Development of Bayesian Inference to Predict household Trip Production (Case Study of Isfahan City)

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Abstract: *Household trip production is not a constant parameter and vary based on socio-economic characteristics. Even households in each category (households with constant socio-economic characteristics) produce several numbers of trips. Purpose of present study is to model the variation of household trip production rate in urban societies. In order to do this, concept of the Bayesian Inference has been used. The city of Isfahan was selected as case study. First, likelihood distribution function was determined for number of household trips, separating odd and even trips. In order to increase precision of the function, the composed likelihood distribution function was utilized. To insert households' socio-economic variables in the process, disaggregate χ^2 calibrated model were used at the likelihood distribution function. Statistical indices and χ^2 test show that likelihood distribution function of numbers of household trip production follows the Poisson distribution. The final composed likelihood distribution was determined based on Bayesian inference. Related function was created with compilation of mean parameter distribution function (Gamma distribution) and numbers of household trip production (Poisson distribution). Finally, disaggregate model was put at final composed probability function instead of mean parameter. Results show that with Bayesian inference method, it would be possible to model the variation of household trip production rate in urban societies. Also it would be possible to put socio-economic characteristics in the model to predict likelihood of real produced trips (not average produced trips) for each household's category.*

Keyword: *χ^2 test, composed likelihood distribution function, even trips, odd trips, Poisson discrete distribution function.*

1. Introduction

The concept of variation of household trip production rate in urban societies is a very paramount and considerable issue which is often overlooked in the modeling process of trip production. The variation of trip production's rate means that a certain household with specific socio-economic characteristics does not necessarily produce a constant rate of trips, but the produced number of trips by the household follows its unique likelihood distribution. This concept could be generalized to any other urban society. That is, in a certain statistical society like an urban area, an internal zone, and a household category in terms of specific socio-economic characteristics, the trip production rate of the members of these statistical societies follows its unique and particular likelihood distribution.

The purpose of present study is based on an attempt to model the variation of household trip production rate, also to develop the composed likelihood distribution function in which the frequency of the rate of trip production would be possible to measure based on the socio-economic characteristics. In fact, by replacing these variables in the related function, it should be possible to assess likelihood of number of trips produced by each household or any other defined statistical society. To achieve this, the concepts of Bayesian inference in probability and statistics have been used.

2. Methodology

The methodology has been divided to three sections in present study:

First, household trip production was divided to two groups (odd trips and even trips).

Second, χ^2 test was utilized for determining the probability function of number of household trips.

Third, based on Bayesian inference, composed probability distribution function has been determined

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and then socio-economic characteristics are inserted to related function based on disaggregate model.

3. Developing the concept of Bayesian inference (composed likelihood distribution function)

Assume that the random variable X has the distribution function of $f(x|\theta)$ in which f is the density function of the random variable X with the unknown parameter of θ. In addition, suppose that there are some extra pieces of information about the unknown variable θ, for example the distribution of parameter is in the form of $\pi(\theta)$. These additional pieces of information about the distribution of parameter of the society help to achieve a better distribution for the random variable X. [1,2]. While this is so, the random variable X has the distribution function of g(x). That is, the new distribution is independent from the parameter- θ. This type of new distribution is called composed distribution. In this article, the number of trip n_k has a Poisson distribution function with the parameter λ for the odd and even trips. Since the parameter λ in Poisson distribution equals the mean of the society, the average for different social categories must be gained and then the distribution of the mean parameter is measured. The mean of society has gamma distributions with the parameters α and β.

4. Results and Discussion

4.1. Determining likelihood distribution function for the number of household trips

In figure no. 1 the frequency distribution for trip production has been presented based on the Isfahan data bank. [3]

As it is observed in figure [1], the frequency of odd trips-red columns-produced is far less than the frequency of even trips-blue columns-produced. According to this, it is needed to compose likelihood distribution function for household trip production separately, once for even trips and once for odd ones. The household trip production likelihood distribution function is a discrete function and the most likely of these functions is Poisson discrete distribution function. [4,5].

4.1.1. Determining the likelihood distribution function (odd trips)

In order to test whether the number of odd trips has the Poisson distribution or not, the below procedure was followed:

Since the number of trips starts from one and is added two by two, and considering the fact that in Poisson distribution the numbers start from zero and is added one by one, this issue will be analyzed by the number of odd trips minus 1 divided by two. To do this, a table consisting of 11 levels in a manner that the expectative frequency in each level equals $e^{-\lambda}\frac{\lambda^x}{x!}\times 2244$ is composed. Table [1] clearly shows this concept. [6], [7].

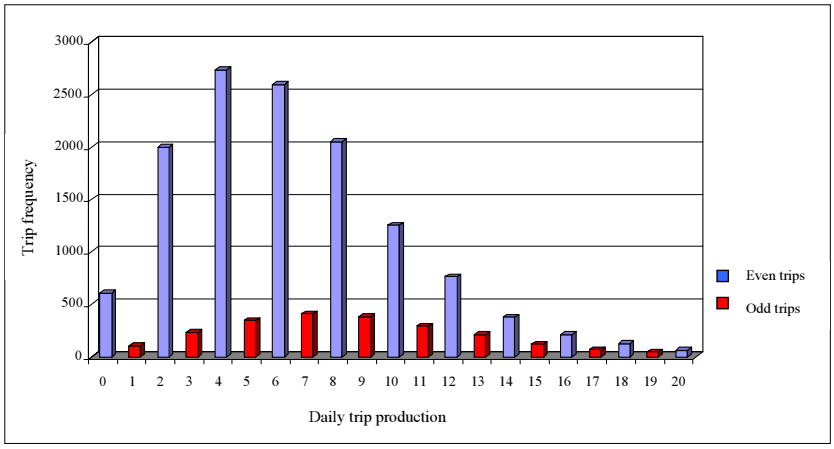

Fig. 1. The Frequency Distribution of Household Trip Production in Isfahan

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Table 1. The analysis of χ^2 test in terms of Poisson distribution (odd trips)

number of odd trips converted	number of odd trips	observed frequency o_i	the expectative frequency e_i	$\frac{(o_i - e_i)^2}{e_i}$
0	1	57	46.21094583	2.518964
1	3	203	179.4277946	3.096783
2	5	345	348.3409924	0.032044
3	37	439	450.8459695	0.311253
4	9	426	437.6360219	0.309383
5	11	320	339.8505044	1.159458
6	13	212	219.9285089	0.285826
7	15	121	121.9911573	0.008053
8	17	65	59.20838644	0.566521
9	19	32	25.54380427	1.631803
10	21	15		
11	23	5		
12	25	3		
14	29	1		
17	35	1		
21 trtips and more		24	15.0159145	5.375217
total		2244	2244	15.2953

The value of χ^2 is equal to $\sum_{i=1}^{11} \frac{(o_i - e_i)^2}{e_i} = 15.2953$ which has a χ^2 distribution with 9 degrees of freedom.

There is k =11 group in which one parameter (p=1) has been estimated and the degree of freedom of χ^2 distribution equals *(k-1-p)*. P - Value equals 0.083137. Accordingly, the hypothesis which mentions the converted amounts of odd trips follow the Poisson distribution will not be rejected at the level of 0.95. That means the number of odd trips following Poisson distribution with the parameter of $\lambda_1 = 3.88279$.

In which λ is the mean of Poisson distribution which has been obtained through torque method. Consequently, the distribution of odd trips ($g_1(n_k \mid \lambda_1)$: Likelihood density distribution of odd trips) would be as follows:

Relation [1]:

$$y = \frac{(n_k - 1)}{2} \sim Poisson(\lambda_1)$$

$$\& \quad f(y \mid \lambda_1) = g_1(2y+1 \mid \lambda_1) = g_1(n_k \mid \lambda_1) =$$

$$: e^{-\lambda_1} \frac{(\lambda_1)^y}{(y)!} \quad \begin{cases} \forall \ \ n_k = 1,3,5,...,2n-1,... \\ or \ \ y = 0,1,2,... \end{cases}$$

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$$g_1(n_k \mid \lambda_1) = e^{-\lambda_1} \frac{(\lambda_1)^{\frac{n_k-1}{2}}}{((n_k-1)/2)!} =$$

$$= e^{-3.882799} \frac{(3.882799)^{\frac{n_k-1}{2}}}{((n_k-1)/2)!} = e^{-3.882799} \frac{(1.970482)^{n_k-1}}{((n_k-1)/2)!}$$

4.1.2. Determining the likelihood distribution function (even trips)

In order to assess whether the even trip have the Poisson distribution or not, the below procedure was followed:

Since the number of even trips starts from zero, but it is added two by two, but in Poisson distribution this number is added one by one; therefore, by converting the number of trips into divided by two, this issue will be considered. To do so, a table consisting of 11 levels in a manner in which the expectative frequency in each level equals $e^{-\lambda} \frac{\lambda^x}{x!} \times 2244$ is made. In table [2], this has clearly been demonstrated. [6,7].

The value of χ^2 is equal to

Table 2. The analysis of χ^2 test in terms of Poisson distribution (even trips)

number of even trips converted	number of even trips	observed frequency o_i	expectative frequency e_i	$\frac{(o_i - e_i)^2}{e_i}$
0	0	560	531.1463537	1.583153
1	2	1688	1691.396211	0.006866
2	4	2779	2693.062959	2.75252
3	6	2786	2858.619821	1.845911
4	8	2224	2275.765756	1.174511
5	10	1409	1449.401488	1.119818
6	12	763	769.251942	0.050367
7	14	373	349.9465451	1.500711
8	16	156	139.2973557	1.972833
9	18	61	49.28690095	2.733546
10	20	20	15.69504844	1.15612
11	22	4		
12	24	3		
13	26	1		
14	28	1		
15	30	1		
22 trips and more		10	6.129620102	2.173016
total		12829	12829	18.06937

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$\sum_{i=1}^{12} \frac{(o_i - e_i)^2}{e_i} = 18.06937$ which has a χ^2 distribution with 10 degrees of freedom. In fact, there is k=12 groups in which one parameter has been assessed, p = 1. The degree of freedom of χ^2 is equal to k – 1 – p. The p - value equals 0.053805. Therefore, the hypothesis that the converted number of even trips follows the Poisson distribution is not rejected at the level of 0.95. Accordingly, the number of even trips follows Poisson distribution with the parameter of $\lambda_2 = 3.1844$.

In which λ_2 parameter is the mean of Poisson distribution which has been obtained through torque method. Consequently, the distribution of

$$y = \frac{n_k}{2} \sim Poisson(\lambda_2)$$

$$\& \quad f(y \mid \lambda_2) = g_2(2y \mid \lambda_2) = g_2(n_k \mid \lambda_2) =$$

$$e^{-\lambda_2} \frac{(\lambda_2)^y}{(y)!} \quad \begin{cases} \forall \ n_k = 0,2,4,...,2n,... \\ or \quad y = 0,1,2,... \end{cases}$$

even trips ($g_2(n_k \mid \lambda_2)$: the density likelihood distribution function of even trips) would be as

$$g_2(n_k \mid \lambda_2) = e^{-\lambda_2} \frac{(\lambda_2)^{\frac{n_k}{2}}}{(n_k/2)!} = e^{-3.18442591} \frac{(3.18442591)^{\frac{n_k}{2}}}{(n_k/2)!}$$

$$= e^{-3.18442591} \frac{(1.784495982)^{n_k}}{(n_k/2)!}$$

follows:

Relation [2]:

4.2. Determining the composed likelihood distribution function

$$g(n_k \mid \lambda_1, \lambda_2) = \begin{cases} g_1(n_k \mid \lambda_1) & \text{with prob.} 1 - r \quad if\ n \\ g_2(n_k \mid \lambda_2) & \text{with prob.} r \quad if\ n \end{cases}$$

if n_k *is odds*
if n_k *is even*

Based on what was already mentioned in section 4-1, the density likelihood of even trips and odd trips would be as follows:

The value of r based on what was said in

$$g(n_k \mid \lambda_k) = g(n_k \mid \lambda_1, \lambda_2) = \begin{cases} e^{-3.882799} \frac{(1.970482)^{n_k - 1}}{((n_k - 1)/2)!} \\ e^{-3.18442591} \frac{(1.784495982)^{n_k}}{(n_k/2)!} \end{cases}$$

with prob.1 – 0.8511 $\quad n_k = 1,3,5,...$

with prob.0.8511 $\quad n_k = 0,2,4,...$

section 4-1, equals the number of even trips divided by the total sum of odd and even trips. This value is equal to: 0.8511

4.2.1. Determining the composed likelihood distribution function

The distribution of n_k / λ_k is in fact the distribution of number of trips, which is shown as $g(n_k \mid \lambda_1, \lambda_2)$, it also has the density likelihood distribution function as mentioned.

λ_k Has the prior gamma distribution with the parameters of shape and scale. Now, considering these additional information, the distribution of the number of trips independently from the parameter λ_k must be obtained. Since the parameters of the density function, the number of trips for odd and even trips is not equal; consequently, the distributions of trips is

$$g(n_k) = (1-r) \int_0^\infty \left(e^{-\lambda_1} \frac{\lambda_1^{\frac{n_k - 1}{2}}}{((n_k - 1)/2)!}\right)\left(e^{-\beta\lambda_1} \frac{\beta^\alpha \lambda_1^{\alpha - 1}}{\Gamma(\alpha)}\right)$$

$$d\lambda_1 \times I_{(n_k \text{ is odds})}$$

$$+ r \int_0^\infty \left(e^{-\lambda_2} \frac{\lambda_2^{\frac{n_k}{2}}}{(n_k/2)!}\right)\left(e^{-\beta\lambda_2} \frac{\beta^\alpha \lambda_2^{\alpha - 1}}{\Gamma(\alpha)}\right) d\lambda_2 \times I_{(n_k \text{ is even})}$$

$$g(n_k) = \int_0^\infty g(n_k \mid \lambda_k)\pi(\lambda_k)\, d\lambda_k$$

$$= (1-r) \int_0^\infty g(n_k \mid \lambda_1)\pi(\lambda_k)\, d\lambda_k \times I_{(n_k \text{ is odds})}$$

$$+ r \int_0^\infty g(n_k \mid \lambda_2)\pi(\lambda_k)\, d\lambda_k \times I_{(n_k \text{ is even})}$$

calculated separately and independently from the society parameter (refer to section 4-1). These

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calculations have been presented below. [1,4]

$$g(n_k) = (1-r)\frac{\Gamma((n_k-1)/2+\alpha)}{\Gamma(\alpha)\Gamma((n_k-1)/2)} \times \left(\frac{\alpha}{\alpha+\lambda_k}\right)^{\alpha} \left(\frac{\lambda_k}{\alpha+\lambda_k}\right)^{\frac{n_k-1}{2}} \times I_{(n_k \text{ is odds})}$$

$$+ r\frac{\Gamma(n_k/2+\alpha)}{\Gamma(\alpha)\Gamma(n_k/2)} \times \left(\frac{\alpha}{\alpha+\lambda_k}\right)^{\alpha} \left(\frac{\lambda_k}{\alpha+\lambda_k}\right)^{\frac{n_k}{2}} \times I_{(n_k \text{ is even})}$$

After doing calculations, the results are as below:

Relation [3]:

In which:

- n_k: number of trips (produced from household k)
- r : probability of even number of trips
- α : shape parameter in gamma distribution ,
- β : scale parameter in gamma distribution ,
- λ_k: is the mean parameter in gamma distribution, (dependent variable in disaggregate household trip production model)
- $I_{(n_k \text{ is odds})}$: For odd trips equals 1, otherwise equals 0.
- $I_{(n_k \text{ is even})}$: For even trips equals 1, otherwise equals 0.

Considering the distribution of average household trip production, and also, a suitable linear model with dependent variable (average number of trips) and four independent variables *(number of drivers, number of employee, household size and auto ownership)*, the composed likelihood distribution function can be presented based on these variables.

The frequency distribution of household trip production (trip information for 15073 households), was determined based on the composed likelihood distribution function (section 4-2-1). Then it has been compared and contrasted with the empirical frequency distribution of household trips-based on the survey conducted in updating studies of comprehensive plan of transportation in Isfahan. The results have been presented in graph 2. [8]

The results demonstrate an acceptable and appropriate concordance between the empirical frequency distribution of household trips and the result of the composed likelihood distribution function in section 4-2-1.

5. Conclusion

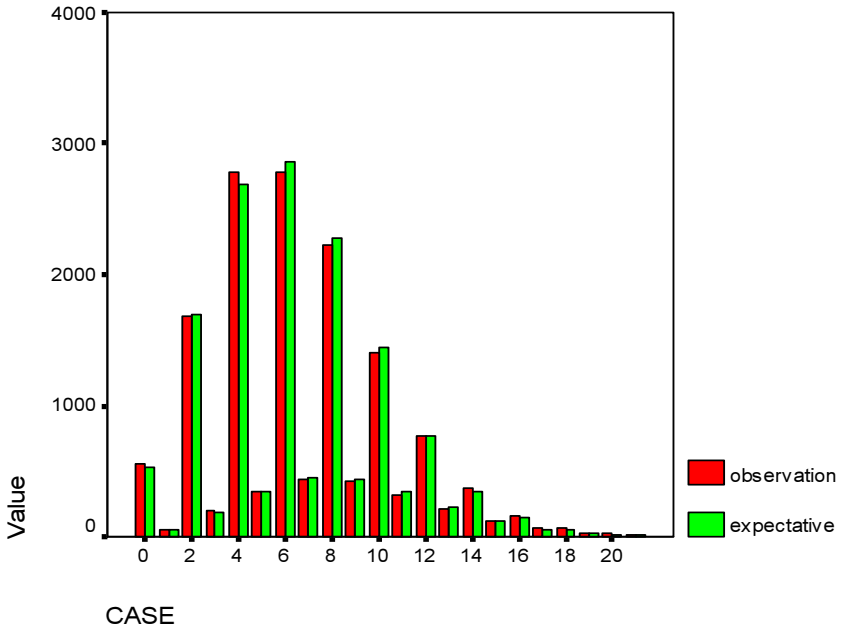

Fig. 2. Comparison of empirical distribution of household trip numbers with the frequency distribution gained through the composed likelihood distribution.

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The χ^2 test in terms of likelihood distribution function of household trip production demonstrated that the parameter of the number of household trip production is following the Poisson discrete distribution function (for odd and even trips).

The final composed probability function shows that it is possible to model the variation of household trip production rate. Comparison between the results of composed probability function and empirical distribution shows that the precision of final function is more than initial functions. (Poisson distribution for odd and even trips) It would be possible to predict likelihood of real produced trips (not average) for each household's category. (By inserting socio-economic characteristics in the composed likelihood distribution function.) In the other hand, it would be possible to estimate and predict for each household's category or specific household, what the likelihood to produce one trip is? How likely it is to produce two trips? And etc.

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