86	-3.19	15.44	-4.22
	SRSS	6.57	-3.07	-2.40	16.04	-3.98
	IMP	6.56	-3.13	-5.33	16.00	-3.98
Tank3	*SUM*	4.85	8.03	-4.93	12.62	-1.11
	SRSS	3.30	6.46	1.50	13.43	4.40
	IMP	2.91	5.04	-8.14	13.21	2.60

Table 8 Errors of MSM (%) on evaluation of maximum response of Overturning Moment respect to the FEM

			EL	IV	CHI	TAB	NOR
Tank1	*Analog Result*	*SUM*	27.7	23.31	2.04	-12.8	8.83
		SRSS	27.6	27.30	-4.50	-9.62	9.23
		IMP	27.5	26.86	-20.6	-9.83	9.22
Tank2	*Analog Result*	*SUM*	4.72	-14.4	-6.90	3.44	-8.02
		SRSS	5.57	-10.6	-3.97	4.25	-7.68
		IMP	5.55	-10.7	-9.89	4.18	-7.68
Tank3	*Analog Result*	*SUM*	11.9	13.84	15.52	14.20	14.80
		SRSS	9.94	13.51	27.77	16.08	29.70
		IMP	8.20	8.64	-1.72	15.28	22.70

refer to the results obtained from time history analysis of simplified MSM. Using ANSYS computational tool for the MSM, impulsive and convective hydrodynamic pressures are explicitly simulated using separate DOFs. Therefore, response magnitudes and their maximum values are computed for each degree. These parameters are identified as “IMP” and “CONV” for the impulsive mass contribution and convective mass contributions, respectively. “SUM” label stands for summation of the maximum computed values of impulsive and convective responses at every time step, “SRSS” stands for the “Square Root of Sum of Squares” rule, which is an alternative to compute the maximum response of the system (based on the maximum values of the impulsive and convective responses).

Table 7 and 8 present the differences between the results of MSM shear force and overturning moment force respect to FEM results. Although for most of the tanks, the error between the results of MSM and FEM for overturning moment and shear force are less than 10 %, the different could increase up to 30 % for some cases. Fig.6 illustrates the overturning moment time history of tanks. It seems that agreements between the FEM and MSM results increase by decreasing the main sloshing period.

Since SRSS rule allows the response spectrum analysis for the cases which only the maximum response of each DOF is computed, it is widely used in design codes (NZSEE [16], API Standard [15]). The results obtained from SRSS method and SUM method present maximum 13% error (Table 9). The differences between SRSS and SUM method increase for the cases that the convective pressure effects highlight. However, SRSS method produces almost conservative result for all the cases.

The responses are mainly affected by the

Table 9 Deviation between SUM and SRSS methods (%) for Overturning Moment [(SUM-SRSS)/SUM*100]

	EL	IV	CHI	TAB	NOR
Tank1	0.10	-3.2	6.41	-3.7	-0.4
Tank2	-0.8	-4.4	-3.1	-0.8	-0.4
Tank3	1.80	0.29	-10	-1.6	-13

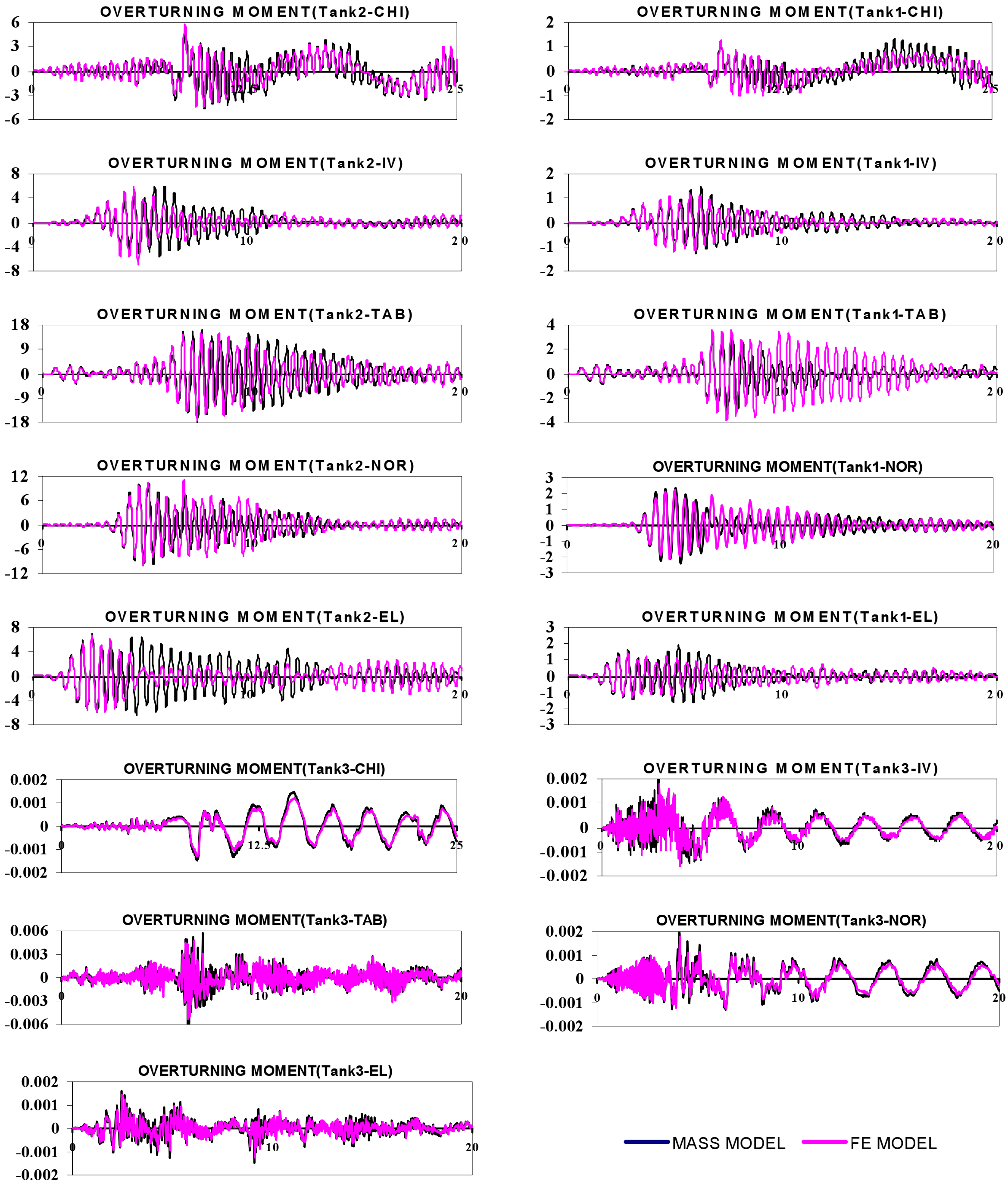


Fig. 6. Overturning moment response of tanks versus the time(The unit of vertical axis is*E+9 N-M)

contribution of the impulsive hydrodynamic pressures exerted on the tank walls. As can be seen in table 10, the convective pressures contribute less than 10% to the total response (except for the CHI earthquake which includes excited higher sloshing mode). According to table 10, neglecting the convective mass and assuming the impulsive mass results as a total

Table 10: Error between IMP and SUM methods (%) for Overturning Moment [(IMP-SUM)/IMP*100]

	EL	IV	CHI	TAB	NOR
Tank1	-0.1	2.88	-22.2	3.50	0.36
Tank2	0.79	4.33	-3.22	0.72	0.37
Tank3	-3.3	-4.5	-14.9	0.95	6.87

seismic response of a tank is acceptable assumption in most of the cases. However, the error of this assumption can increase up to 22 % for the cases with long periods motions (See table 10 for CHI earthquake).

From the MSWH point of view in MSM the wave height is generally calculated based on the absolute acceleration of the convective mass $A_{con}(t)$. Considering only the 1st sloshing mode, the sloshing wave height, h, could be obtained by:

$$h(t) = 0.84 \cdot R \cdot \frac{a_{con}(t)}{g} \quad (3)$$

Where R is the radius of the tank and g is the gravitational acceleration [3].

The results calculated from MSM (Eq.3) and obtained from FEM of tanks are tabulated and compared in table 11. It is noticeable that the Eq.3 only considers the first sloshing mode. Considering the shape of first sloshing mode, maximum sloshing wave height calculated by this equation occurs at the tank wall.

Table 11 shows that simplified *Eq.3* gives underestimated maximum wave height for most of the tanks. As can be seen, MSWH occurs for the tanks subjected to CHI earthquake due to the special nature of excitation (including long periods of motions). According to the result of table 11, the errors are in acceptable range (less than 8%) for slender tank (Tank 3) due to the small natural period of first sloshing modes of such a tank.

In addition, table 11 shows that the (MSWH) occurring in the middle of the liquid tank free surface could increase up to 70% (larger than that occurs in the side wall of the same tanks).

For all the applied tanks, the time histories of sloshing wave height obtained by both models are plotted in Fig.9. Although the trends of time history results are similar, the MSWH obtained from FEM analysis are more than two times of those evaluated by MSM. Therefore, it can be concluded that the simplified MSM (Eq.3) may

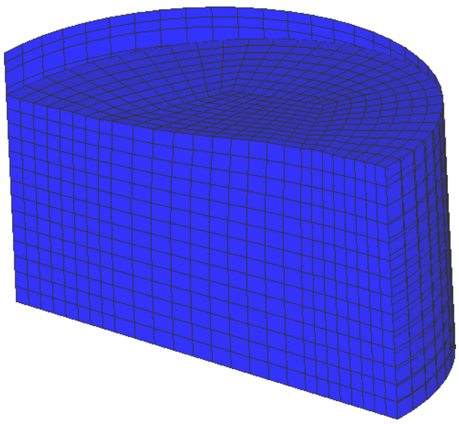


Fig. 7. Tank2 under TAB Earthquake
(Maximum wave height occurs in the middle of tanks)

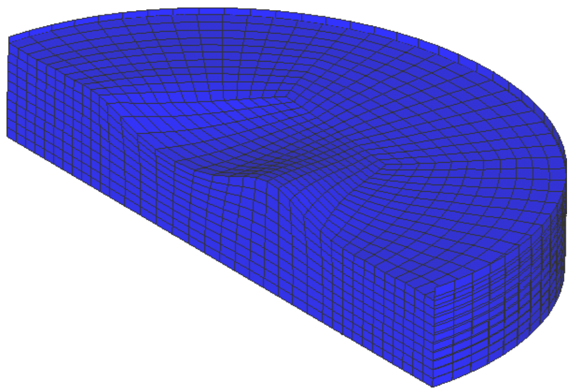


Fig. 8. Tank1 under CHI Earthquake
(Maximum wave height occurs in the side wall of tanks)

Table 11 Comparison of MSWH computed by FEM and MSM

		EL		IV		CHI		TAB		NOR	
		Middle	Side	Middle	Side	Middle	Side	Middle	Side	Middle	Side
Tank1	*FEM*	-	0.35	1.82	1.45	-	3.48	3.00	1.73	-	0.67
	MSM (Eq.3)		0.15		0.66		3.59		1.26		0.16
	Error (%)		**133.3**		**119.7**		**-3.1**		**37.3**		**309**
Tank2	*FEM*	-	0.56	2.5	1.97	-	8.00	3.35	3.00	-	0.85
	MSM (Eq.3)		0.58		1.33		7.64		2.65		0.32
	Error (%)		**-3.4**		**48.1**		**4.7**		**13.2**		**166**
Tank3	*FEM*	-	0.46	-	1.09	-	2.01	-	1.21	-	1.36
	MSM (Eq.3)		0.50		1.06		1.86		1.30		1.33
	Error (%)		**-8.0**		**2.8**		**8.1**		**-6.9**		**2.3**

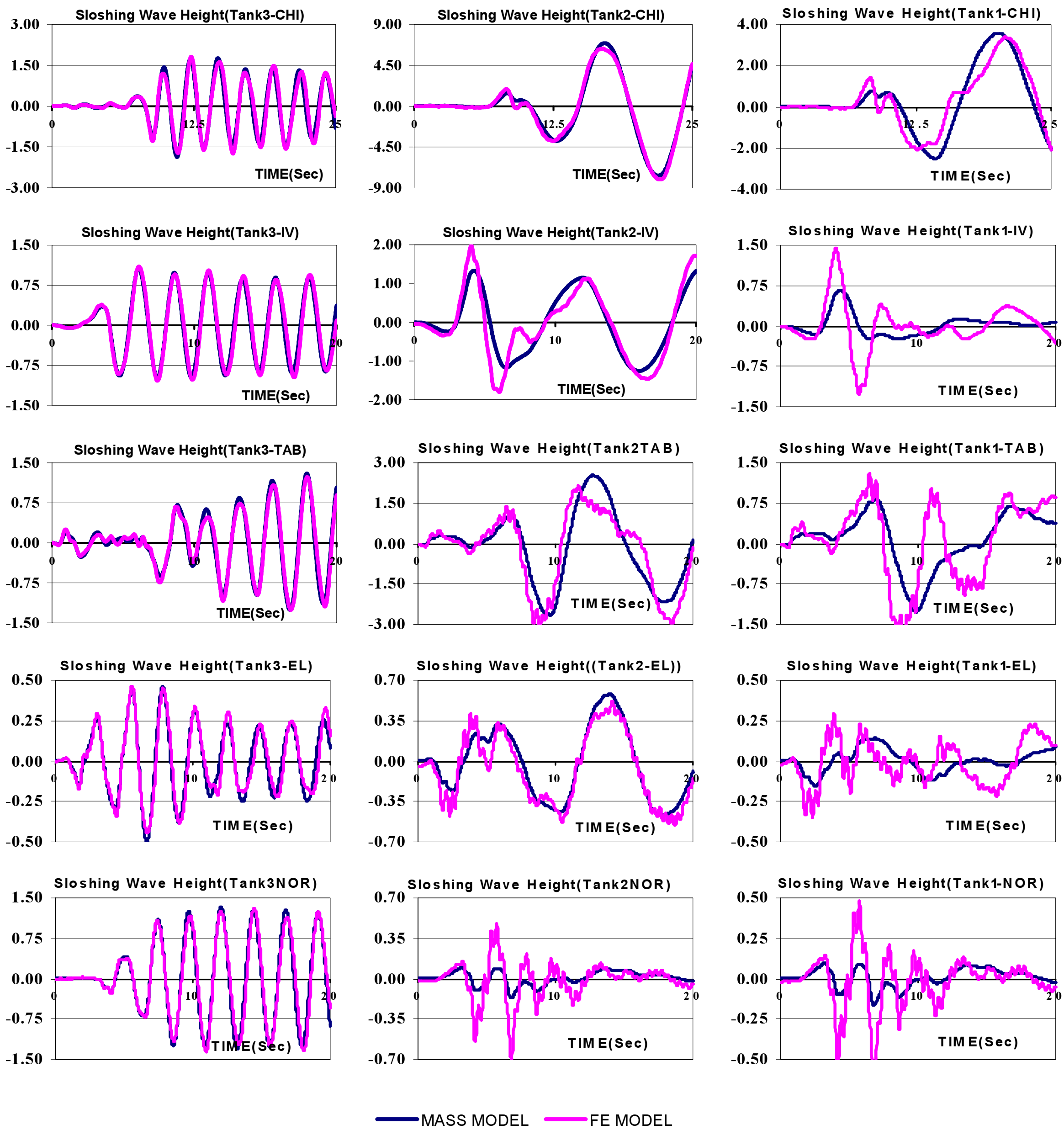


Fig. 9. Time history results of sloshing wave height (m)

give underestimations on prediction of MSWH for the case with small aspect ratio (H/R=0.3).

Furthermore, considering the effect of higher sloshing modes in FEM shows that the maximum wave height may takes place either at the wall sides of the tanks (Fig .7) or in the middle of free surface (Fig.8). The maximum sloshing wave displacements which are computed by FEM in the middle of free surface are also tabulated in table 11.

5. SUMMERY AND CONCLUSIONS

In this paper, the quality of the simplified Mass Spring Models (MSM) that have been proposed for the preliminary analysis and design of vertical cylindrical liquid storage tanks was assessed using Finite Element Modeling (FEM) analysis. The FEM was used to perform modal and nonlinear response-history analysis of three vertical cylindrical tanks in the three-dimensional space. The numerical simulation of the MSM was also conducted using structural mass element of the utilized software. The results obtained from

FEM analysis of the tanks were compared with those obtained from the corresponding simplified MSM and an investigation of the accuracy and validity of the simplified MSM was carried out. The key conclusions of the analyses described in this study are listed below.

1- There are good agreements between Malhotra's MSM (Eq. 1, 2) predictions for the period of convective and impulsive natural modes and the results obtained from modal analysis of FEM. The periods of the sloshing mode is not affected by the flexibility of the tanks. The results also show that the accuracy of natural periods predicted by simplified MSM is reduced by decreasing the natural period of the sloshing mode.

2- Although for the most of the considered tanks, the differences between the results of FEM and MSM for the overturning moment and shear are less than 10 %, this variation could increase up to 30 % for broad tank.

3- The maximum deviation between the results of SRSS and SUM methods which is used for computing the total seismic response of tanks (using the combination of convective and impulsive mass results) is 13%. However, for all the cases SRSS method produces more conservative results.

4- The seismic responses of considered tanks are mostly affected by the contribution of the impulsive hydrodynamic pressure. Neglecting the convective part of seismic response would produce acceptable results (with less than 10% error). However, by applying this assumption, the maximum error of 22% obtained from FEM analysis.

5- MSM (Eq.3) gives underestimated MSWH for the considered tanks. Deviations between FEM and MSM results are less than 8% for slender tank. However, MSWH obtained from FEM analysis could increase more than two times of those obtained by MSM for wider tanks. It can be stated that the accuracy of MSM results (which does not consider the nonlinear behavior of the fluid for predicting of MSWH) is not satisfactory for the broad tanks (with $H/R<0.5$).

6- The results of FEM show that MSWH in the middle of free surface may be computed 70% larger than those occurs in the side walls of the tank. Therefore, the effects of higher sloshing modes should be taken in to account in the MSWH analyses.

7- CHI earthquake created MSWH at the tank wall of all three tank types. This is due to the fact that MSWH is mainly affected by the nature of earthquakes motion (ie long period motion), while other seismic characteristics of earthquakes have minor effects.

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