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Optimal resource allocation in urban transportation networks considering capacity reliability and connectivity reliability: a multi-objective approach

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Abstract

Since the 1990's, network reliability has been considered as a new index for evaluating transportation networks under uncertainty. A large number of studies have been revealed in the literature in this field, which are mostly dedicated to developing relevant measures that can be utilized for the evaluation of vulnerable networks under different sources of uncertainty, such as daily traffic flow fluctuations, natural disasters, weather conditions, and so fourth. This paper addresses the resource allocation problem in vulnerable transportation networks, in which multiple performance reliability measures should be met at their desired levels, while the overall cost of upgrading links' performances should be minimized simultaneously. For this purpose, a new approach has been considered to formulate the two well-known performance measures, connectivity and capacity reliability, along with their application in a bi-objective nonlinear mixed integer goal programming model. In order to take into account the uncertain conditions of supply, links' capacities have been assumed to be random variables and follow normal distribution functions. A computationally efficient method has been developed that allows calculating the network-wise performance indices simply by means of a set of functions of links' performance reliabilities. Using this approach, as the performance reliability of links are themselves functions of the random links' capacities, they can be simply calculated through numerical integration. To achieve desirable levels for both connectivity reliability and capacity reliability (as network-wise performance reliability measures) two distinct objectives have been considered. One of the objectives seeks to maximize each of the measures regardless of what is happening to the other objective function which minimizes the budget. Since optimization models with two conflicting objectives cannot be solved directly, the well-known goal attainment multi-objective decision-making (MODM) approach has been adapted to formulate the model as a single objective model. Then the resultant single objective model has been solved through the generalized gradient method, which is a straightforward solution algorithm coded in existing commercial software such as MATLAB programming software. To show the applicability of the proposed model, numerical results are provided for a simple network. Also, to show the sensitiveness of the model to decision maker's direction weights, the results of sensitivity analysis are presented..

Keywords: *Network design, Resource allocation, Multi-objective programming, Urban transportation networks, Connectivity reliability, Capacity reliability.*

1. Introduction

Because of a broad range of external or internal events, influencing transportation networks since the 1990's, researches has been motivated to evaluate the performance of networks accounting for uncertainties. 'Reliability' can keep the meaning of performance measures as well as in certain cases while providing a quantitative tool to consider uncertainties. There are different definitions for 'reliability'. For instance, reliability can be defined as the ability of an item to perform a required function, under given environmental and operational condition and for a stated period of time [1]. In statistical terms, 'reliability' is the probability that a system, possibly consisting of many components, will function correctly [2].

This paper presents a new resource allocation problem in vulnerable transportation networks. Multiple performance reliability measures can be considered to be improved in the resource allocation problem, among which the capacity

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reliability and the connectivity reliability are known as the supply-side measures and considered in this paper. One of the major contributions of this paper relates to computing the measures with a computationally efficient method. The other contribution are concentrated on developing a bi-objective mathematical programming model that seeks to maximize each of the reliability measures while minimizing the budget needed. In continue, we review the literature on 'reliability measures' and 'resource allocation', each in a separate part, and at the same time try to clarify what this paper devises as new contributions in each part.

1.1. Transportation reliability measures

Urban transportation networks can be classified as multi-component systems. To improve reliability measures in such networks, it will be important to define suitable performance measures for different sources of unreliability (uncertainty).

Existing reliability studies on road networks mainly contain the following four aspects: travel time reliability, connectivity reliability and capacity reliability. Maybe the major impulse for establishing serious researches on road network reliability has been natural disasters (such as earthquake). These events can severely disrupt the normal functioning of network by disconnecting paths. Hence, connectivity reliability was the first measure of performance reliability taken into account to evaluate performance of degradable transportation networks. Connectivity reliability considers the probability that a pair of nodes in a network remains connected. A special case of this index is the terminal reliability that is concerned with the existence of at least one path between each origin-destination (OD) pair [3]. Some methods for analyzing the connectivity reliability of transport networks have been presented in [4 to 7]. In these studies a probabilistic and binary state has been assumed for the performance of network's components (i.e. network's links). The state of a link is usually expressed by a binary variable, which is equal to 1 if the link operates normally and 0 if it fails. This state variable may also has more than two states so that it follows an integer variable as assumed in [7].

As discussed above, previous methods of computing connectivity reliability mostly has the limitation of binary states for links. This paper presents an alternative approach for computing the states of links, whereby the performance reliability of a link is defined as a function of the probability density function of its capacity. This new definition of link performance reliability allows us to formulate connectivity reliability by means of the classical methods frequently used in the literature of reliability, i.e. minimal cut/path set method.

Another well-known measure for transportation network reliability is travel time reliability which is defined as the probability that a trip between a given origin-destination (OD) pair can be completed successfully within a specified time interval [8]. This measure is useful to evaluate network performance under normal daily flow variations [9]. As the focus of this paper is not on travel time reliability measures, further descriptions on this measure is not presented here, and interested readers can refer to related works (e.g. [10] to [14]) and references therein.

The capacity reliability has been defined as the probability that a network can accommodate a certain traffic demand at a required service level (see [15] and [16]). This measure may also be defined as the probability that reserve capacity (largest multiplier applied to a given basic OD demand matrix that can be allocated to a network without violating the link capacity) is greater than or equal to the required demand for a given capacity loss due to degradation. A Monte Carlo simulation framework was presented to compute the capacity reliability in a simple network (ref. [9]). But the simulation method requires large computational effort that makes the solution to be inefficient especially in real-word large-scale instances. An alternative approximate method has recently been developed that utilizes the link performance functions to put simply in a closed formula [17 and 18]. We will incorporate this method to our resource allocation model in Section 4 along with a brief explanation on it in Section 2.

1.2. Resource allocation and reliability optimization researches

The main goal of this paper is to develop a multi-objective model that facilitate planners to minimize construction/rehabilitation expenditures while maximizing reliability measures. Obviously, these two objectives should be considered simultaneously within a single programming model in order to guarantee the optimality of solutions. Hereafter, we will use 'network design problem' and 'resource allocation problem' interchangeably.

There are a large number of studies on the transport network design problem under uncertainty. A newly published review paper has categorized these studies into six major classes of models [19].: expected value models (e.g. [20] to [23]), mean–variance models (e.g. [24] to [29]), chance-constrained models (e.g. [30] and [31]) , probability models (e.g. [32] to [35]), min–max models and alpha-reliable models (e.g. [36] to [38]).

Transportation network design problem (NDP) is inherently multi-objective in nature, because it involves a number of stakeholders with different needs [39]. Despite this fact, the literature on the stochastic multi-objective network design problem is limited. One of the first attempts to this issue have been to develop a simulation-based multi-objective genetic algorithm procedure to solve the build-operate-transfer network design problem with multiple objectives, which are to determine the optimal toll and capacity in a roadway subject to demand uncertainty [40]. A pareto-optimal multi-objective optimization was proposed for robust transportation network design problem to minimize two objectives: the expected total system travel time and the higher moment for total system travel time [29]. Chen et al. [39] developed three stochastic multi-objective models: the expected value multi-objective programming model, chance constrained multi-objective programming model, and dependent chance multi-objective programming model, using different measures of travel time reliability. They developed a simulation-based multi-objective genetic algorithm (SMOGA) solution procedure, consisting of a traffic assignment algorithm, a genetic algorithm, a Pareto filter and a Monte-Carlo simulation, to solve the models. Later, Chen et al. [19] extend the SMOGA solution procedure to solve a new model that maximizes travel time reliability and

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capacity reliability by simultaneously generating a family of optimal solutions known as the Pareto optimal solution set. Recently, Chen and Xu [41] have used a goal programming approach to solve the three stochastic multi-objective models developed in [39].

As discussed, none of the abovementioned multi-objective models has considered the connectivity reliability as a measure to be optimized (maximized). As the connectivity reliability is an important measure for evaluating transportation networks under supply uncertainty (e.g. in the aftermath of natural disasters), we develop a new multi-objective programming model accounting for this measure. Besides, the previous stochastic multi-objective models mostly suggest that the budget limitation should be considered as a constraint. Considering the fact that a network design problem is set up well if satisfies the goal of different stakeholders, the construction/rehabilitation budget should be minimized from the decision maker's perspective. Therefore, we have postulated that this objective also should be embedded in the network design problem. Summarily, this paper presents a bi-objective goal programming model with the two following embedded objectives:

i) to minimize the resource or budget that is important from decision makers' perspective because of economical issues and resource limitations; and

ii) to maximize capacity reliability or connectivity reliability from an operational perspectives and to satisfy travelers' convenience.

As this optimization model with the two conflicting objectives cannot be solved directly, the well-known goal attainment multi-objective decision-making (MODM) approach is adapted to formulate the model as a single objective model.

2. Proposed methods for evaluating the reliability measures

This section presents a new approach to evaluate the connectivity reliability and the capacity reliability in urban transportation networks. The main key that makes the proposed methods superior than the existing methods is that here the link performance reliabilities are determined considering the probability density functions of link capacities, rather than the pre-assumed binary state or integer state variables considered in similar studies (e.g. [3] and [7]). This treatment to link performance reliabilities can provide more flexible tools to evaluate network-wise reliability measures and to be embedded in an optimization framework as well. Our method for computing the link performance reliabilities has been presented before in [18], but we will discuss it briefly in Section 3 for complete throughput.

2.1. Connectivity Reliability

A transport network is complex system consisting of components not necessarily connected to each other in series-parallel order. Exact computation methods for connectivity reliability are not tractable in large-scale networks and will fail once the number of components exceeds 5. This fact have been expressed in related textbooks such as [2] and [42]. For this reason, usually approximate methods are used to evaluate the connectivity reliability for communication and electronic networks instead of exact ones. Here, we adopt a repeatedly used minimal cut/path set approach to measuring the connectivity reliability of urban transportation networks as conceptually used in [3]. However, as we have not assumed pre-specified link performance reliabilities in our method, our method differs from that of [3] in its detail application.

Let be:

It can be shown that the following formulas are true for the

$R_{net.}^{connectivity}$: the network connectivity reliability, the probability that all OD pairs in the network remain connected under a given source of uncertainty;

$Links(w)$ *:* the set of links that can play role in connecting OD pair *w;*

$\boldsymbol{R}$ $:=\{R_1,R_2,...,R_i,...,R_n\}$: the vector of performance reliability of all network's links (*n* is the number of links of the network); link performance reliability is defined as the probability that the link performs well, given a desired level of service; the explanation of link performance reliability calculation is presented in Section 3;

$\boldsymbol{R}_w$: $=\{R_1 | i\in Links(w)\}$: the vector of performance reliability of those links play role in connecting OD pair *w*; ($\boldsymbol{R}_w$ is a subset of $\boldsymbol{R}$)

$R_w^{connectivity}$: the connectivity reliability of OD pair *w*, as a function of links' performance reliabilities, $R_w^{connectivity}(\boldsymbol{R}_w)$; the connectivity reliability for each OD pair is defined as the probability that it remains connected under a given source of uncertainty;

R_w^{upper} : the upper bound of the connectivity reliability of OD pair *w*;

R_w^{lower} : the lower bound of the connectivity reliability of OD pair *w*;

$A(w)$: $=\{A_1^w,A_2^w,...,A_U^w\}$: the minimal path set of OD pair *w* (A minimal path set is a minimal set of components whose functioning ensures the functioning of the system);

U : the number of minimal paths;

$C(w)$: $=\{C_1^w,C_2^w,...,C_L^w\}$: the minimal cut set of OD pair *w* (A minimal cut set is a minimal set of components whose failure ensures the failure of the system); and

L : the number of minimal cuts;

upper bound and lower bound connectivity reliability (see [2], Section 9.4.2):

$$R_w^{upper}=1-\prod_{u=1}^{U}\left(1-\prod_{i\in A_u^w}R_i\right) \quad (1)$$

, and

$$R_w^{lower}=\prod_{l=1}^{L}\left[1-\prod_{i\in C_l^w}(1-R_i)\right]. \quad (2)$$

It is to be noted that the upper bound should be close to the actual $R_w^{connectivity}$ if there is not too many joint links in the minimal path sets, and the lower bound to be close if there is not too many joint links in the minimal cut sets.

The interpretation of lower and upper bounds obtained from

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minimal cut and path sets is as follows. Connecting minimal paths in series order and then computing the reliability of this series system gives an upper bound for the connectivity reliability of the system, since a system will function if and only if all the components of at least one minimal path set are functioning. In the other direction, connecting minimal cuts in parallel order and then computing the reliability of this parallel system gives a lower bound for the connectivity reliability of the system, since a system is not functioning if and only if all the components of at least one minimal cut set are not functioning.

To achieve a single measure for connectivity reliability we have used the simple arithmetic mean of R_w^{upper} and R_w^{lower} as the overall connectivity reliability of OD pair :

$$R_w^{connectivity} = (R_w^{lower} + R_w^{upper})/2 \tag{3}$$

To be applicable for network-wide assessment, the weighted average of OD's connectivity reliability with respect to their trip demands, q_w, are accounted. Therefore, the whole network connectivity reliability is a function of R:

$$R_{net.}^{connectivity} = connect(\mathbf{R}) = \frac{\sum_{w \in W} (q_w \cdot R_w^{connectivity})}{\sum_{w \in W} q_w}. \tag{4}$$

Remark 1. An important point to note is that in the application to large networks the determination of all paths connecting an OD pair is not necessary because the travelers use only a few common paths; thus, the *Links*(w) set can be obtained by using k−shortest path algorithms, such as those discussed in [43].

Remark 2. The most important advantage of employing the above closed formula to evaluate network performance reliability is that it is simple to use in optimization techniques and it has very little computation procedure elapsed time compared with simulation based techniques such as Monte Carlo and Latin Hypercube Sampling (LHS).

2.2. Capacity Reliability

The capacity reliability of a transportation network is the probability that the maximum network capacity is greater than or equal to a required demand level. The interpretation of the capacity reliability is that the network can successfully accommodate a required demand if the capacity of even one of the links is greater than or equal to its flow volume. Given the performance reliability of a link as the probability that the capacity of the link is not less than the flow volume assigned to it, the non-degradation probability (or capacity reliability) in a network with independent link capacities can be obtained as follows.

Define D_i the event that link i performs normally ($\Pr(D_i)=R_i$). Then the event that the whole network performs normally can be defined as the intersection of D_1, D_2 , …, and D_n, or $\bigcap_{i=1}^{n} D_i$. By definition, the capacity reliability is $\Pr(\bigcap_{i=1}^{n} D_i)$. Under the assumption of independent performance for the network's links one can write:

$$R_{net.}^{capacity} = cap(\mathbf{R}) = \prod_{i=1}^{n} R_i . \tag{5}$$

This formula provides an approximate value for the capacity reliability, because in real situations the link performances are dependent on each other. Although this method is not exact, in situations where the stochastic event influence links independently and travelers do not change their paths (for example, in accidents) the performance reliability of links will be independent of each other and the proposed formula leads to exact values for the capacity reliability. Application to the schematic network of reference [9] in Section 5 shows good approximations by the proposed method. Note that this approximation is very valuable with respect to its little effort compared with exact methods such as Monte Carlo simulation or LHS methods.

The only unknown variable in Equations (1) to (5) is R_i which will be dealt with in the next section.

3. Link performance measure

Suppose that the capacity of each link is a random variable C_i which follows a certain distribution function (for example a normal distribution $C_i \sim N(\mu_i, \sigma_i^2)$ where μ_i and σ_i^2 are respectively the mean and variance of the distribution). It should be noted that the selected distribution function (or, probability density function, PDF, in continuous cases) should be bounded in left tale to the minimum possible capacity C_i^{min} (that is zero at its extreme) and in the right tale to the maximum possible capacity C_i^{max} . Therefore, we propose the following justification to be used in distributions with unbounded PDF cases. Define $h_i(c)$ the pdf function of capacity of link i, and $f_i(c)$ the justified PDF of capacity of link i:

$$f_i(c) = f_i(x) + C_i^0$$

where

$$C_i^0 = \frac{1 - \int_{c_i^{\min}}^{c_i^{\max}} h_i(x)dx}{c_i^{\max} - c_i^{\min}} \tag{6}$$

Regardless of the type of the selected distribution, $f_i(c)$ is a function that holds the main characteristics of a probability distribution function as follows [17]:

$$\begin{aligned}\int_{c_i^{\min}}^{c_i^{\max}} (h_i(x) + C_i^0)dx &= \int_{c_i^{\min}}^{c_i^{\max}} C_i^0 dx + \int_{c_i^{\min}}^{c_i^{\max}} h_i(x)dx \\ &= C_i^0 (c_i^{\max} - c_i^{\min}) + \int_{c_i^{\min}}^{c_i^{\max}} h_i(x)dx \\ &= \frac{1 - \int_{c_i^{\min}}^{c_i^{\max}} h_i(x)dx}{c_i^{\max} - c_i^{\min}} (c_i^{\max} - c_i^{\min}) + \int_{c_i^{\min}}^{c_i^{\max}} h_i(x)dx \\ &= 1\end{aligned}$$

The new PDF ($f_i(c)$) can be interpreted as the truncated from of $h_i(c)$ with lower and upper bounds of C_i^{min} and C_i^{max} instead of $-\infty$ and $+\infty$.

Now, the link performance reliability can be defined as the probability that the capacity of link i is greater than or equal to its flow volume, or, mathematically:

$$R_i = P(C_i \geq v_i) = \begin{cases} \int_{v_i}^{c_i^{\max}} f_i(c)\,dc & , v_i \geq c_i^{\min}; \\ 0 & , c_i^{\max} < v_i. \end{cases} \tag{7}$$

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In fact, when link's flow volume is less than the upper bound of capacity, the integration of $f_i(c)dc$ from v_i to C_i^{max} expresses the link reliability; otherwise, the link reliability vanishes; namely, there is no chance for the link's capacity to be greater than or equal to the flow volume assigned to the link (see Fig. 1).

Furthermore, the proposed link performance reliability can be computed in different service levels in terms of $(v/C)_i$ ratio (or α_i) as the probability that the capacity of link i is greater than or equal to v_i/α_i:

$$R_i = P(C_i \geq v_i/\alpha_i) = \begin{cases} \int_{v_i/\alpha_i}^{c_i^{\max}} f_i(c)\,dc & , v_i/\alpha_i \geq c_i^{\min}; \\ 0 & , c_i^{\max} < v_i/\alpha_i. \end{cases} \quad (8)$$

This definition may help us to evaluate the network performance reliability at a special level of service. It is to say, α_i=1 indicates that all links of the network can operate with their maximum capacities. Note that for being meaningful in connectivity reliability assessment the α_i ratio must be large enough so as link i is considered as a connected link if $C_i \geqq v_i/\alpha_i$ and disconnected if $C_i < v_i/\alpha_i$.

The only unknown in Equation (8) is v_i that is obtained for each set of link capacities (i.e. $\{C_1=c_1, C_2=c_2, \ldots, C_i=c_i, \ldots, C_n=c_n\}$, where C_i is the random variable for capacity of link i, and C_1 denotes one realization of C_i) from the following well-known user equilibrium assignment problem:

$$Minimize \sum_{i \in I} \int_0^{v_i} g_i(x, c_i)\,dx \quad (9)$$

subject to:

$$\sum_{r \in R_w} f_r = q_w \qquad , \forall w \in W \quad (10)$$

$$v_i = \sum_{r \in Paths} f_r X_i^r \qquad , \forall i \in I \quad (11)$$

$$f_r \geq 0 \qquad , \forall r \in Paths \quad (12)$$

where is the set of network paths, w the set of OD pairs, p_w the set of paths and q_w flow between OD pair $w \in W$, f_r user equilibrium flow on path $r \in Paths$, and X_i^r is 1 if link i belongs to path r; otherwise 0. $g_i(.)$ is a cost function for link i.

If the stochastic event that influences the link capacities is assumed to be low frequent (namely, the travelers have not so much information about the event to switch their paths after the event happens) then the objective of the assignment problems is substituted by

$$Minimize \sum_{i \in I} \int_0^{v_i} g_i(x, c_i^{\max})\,dx \quad (13)$$

Due to the fact that the event is low-frequent and the travelers make their decisions by accounting for the most experienced network state, i.e. $\{C_1=c_1^{max}, C_2=c_2^{max}, \ldots, C_i=c_i^{max}, \ldots, C_n=c_n^{max}\}$.

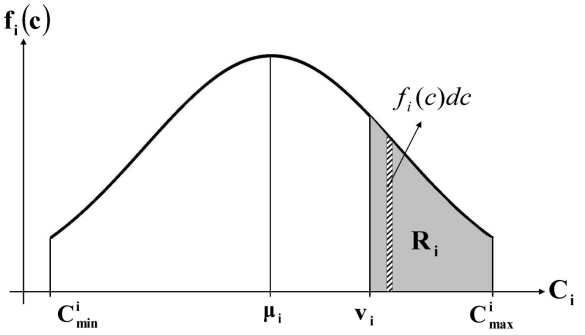

Fig. 1 Probability density function for capacity of link and the related parameters required for computing the link performance reliability

4. The resource allocation model

4.1. Preliminaries

We have assumed that the proposed resource allocation model assigns the level of investment in network's links in a discrete manner. Thus, this model can be called discrete reliable network design problem (DRNDP). The problem, therefore, needs an investment function indicating that if a certain amount of budget is assigned to a link what would be the resultant change in the link's capacity. Since the link capacities are random variables with parameters σ_i and μ_i. We have assumed that a certain amount of budget E_i (where j shows the investment level) can enhance the capacity of link i to a new capacity C_{ij} characterizing by parameters σ_{ij} and μ_{ij}. Hence, the performance reliability of link i at investment level j is obtained as

$$R_{ij} = P(C_{ij} \geq v_i/\alpha_i) = \begin{cases} \int_{v_i/\alpha_i}^{c_{ij}^{\max}} f_{ij}(x)\,dx & , v_i/\alpha_i \geq c_{ij}^{\min}; \\ 0 & , c_{ij}^{\max} < v_i/\alpha_i. \end{cases} \quad (14)$$

where f_{ij}(c) is the PDF of link i's capacity at investment level j with parameters σ_{ij} (standard deviation) and μ_{ij} (mean). c_{ij}^{max} and c_{ij}^{min} denote respectively the upper and the lower bounds of link i's capacity at investment level j.

4.2. The bi-objective model

The model has two levels. In the upper level the desired objectives –minimization of budget and maximization of network reliability- are embedded. As discussed in Section 3, the link performance reliabilities are functions of the traffic volumes assigned to the links. On the other hand, the link volumes may change once the link capacities are changed. Consequently, the link volumes should be, in long-run analyses, determined from the assignment problem. Here, the assignment problem is called the 'lower level' problem. Interested readers can find more in-depth discussions about bi-level models and their applications in transportation network design problems in [44].

The model may be formulated as follows:

(The upper level problem)

$$\underset{\mathbf{Z}}{Maximize} \quad R_{net.} = f(\mathbf{R}(\mathbf{Z}, v^*(\mathbf{Z}))) \quad (15)$$

$$\underset{\mathbf{Z}}{Minimize} \quad B = \sum_{j \in J} \sum_{i \in I} E_{ij}.Z_{ij} \quad (16)$$

subject to:

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$$\sum_{j\in J} Z_{ij} = 1, \quad i \in I \tag{17}$$

$$Z_{ij} \in \{0,1\}, \quad i \in I, \quad j \in J \tag{18}$$

where R_{net} is the selected network reliability measure which can be either $R_{net.}^{connectivity}$ or $R_{net.}^{capacity}$; consequently, the function $f(.)$ can be selected from Equation (3) or (5) with respect to the selected network reliability measure, i.e. *connect (**R**)* for the case of selecting $R_{net.}^{connectivity}$ and *cap(**R**)* for $R_{net.}^{capacity}$. The second objective (Equation (16)) is to minimize the total required investment, B. The variable Z_{ij} is the decision variable to reflect how much budget is assigned to link i; Z_{ij} equals 1 if investment level $j \in J$ is assigned to link $i \in I$ and equals 0 otherwise. The matrix of Z_{ij} s is denoted by $\mathbf{Z}$. The reason of using the function form of R, i.e. $\mathbf{R(Z,v^*(Z))}$, is that it is itself a function of links' capacities and volumes (see Equation (14)) in the current state of the network ($\mathbf{Z}$). Since the current state of the network (the vector of link capacities) is determined by decision variables Z_{ij}, the vector $\boldsymbol{R}$ will be a function of $\mathbf{Z}$ and $\mathbf{v^*(Z)}$. As $\mathbf{v^*(Z)}$ is the vector of link volumes when the state of network is defined by $\mathbf{Z, v^*(Z)}$ must be obtained from the assignment problem, which is formulated as:

(The lower level problem)

$$\underset{\mathbf{v}}{Minimize} \quad \sum_{i\in I} \int_0^{v_i} g_i\left(x, \sum_{j\in J} Z_{ij}^{*} . c_{ij}^{\max}\right) dx \tag{19}$$

subject to Constraints (10) to (12).

where v is the vector of link volumes and the decision variable of the lower level problem. Z_{ij}* is a function of v and is determined from the upper level problem.

4.3. The application of goal attainment programming

In the previous section, a multi-objective decision making (MODM) model was developed to formulate the proposed bi-objective model. It is obvious that each of the tow objectives is against the other; for instance, increasing the network reliability, that is the first objective, would lead to increasing the investment values. Hence, it is necessary to apply a model in a manner that the objectives can be improved simultaneously. Moreover, since the decision variables are binary, the MODM model must be capable of considering integer variables (notice that binary variables are special cases of integer variables).

A goal attainment approach, which is one of the most applicable types of discrete-based approaches (see [45]) in MODM, is utilized to solve the proposed bi-objective problem. In order to apply a goal attainment model, we have employed a reference point $\boldsymbol{G} \in \boldsymbol{R}^p$ (here p is equal to 2 and R^p denotes the -dimensional Euclidean space), a direction $\boldsymbol{D}$ in the objective space along which the search is performed, and a real variable γ measuring the progress along the direction $\boldsymbol{D}$. Given a goal vector $\mathbf{G}=(R_{net}^{\ 0}, B^0)$, where $R_{net}^{\ 0}$ and R^0 are respectively the goals for the objectives, and a direction $\mathbf{D}=(D_R, D_E)$, the goal attainment approach can be formulated as follows:

$$Minimize\ \gamma \tag{20}$$

subject to:

$$R_{net.}^{0} - R_{net.} \leqslant \gamma D_R \tag{21}$$

$$\sum_{i\in I}\sum_{j\in J} E_{ij} Z_{ij} - B^0 \leqslant \gamma D_E \tag{22}$$

$$\sum_{j\in J} Z_{ij} = 1, \quad \forall i \in I \tag{23}$$

$$Z_{ij} \in \{0,1\}, \quad i \in I, \quad j \in J \tag{24}$$

$$\gamma \geqq 0 \tag{25}$$

D_R and D_E can be interpreted as weights for each objective, in terms of combined effect of the objectives including their importance degrees and their normalization factors. The normalization factor is a factor that can neutralize the objective dimension effect. It should be noted that the direction vector has not a fixed and predefined quantity, but it depends on the worthiness degree of the objective for the decision maker; therefore, the analyst must examine different directions in order that the decision maker can achieve his/her goals with acceptable deviations.

5. Numerical Results

In this section, the presented method is applied to a simple network analyzed repeatedly in the preferable related researches (e.g. [9]) to facilitate comparing the results of the calculation of capacity reliability and connectivity reliability provided here with those of previous works. This simple test network is shown in Fig. 2. As explained in previous sections, this paper deals with two aspects of network reliability evaluation; performance reliability evaluation and resource allocation with respect to reliability optimization. The subjects of those parts dealing with reliability analysis are similar in some existing works; therefore, for the sake of comparing the numerical results obtained from proposed method and other methods, we have examined the test network. As shown in Fig. 2, the network consists of five nodes, seven links, and two OD pairs. The base demand for OD pairs (1,4) and (1,5) are 20 and 25, respectively. The Bureau of Public Road (BPR) link travel time function is used:

$$t_i = t_i^f \left(1 + .25\left(v_i / C_i\right)^4\right) \tag{23}$$

where v_i, t_i^f, and C_i are the flow, free-flow travel time, and random capacity on link i, respectively. Table 1 gives the results of user equilibrium traffic assignment and general information about the network links. As mentioned before, traffic volumes are assigned to the links based on expected values of the link capacities.

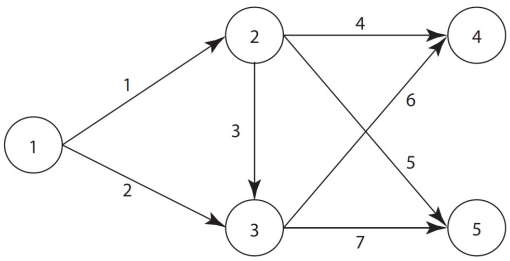

Fig. 2 Example network

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Table 1. General Properties of the Links and Assigned Flow Volumes

Link No.	Free-flow travel time	Statistical properties of link capacities				Flow volume
		Mean	Standard deviation	Lower bound	Upper bound	
1	4.0	18.75	3.61	12.5	25.0	24.95
2	5.2	18.75	3.61	12.5	25.0	20.05
3	1.0	11.25	2.17	7.5	15.0	5.86
4	5.0	11.25	2.17	7.5	15.0	7.84
5	5.0	11.25	2.17	7.5	15.0	11.25
6	4.0	11.25	2.17	7.5	15.0	12.16
7	4.0	11.25	2.17	7.5	15.0	13.75

5.1. Reliability measures

Because the Monte Carlo simulation is a powerful technique to analyze probabilistic networks, especially when enough samples can be drawn, it may result in an acceptable solution. For comparisons, therefore, we will consider the Monte Carlo simulation method.

Reliability assessment procedure presented in this paper has been applied to the test network with the assumption of $(v/C)_i$=1. Fig. 3 compares the connectivity reliability of OD pair (1,4) obtained from the proposed method and the Monte Carlo simulation (with 5000 samples) for different demand levels. This figure shows that the result of the proposed method is relatively similar to the results of the simulation method, while needing less computational effort due to the analytical closed formula used.

Fig. 4 shows that for the capacity reliability, both of the proposed and Monte Carlo simulation methods have similar results for high of demand levels (namely, levels over than 0.7). For demand levels under 0.7, there exist considerable differences between the results obtained from the proposed method and the Monte Carlo simulation method. This difference is due to the different traffic assignment approaches adopted in the methods.

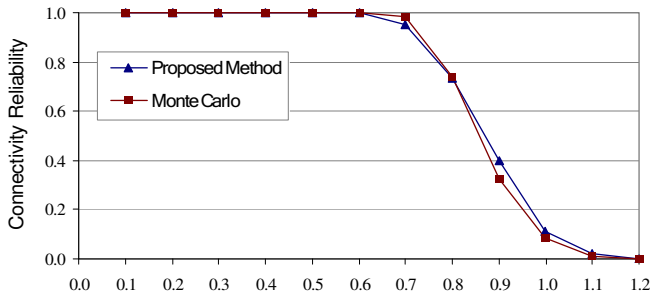

Fig. 3 Connectivity reliability of OD pair (1,4) at various levels of demand

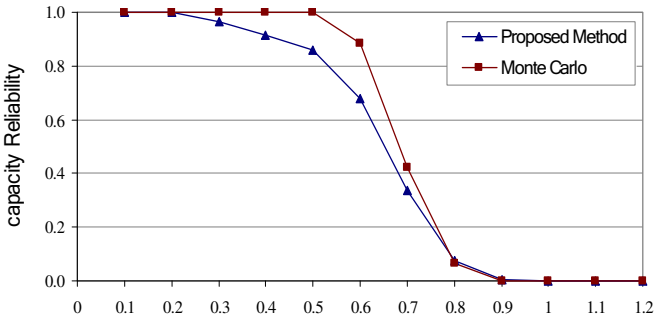

Fig. 4 Capacity reliability of the network at various levels of demand

More specifically, in [9] it has been assumed that all travelers have a complete knowledge about the events and they can always realize the shortest possible paths under the occurrence of any combination of link capacity degradations due to the stochastic events (i.e. users has been assumed to be 'adaptive'). On the contrary, as stated earlier in the last paragraphs of Section 3, in the proposed method it is assumed that travelers do not have such complete knowledge, or even if they do, they are not so familiar with the network statuses that can switch their paths to new shortest (better) paths (i.e. users has been assumed to be 'non-adaptive'). As a result, we can distinctly differentiate between the capacity reliability for the cases with well-informed (adaptive) and non-well-informed (non-adaptive) travelers. In this sample case, the difference can vividly be seen in low-congested situations wherein the demand multiplier changes between the range of [0.3, 0.7] (see Figure 1). In addition, within this range, the higher amounts of network capacity reliability computed through the Monte Carlo simulation method are rational and expectable because in this method the travelers can make their trips successfully on the shortest possible paths. Namely, in the case of adaptive travelers, the network will be more successful to accommodate higher levels of demand because of better reaction of travelers to the consequences of the stochastic events.

Another important finding in Figure 4 is that the capacity reliability of the two cases (adaptive and non-adaptive) cases is not so different when the demand multiplier is over 0.7, in this special case. As a by-product, one may conclude that the proposed expression can also be used as a good approximation for the case of adaptive travelers when dealing with highly congested networks (where capacity analysis becomes more crucial). This would result in very fast evaluation of network's capacity reliability without need of implementing Monte Carlo simulation with very large number of replications (as well as performing UE traffic assignment in each replication) in large network instances.

5.2. Goal attainment Results

Results of application of goal attainment method to the test network are presented in Tables 2 and 3. Table 2 gives the connectivity reliability values and investment levels for different combinations of goals' importance factors, that are interpreted as the direction vector $\mathbf{D}=(D_R,D_E)$, when the goal vector is (8, 0.95). For instance, in condition 1, D_R,D_E are both equal to 1 and the amount of network connectivity reliability and investment level are respectively 0.8677 and 7 with an objective value of 0.0823. An important issue, when using the goal attainment programming, is to be aware of selecting adequate directions, because different directions would result in different quantities for the reliability measure. For example, consider columns 3 and 4. In these columns, by altering D_E from 20 to 40 the connectivity reliability is decreased only by 0.0001 (i.e. from 0.9245 to 0.9244). This shows that it would

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Table 2 Application of different direction combinations to the goal attainment model, R_{net}^{0}=0.95 and B^{0}=10

	Condition	1	2	3	4	5	6	7
Connectivity Reliability	D_R	1	1	1	1	1	0.01	1
	Value	0.8677	0.9048	0.9244	0.9245	0.9400	0.9451	0.9570
Investment	D_E	1	8	20	40	100	1	400
	Value	7	7.5	8.5	8.5	9.5	10.5	10
Objective	γ	0.0823	0.0452	0.0256	0.0255	0.0150	2.5	0.005

Table 3 Application of different direction combinations to the goal attainment model R_{net}^{0}=0.50 and B^{0}=8

	Condition	1	2	3	4	5	6	7
Connectivity Reliability	D_R	1	1	1	1	1	0.01	1
	Value	0.8677	0.9048	0.9244	0.9245	0.9400	0.9451	0.9570
Investment	D_E	1	8	20	40	100	1	400
	Value	7	7.5	8.5	8.5	9.5	10.5	10
Objective	γ	0.0823	0.0452	0.0256	0.0255	0.0150	2.5	0.005

be very important for the analyzer how to select relevant directions to avoid misleading results, because the optimal solution is made an optimal decision based on these directions. The other notable fact is that the ratio of the weights (D_R,D_E) does not play an important role in determining reliable solutions, because there may be some conditions that the ratios are analogous while the model results are quite different. For example, in columns 5 and 6, it is shown that D_R/D_E is equal to 0.01, and the value of both investment and connectivity reliability are different in each column.

Table 3 presents results of application of goal attainment method to the case of capacity reliability with goal vector (10, 0.50). As seen in this table, the values of capacity reliability with respect to the associated investment levels behave differently in comparison to the results of Table 2. For example, changing direction **D**=(D_R,D_E) from (1, 1) to (100, 1) in columns 1 and 4 respectively, results in similar values for capacity reliability with a constant investment level 9.5. Namely, the quantity of capacity reliability shows no changes when the expended investment is 9.5 units. Analogous conditions are also occurred in columns 2 and 4. Therefore, an important result derived from Table 3 is that in some especial conditions the model is not so sensitive to selected direction vectors. This issue may imply the goodness of the values achieved for the required capacity reliability with an especial investment level and, therefore, for investment levels 9.5 and 11, probably we cannot achieve better directions, by altering the weights, compared with those presented in Table 3.

It should be noted that all of the results presented in this work is a short-run analysis of the proposed method in which the link volumes is assumed to be constant in the upper level problem.

6. Conclusions

In this paper, first, a framework was developed for evaluating the two well-known transportation network reliability measures, connectivity reliability and capacity reliability. The proposed framework provides a closed formula for each of the measures.

To enhance the performance reliability of degradable transportation networks, this paper presents a bi-objective goal programming model with the two following embedded objectives:

i) to minimize the resource or budget that is important from decision makers' perspective because of economical issues and resource limitations; and

ii) to maximize capacity reliability or connectivity reliability from an operational perspectives and to satisfy travelers' convenience.

The two objectives are conflicting so that increasing one of the objectives would result in decrease in the other one. Therefore, the well-known goal attainment multi-objective decision-making (MODM) approach was adapted to formulate the model as a single objective model. Results show that it would be very important for the analyzer how to select relevant directions to avoid misleading results. Also, it was shown that the ratio of the weights (D_R/D_E) does not play an important role in determining reliable solutions.

In this paper, two objectives, to achieve a specified value of performance reliability and to expend a limited investment, have been incorporated into a goal attainment model which have a single objective. However, somewhere we might need to account for other objectives, like maintaining the level of service at a required value. These objectives can also be incorporated into the proposed framework, but it requires elaborated assumptions and more applicable test networks. This topic can be a good area for future research. The other possible extension of this work is to provide methods that can consider a set of more cooperative paths for calculating lower and upper bounds in connectivity reliability, because in the method presented here, all of the paths connecting OD pairs are considered in minimal cut/path method. Therefore, when a large network is to be analyzed, the calculation procedure tends to be very complicated and time-consuming. Moreover, the application of its associated non-linear function as the objective of the model leads to a complex programming that is a NP-complex problem. Thus, an approximate method by which a number of more cooperative paths between each OD pair can be distinguished is valuable, or thinking on the application of meta-heuristic optimization methods should be made to solve the problem.

Furthermore, the examination of other multiple objective decision making methods can be considered as an important issue for future works in the field of transportation network reliability analysis.

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