International Journal of Civil Engineering

Structural Engineering

Prediction of extent and liklihood of corrosion-induced cracking in reinforced concrete bridge decks

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Received: October 2009, Revised: October 2010, Accepted: November 2010

Abstract

Corrosion of reinforcement due to frequently applied deicing salts is the major source of deterioration of concrete bridge decks, e.g. severe cracking and spalling of the concrete cover. Since crack width is easily recordable in routine visual inspections there is a motivation to use it as an appropriate indicator of condition of RC bridge elements in decision making process of bridge management. While few existing research in literature dealing with spatial variation of corrosion-induced cracking of RC structures is based on empirical models, in this paper the extent and likelihood of severe cracking of a hypothetical bridge deck during its lifetime is calculated based on a recently proposed analytical model for corrosion-induced crack width. Random field theory has been utilized to account for spatial variations of surface chloride concentration, as environmental parameter, and concrete compressive strength and cover depth as design parameters. This analysis enables to track evolution of cracking process, spatially and temporally, and predict the time for the first repair of bridge deck based on acceptable extent of cracked area. Furthermore based on a sensitivity analysis it is concluded that increasing cover depth has a very promising effect in delaying corrosion phenomenon and extension of the service life of bridge decks.

Keywords: Bridge, Corrosion, Crack Width, Random Field.

1. Introduction

Bridges are prone to various deterioration mechanisms. Amongst others corrosion-induced cracking is the most prevalent damage in reinforced concrete decks. This phenomenon which is subject to applied deicing salts is recordable during routine inspections. Based on condition ratings collected during inspections Bridge Management Systems (BMSs) are developed for optimal allocation of limited budgets [1]. In bridge decks ingress of chlorides from de-icing salts through the concrete cover deactivates the natural protective oxide layer formed around the reinforcements by the strong alkalinity of pore solution. Once the protective layer has dissolved, if chloride concentration exceeds a threshold value and enough oxygen and moisture are present, corrosion is initiated. The corrosion products have a volume of three to six times greater than the original steel, leading to tensile stresses within the concrete and resulting in longitudinal cracking and spalling at the surface.

The complexity of prediction of corrosion of steel bars is not only because of complicated associated mechanical and chemical phenomena but also because of their environmental nature. There is indeed a high degree of uncertainty associated with the environmental parameters, physical properties of materials and loading. This was the motivation for most researchers in the past decade to apply probabilistic methods for evaluation of deterioration of RC bridges and many models have been developed for bridge deck deterioration due to chloride diffusion in concrete [2,3]. However, most of these models assume that "failure" occurs when corrosion is initiated. In propagation phase after initiation of surface cracking their width will increase with time to a limit that spalling of concrete is prone. Most bridges prior to becoming structurally deficient become unserviceable due to severe cracking and spalling of concrete and need routine repair and maintenance interventions [4,5]. The reason for this fact is due to higher safety factors used in design for strength limit states rather than serviceability limit states. Cost-effective maintenance strategies can be attained if the extent and likelihood of deterioration during lifetime of a bridge can be predicted in a probabilistic framework. It is recognized that the material and dimensional properties of a concrete structure

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will not be homogeneous due to the spatial variability of workmanship, environmental and other factors and, as such, corrosion damage will occur spatially over any exposed surface. In recent years several efforts have been made to predict in a probabilistic manner the extent of damage [?6-?15]. In this paper a two-dimensional spatially variable time-dependent reliability analysis is developed to predict the likelihood and extent of cracking for RC deck top surface exposed to chloride ion attack. This model will consider the random spatial variability of concrete material properties, concrete cover and surface chloride concentration which in turn will influence the spatial variability of dependent variables such as corrosion initiation and cracking.

1.1. Corrosion initiation

Numerous research have found that the penetration of chlorides through concrete can be best represented by a diffusion process if the concrete is assumed to be relatively moist [2,16-17]. In this case, the penetration of chlorides is given empirically by Fick's second law of diffusion if the diffusion is considered as one- dimensional in a semi-infinite solid; this is expressed as Eq. 1:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial^2 x} \tag{1}$$

Crank's solution to Fick's second law of diffusion is represented as below [2]:

$$C(x,t) = C_0 + (C_{sa} - C_0)\left[1 - erf\left(\frac{x}{2\sqrt{D_a t}}\right)\right] \tag{2}$$

In general, the chloride concentration profiles obtained under different climates are used in mathematical models for obtaining parameters of Eq. 2. A plot of chloride concentration vs. penetration depth can often be closely described by Crank's solution to Fick's second law of diffusion. The curve-fitting results in these parameters: apparent diffusion coefficient, D_a, apparent surface chloride concentration, C_{sa} and the original chloride concentration, C_0 then $C(x,t)$ express the chloride content in depth of x at time t. Apparent diffusion coefficient (D_{ap}) reflects the influence of all possible transport mechanisms that have contributed to the chloride profile and it is the mean value of the actual coefficient over the period between the initial exposure to the chloride-laden environment to sampling.

The corrosion of reinforcements is initiated when the chloride content exceeds a threshold value, C_{cr}, which depassivates the steel embedded in the concrete provided that sufficient moisture and oxygen are present. If the original chloride concentration, C_0, be neglected the probability of corrosion initiation with time can be expressed with the following limit state:

$$g(.)=C_{cr}-C(x,t) \tag{3}$$

Prob (Corrosion)=Prob (g(.)≦0)=*Prob* ($C(x,t)$≧C_{cr})=

$$Prob\ (C_{sa}\left[1 - erf\left(\frac{X_c}{2\sqrt{D_a t}}\right)\right] \geq C_{cr}) \tag{4}$$

The influence of several factors such as concrete mix proportions, cement type, C_3A content of cement, materials incorporated, w/c ratio, relative humidity, and temperature are amongst the factors contributing to variation of governing parameters, e.g. D_a, C_{cr} , within a large range [18,19] but in general the uncertainty of governing parameters can be handled with random variables. Although accurate statistical distributions cannot be derived without sufficient experimental data from existing structure, Dupart [18] concluding of various measurements in the literature for various environmental and workmanship classifications made some general propositions. The descriptive parameters of random fields and random variables in Table 1 are based on such propositions.

1.2. Corrosion Propagation

The expansive corrosion products create tensile stresses on the concrete surrounding the corroding steel bar. This can lead to cracking and spalling of concrete cover as a usual

Table 1. Statistical description of variables

Variable	*Scale of Fluctuation (m)* $\theta_x = \theta_x$	*Mean*	*C.O.V*	*Distribution*	*Reference*
Concrete Cover (mm)	*3.5*	*50*	*0.2*	*Normal*	*[12,18]*
Concrete Comprssive Strength, f_c, MPa	*3.5*	*5.725*	*0.2*	*Lognormal*	*[18]*
Surface Chloride Concentration (kg/m^3)	*3.5*	*3.5*	*0.6*	*Lognormal*	*[12,18]*
Critical Chloride Concentration (kg/m^3)	-	*0.6-1.2*		*Uniform*	*[18]*
Reinforcement Size (mm)	-	*12*	*0.15*		*[24]*
Workmanship Coefficient (k_w)		*0.87*	*0.06*	*Normal*	*[12]*
Porous Media around Reinforcement (d_0), μm	-	*12.5*	-	-	*[24]*
α_{rust}	-	*0.57*	-	-	*[22,24]*
ρ_{rust} (kg/m^3)		*3600*	-	-	*[22,24]*
ρ_{steel} (kg/m^3)		*7850*	-	-	*[22,24]*
Poisson coefficient (ν)		*0.18*	-	-	*[22,24]*

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consequence of corrosion of steel bars in concrete. Considerable research has been undertaken on corrosion-induced cracking process, with perhaps more numerical and experimental investigations than analytical studies [20,21]. For reliability calculations a closed form mathematical model is of paramount importance. In this regard some models for the prediction of time to crack initiation of corroding reinforced concrete (RC) structures have been developed [22,23] but little has been on crack width growth with time. Some empirical models based on mathematical regression of experimental results exist in the literature. For example Vu et al [21] developed an empirical model to predict the time for a crack to propagate to a width of 1 mm. This model is based on accelerated corrosion tests in laboratory.

In this paper the analytical model developed by Li et al. [24] will be used for simulation of corrosion-induced crack width variation with time. The merit of this model is that it is directly related to the factors that affect the corrosion such as concrete geometry and property and corrosion rate.

In this model as shown schematically in Fig.1 concrete with embedded reinforcing steel bars is modeled as a thick-wall cylinder, where D is the diameter of steel bar, d0 is the thickness of the annular layer of concrete pores (that is, a pore band) at the interface between the steel bar and concrete, and X_c is the concrete cover. The inner and outer radii of the thick-wall cylinder are $a=(D+2d_0)/2$ and $b=X_c+(D+2d_0)/2$. When the steel bar corrodes in concrete, its products fill the pore band completely and a ring of corrosion products forms, the thickness of which, $d_s(t)$ (Fig.1), can be determined from Eq. 5 as below [22]:

$$d_s(t) = \frac{W_{rust}(t)}{\pi(D+2d_0)}\left(\frac{1}{\rho_{rust}} - \frac{\alpha_{rsut}}{\rho_{st}}\right) \quad (5)$$

Where α_{rust} is a coefficient related to the type of corrosion products, ρ_{rust} is the density of corrosion products, ρ_{st} is the density of steel, and $W_{rust}(t)$ is the mass of corrosion products. Obviously, $W_{rust}(t)$ increases with time and can be determined from Eq. 6 [22]:

$$W_{rust}(t) = \left(2\int_0^t 0.105(1/\alpha_{rust})\,\pi D \times i_{corr}(t)dt\right)^{1/2} \quad (5)$$

Where icorr(t) is the corrosion current density (in ?A/cm2). In general formation of rust products on the steel surface will reduce the diffusion of the iron ions away from the steel surface resulting in reduced corrosion rates with time. For example Vu and Stewart [13] suggested the following time dependent equation suggested for icorr(t):

$$i_{corr}(t) = i_{corr}(1) \times 0.85(t-T_i)^{-0.3} \quad (8)$$
$$where \qquad t - T_i \gg 1$$

Where T_i is the time to corrosion initiation and i_{corr} (1) is the corrosion current density in the first year after corrosion initiation which is based on concrete quality and can be calculated using Eq. 9 as below [13]:

$$i_{corr}(1) = \frac{27(1-w/c)^{-1.64}}{X_c} \quad (9)$$

According to Li et al. [24] the growth of the ring of corrosion products (known as a rust band) exerts an outward pressure on the concrete at the interface between the rust band and concrete. Under this expansive pressure, the concrete cylinder undergoes three phases in the cracking process: 1) not cracked; 2) partially cracked; and 3) completely cracked. In the phase 1 (no cracking), the concrete can be considered to be elastically isotropic so that the theory of elasticity can be used to determine the stress and strain distribution in the cylinder For a partially cracked concrete cylinder, cracks are considered to be smeared and the concrete to be a quasi-brittle material, so that the stress and strain distribution in the cylinder can be determined based on fracture mechanics When the crack penetrates to the concrete surface, the concrete cylinder fractures completely. Knowing the distribution of stress and strain, the crack width at the surface of the concrete cylinder can be determined simply as below:

$$w_c = 2\pi b[\varepsilon_\theta(b) - \varepsilon_\theta{}^{e,m}(b)] \quad (10)$$

where $\varepsilon_\theta(b)$ is the tangential strain at the surface, that is, at $r=b$ (Fig. 1) and equal to:

$$\varepsilon_\theta(b) = \frac{2d_s/b}{(1-\nu_c)(a/b)^{\sqrt{\alpha}} + (1+\nu_c)(b/a)^{\sqrt{\alpha}}} \quad (11)$$

Where ν_c is Poisson's ratio of concrete and α(<1) is the tangential stiffness reduction factor. According to Li et al. [24] it is assumed that the residual tangential stiffness is constant along the cracked surface, that is, on the interval [a, r0], and represented by αE_{ef} , where E_{ef} is the effective elastic modulus of concrete which can be calculated as per Eq.(12) where ϕ_{cr} and E_c represent creep coefficient and elastic modulus of the concrete [22]:

$$E_{ef} = \frac{E_c}{1+\varphi_{cr}} \quad (12)$$

Furthermore $\varepsilon_\theta{}^{e,m}$(b) is the maximum elastic strain at $r=b$ and equal to:

$$\varepsilon_\theta{}^{e,m}(b) = \frac{f_t}{E_{ef}} \quad (13)$$

Thus, the crack width can be expressed as:

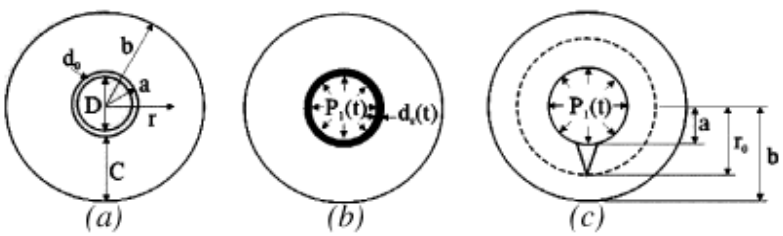

Fig. 1. Schematic representation of cracking process[24]

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$$w_c = \frac{4\pi d_s}{(1-\nu_c)(a/b)^{\sqrt{\alpha}}+(1+\nu_c)(b/a)^{\sqrt{\alpha}}} - \frac{2\pi b f_t}{E_{ef}} \quad (14)$$

In Eq. (14), the key variables are the thickness of corrosion products d_s, which is directly related to the corrosion rate (i_{corr}), and the stiffness reduction factor α, which is related to stress conditions and concrete property and geometry. Eq.14 which is used in this paper has been verified by both numerical and experimental results [24]. In using Eq. 14 one needs to calculate time dependent variables α and d_s. The former is determined solving simultaneous equations derived in Li et al. [24], which are not repeated here for briefness purposes, while the later is calculated according to Eq. 6.

It should be noted that in this formulation time-dependent drying and shrinkage effects is not considered.

1.3. Effect of Concrete Quality

Various mechanical properties of concrete are usually correlated to compressive strength of concrete, a parameter which can be easily measured in practice. On the other hand design codes use compressive strength to define the concrete class. In this paper the same approach is followed. Refer to ACI 318-08 the concrete tensile strength and modulus of electricity is related to compressive strength as:

- Concrete Tensile Strength: $f_t = 0.53\sqrt{f_c'}$ (MPa) (15)

- Concrete Modulus of Elasticity: $E_c = 4600\sqrt{f_c'}$ (MPa) (16)

The other important influencing parameter in studying corrosion in RC structures is the chloride diffusion coefficient. Several models have been developed to consider the influence of mix proportions on chloride diffusion coefficient [25-27]. But Stewart and Vu [13,28] comparing reported data in literature concluded that the model developed by Papadakis et al. [27] appears to be the best fit to available literature which is represented as:

$$D = D_{H_2O}\,0.15\,\frac{1+\rho_c\frac{c}{w}}{1+\rho_c\frac{w}{c}+\frac{\rho_c a}{\rho_a c}}\left(\frac{\rho_c\frac{w}{c}-0.85}{1+\rho_c\frac{w}{c}}\right)^3 \quad (17)$$

In this model a/c is the aggregate-to-cement ratio, ρ_c and ρ_a are the mass densities of cement and aggregates respectively and D_{H2O} is the chloride diffusion coefficient in an infinite solution (=1.6×10^{-5} cm^2/s for NaCl). The water-cement ratio is estimated from Bolomey's formula for Ordinary Portland Cement (OPC) concretes as below [25]:

$$w/c = \frac{27}{f_c'+13.5} \quad (18)$$

Where f_c' is the concrete compressive strength of a standard test cylinder in MPa.

1.4. Random fields

The fact that many parameters show spatial random variability, so-called stochastic fields, which is caused by the variability of exposure environment, materials, and workmanship, is not included in most of the studies. Spatial variability of physical properties includes systematic spatial variation (variation of the mean value and standard deviation) and random spatial variation. Random spatial variability of continuous media can be represented by the use of random fields [7]. In the case of a large surface a 2D random field would be used [9-12], or in the case of beam elements a 1D random field may be more appropriate [6,8,13-15].

A simple model for a random field is a homogenous isotropic Gaussian field, where the random variables have a Gaussian distribution that does not change with direction or location therefore the interdependency between two random variables defined at two points depends only on the distance between them.

The correlation function $\rho(\tau)$ determines the correlation coefficient between two elements separated by distance (τ) and is representative of the spatial correlation between the elements. As the distance between correlated elements becomes smaller the correlation coefficient approaches unity as defined by the correlation function, and likewise as the distance increases the correlation coefficient reduces. The Gaussian (or squared exponential) correlation function used herein is defined as:

$$\rho(\tau) = exp\left[-\left(\frac{|\tau_x|^2}{{d_x}^2}\right)-\left(\frac{|\tau_y|^2}{{d_y}^2}\right)\right] \quad (19)$$

$$\text{Where:}\quad d_x = \theta_x/\sqrt{\pi} \quad \text{and} \quad d_y = \theta_y/\sqrt{\pi} \quad (20)$$

θ_x and θ_y are the scales of fluctuation for a two dimensional random field in x and y directions, respectively [29]; and τ_x=x_i - x_j and τ_y = y_i - y_j are the distances between centroid of element i and element j in the x and y directions, respectively.

The scale of fluctuation for a 1D random field is defined as per Eq. 21: [29-30]:

$$\theta = \int_{-\infty}^{+\infty} \rho(\tau)d\tau \quad (21)$$

The parameters of the Gaussian random field are the mean value μ , the standard deviation σ and the correlation length d_x and d_y.

A Gaussian random field H(x) can be represented by the KL expansion in the form [31]:

$$j=1 \quad (22)$$

Where ζ_j are uncorrelated standard normal random variables independent of x. The deterministic terms μ_j and ψ_j are the eigenvalues and eigenvectors, respectively, of the covariance function C of the continuous random field.

Various methods of discretization of random fields have been proposed [29-30].In midpoint method the random field needs to be discretised into *N* elements of identical size and shape. The random field within each element is represented by a single random variable defined as the value of the random field at the centroid of the element and this value is assumed to be constant within the element. Each of the random variables within the random field are statistically correlated based on the correlation function of the corresponding random field.

Following the discretisation the covariance function can be

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replaced by a n×n covariance matrix C, for which the (i, j)th element is given by:

$$C_{i,j}=\rho_{i,j}\times\sigma^2 \tag{23}$$

The matrix C is a symmetric completely positive matrix and the values on the diagonal refer to the autocorrelation and are equal to the variance of the Gaussian variable. The eigenvalue problem of the covariance matrix is C:

$$C\psi_j = \mu_j\psi_j \tag{24}$$

A discretised random field, given by a vector p of length n, can be represented by the KL expansion in the form:

$$H(x) = \overline{H(x)} + \sum_{j=1}^{r\le n}\psi_j\sqrt{\mu_j}\zeta_j \tag{25}$$

Where $\overline{H(x)}$ denotes the mean, ζ_j are uncorrelated standard normal (zero mean and unit variance) random variables. The mean $\overline{H(x)}$ and the eigensolutions μ_j and ψ_j are deterministic. The randomness of the field is only included in ζ_j There are n eigensolutions, but in general it is sufficient to consider only the $r < n$ eigenfunctions with the largest eigenvalues, which give a good approximation of the random field.

The midpoint method has the advantage that the covariance matrix $C_{i,j}$ is convenient to calculate and can be used for non-Gaussian distributions. However, a disadvantage is that the discretisation size needs to be relatively small in relation to the scale of fluctuation to consider the random field within an element to be constant. For example Sudret and Der Kiureghian [30] showed that the error estimator for a given element of a random field is less than 5% if the ratio Δ/d < 0.2, using a Gaussian correlation function.

If the distribution type is non-Gaussian first a transformation into the Gaussian space is performed using the Nataf model [32]. In this paper regression formulas developed by Li and Der Kiureghian [33] will be used for the mapping of the correlation coefficients to Gaussian space.

2. Description of the Methodology

The extent of distress is an appropriate criterion in decision making about repair and maintenance strategy in practice. A spatial time-dependent reliability analysis is developed for a hypothetical RC bridge deck with 12 m length and 10 m width exposed to de-icing salts. The analysis considers corrosion initiation and then propagation of corrosion-induced upper cover cracking until a crack width of 0.3 mm is reached, which is the prescribed limit crack width in Duracrete [34] and ACI [35]. A 2D homogenous Gaussian random field is applied to the RC bridge deck considering the spatial variability concrete compressive strength. This means that related properties of concrete, e.g. chloride diffusion coefficient, concrete tensile strength and concrete effective modulus of elasticity, water-cement ratio and corrosion density rate follow the variation of the same field of compressive strength of concrete. Furthermore concrete cover depth and surface chloride concentration are represented with random fields to account for spatial variations of these parameters.

In this study the whole deck area is subdivided to NT equal elements and then it can be considered as a series system of elements in serviceability reliability analysis:

$$G_{sys}(X,t) = min\,(G_j(X,t)-w_{lim}) \qquad for\ \ j=1\ to\ N_T \tag{26}$$

In this formulation failure means that at least in one element the crack width is greater than $w_{lim}=0.3$ mm. The vector X represents random variables of every element and t is the time (age) of the concrete deck.

According to Stewart and Mullard [12] the scale of fluctuation of these random fields is approximately 3.5 m ($d_x = d_y$ = 2.0 m). In Table (1) full statistical description of random fields, random variables and deterministic variables are illustrated.

The 2D random field is discretized into square elements of size Δ=0.5 m, resulting in 480 elements. The developed code in MATLAB carries out Monte Carlo Simulation in the space of independent standard normal variables of discretized random fields. Fig. 2 represents a random realization of spatial variability of concrete cover depth as a Normal random field.

3. Results

It is assumed that the service life of the bridge element includes the initiation period and the propagation period until

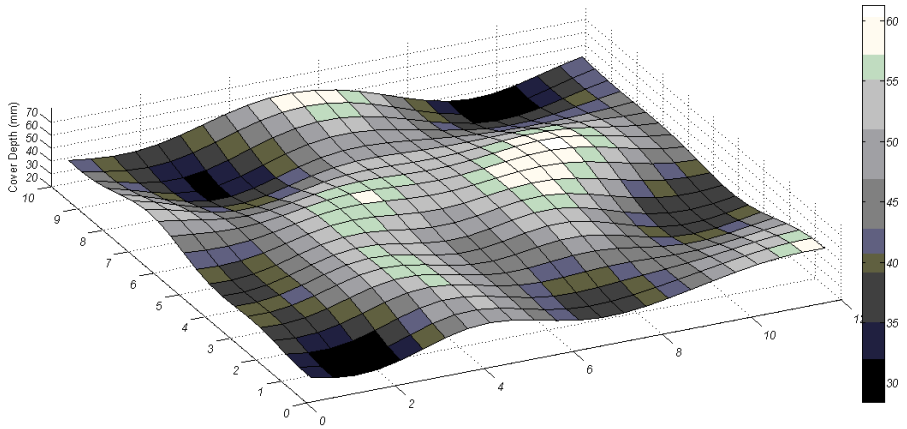

Fig. 2. Random realization of spatial variation of concrete cover depth

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inception of crack in cover and then increase of crack with to reach a limit state. In this study Duracrete prescribed limit for crack width equal to 0.3 mm is used for representation of severe cracking of concrete. For every discretized RC deck element the Monte Carlo simulation analysis is performed on annual time steps to compute the statistical variation of crack width with time based on Eq. 14. The results of this analysis for one element is represented in Fig. 3 where the simulated mean crack width increase with time for a conventional concrete with mean compressive strength equal to 35 MPa and 50 mm cover depth is depicted. Upon the results of this analysis for a RC bridge deck subject to deicing salts if depassivation occurs and corrosion triggers the first hairline cracks will be formed just a couple of years after corrosion initiation and then the limit crack width of 0.3 mm is predictable by about 8 years. In Fig. 4 three simulated random variables and fitted lognormal probability density functions corresponding to time of corrosion initiation, crack initiation and limit crack width are illustrated. Again this illustration apparently emphasizes the substantial role of corrosion initiation period in the service life design of RC bridge decks subject to chloride ingress.

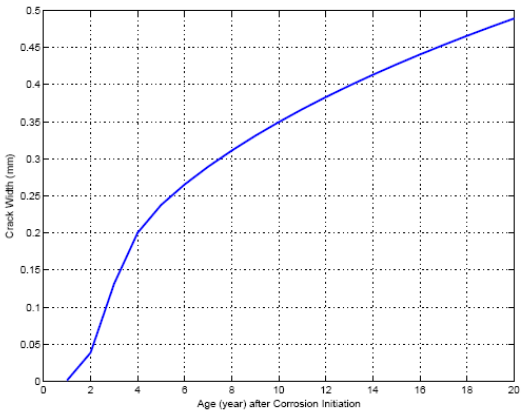

Fig. 3. Corrosion-induced crack width increase with time after corrosion initiation

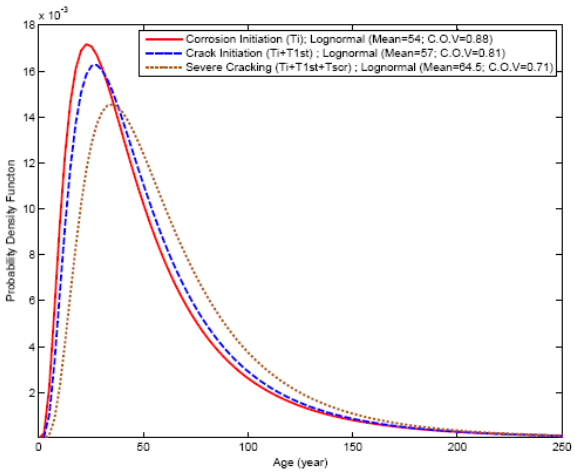

Fig. 4. Simulation of time to corrosion initiation and cracking for case 1

3.1 Spatial variation

In the hypothetical example of this study the whole 120 m2 area of bridge deck is subdivided to 480 equal 0.5 m × 0.5 m square elements for the purpose of considering spatial variation of some governing parameters of corrosion, e.g. concrete properties, cover depth and surface chloride concentration. For every discretized element the crack width was calculated 1000 times to simulate the statistical description of extent and likelihood of failure. According to Eq. 26 reliability analysis in the series system of these elements will result in the probability of failure in at least one element. This sophisticated analysis will calculate the probability of inception of deck failure but with due consideration to spatial variation of corrosion phenomenon parameters. As per Fig. 5 the probability of severe cracking considering the spatial variation is extremely higher than the case of considering the whole deck as one element. In the other words ignoring spatial variability will result to substantial underestimation of the probability of failure. For example in year 20 the simple reliability analysis predicts that there is just less than 10 percent probability of failure, while considering spatial variation will result in more than 90 percent of failure probability.

The result of this Monte Carlo analysis is depicted again in Fig .6 where a random realization of the crack width increase in the elements of the bridge deck system is represented. It is obvious that in every cycle of Monte Carlo simulation another realization may occur. The proportion of the failed area to whole deck area in every cycle of Monte Carlo simulation results in a random variable X which is representing extent of damage. Fig. 7 shows the mean, the upper and lower band, e.g. 95 and 5 percentiles respectively of this random variable during service life of the bridge deck. An interesting result of this analysis is that variation of the simulated extent of damage is not very high and can be best described with a fitted Normal probability density function. This is presented in Fig. 8 where the simulated histograms and fitted density function of the extent of failed area of bridge deck are illustrated in 30, 50, 70 and 100 years of its lifetime.

As another important result of time-dependent spatial

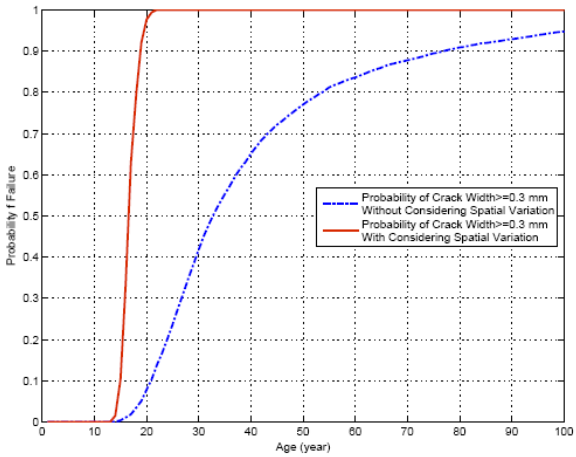

Fig. 5. Effect of considering spatial variability on probability of severe cracking (w_{lim}=*0.3* mm) of concrete

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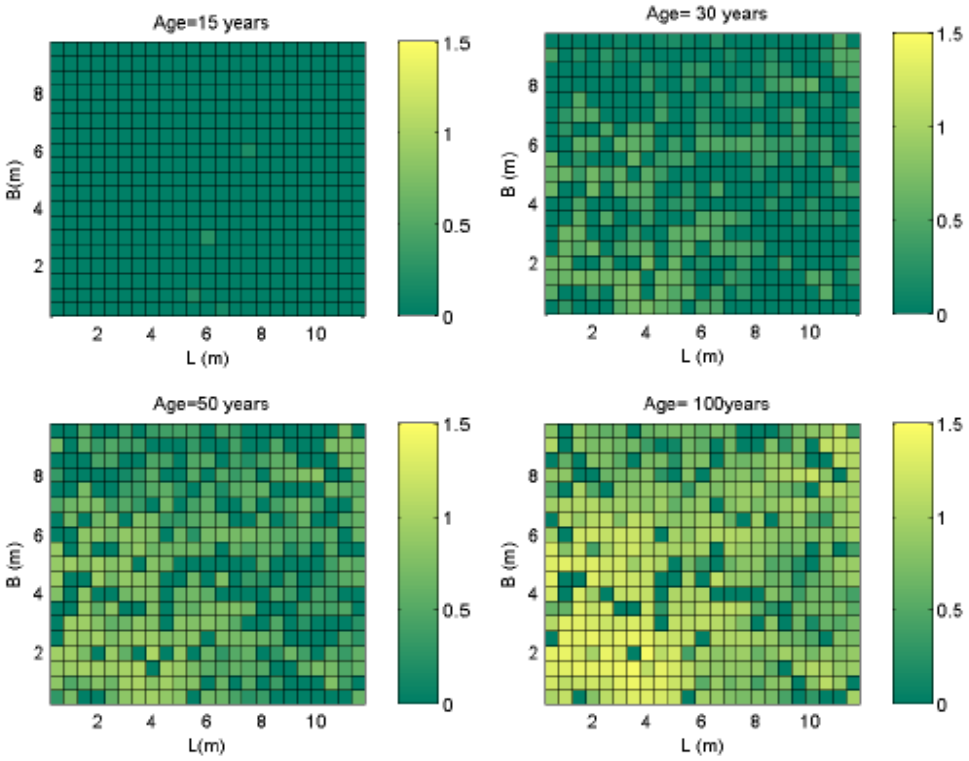

Fig. 6. Random realization of the extent of severe cracking during life cycle of the bridge deck

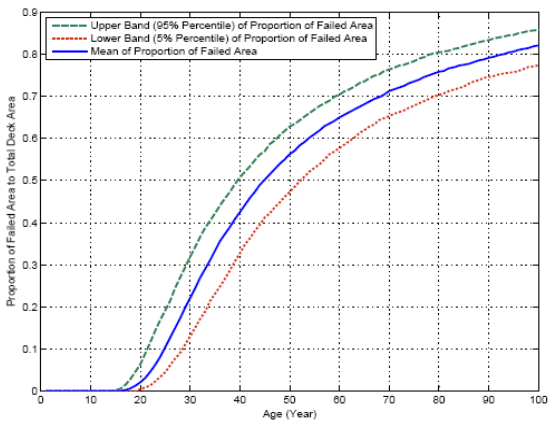

Fig. 7. Mean, upper band and lower band of the extent of corrosion-induced cracks(w_{lim}=*0.3* mm)

reliability analysis is that the probability of exceeding the extent of failure from an acceptable threshold can be calculated. In Fig. 9 the results of such an analysis is represented for various acceptable limits of X. In risk-based maintenance management of bridges this can be a rational criterion in selecting optimum life cycle repair and maintenance strategies. For example if X=30% is the acceptable extent of failed area while there will be negligible risk prior to age 20 years it will increase dramatically and in age 30 years this probability will be about 95 percent. Such an instantaneous increase in calculated probability for every acceptable threshold X can be described in reference to low coefficient of variation of extent of failure in every time as presented in Fig. 8 .

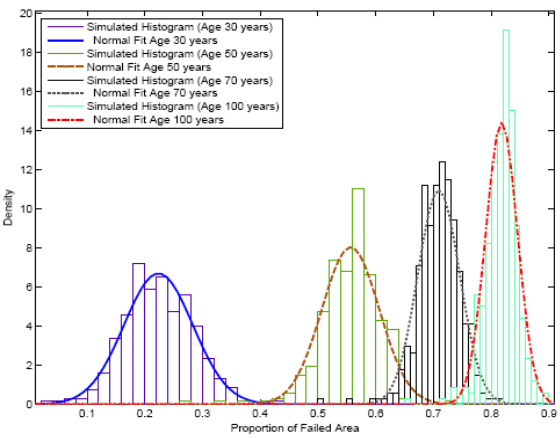

Fig. 8. Simulated histograms of proportion of failed area during lifetime of the bridge deck

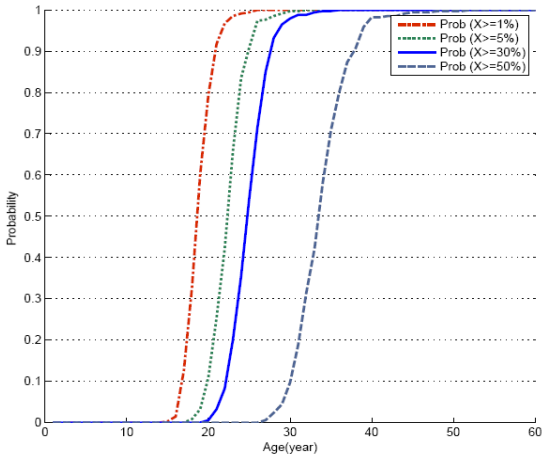

Fig. 9. Simulation of time to corrosion initiation and cracking for case 1

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3.2. Sensitivity analysis

Among the governing parameters of the corrosion process in ordinary reinforced concrete bridge decks the cover depth and concrete class, e.g. compressive strength can be controlled and selected in design and construction stage. In fact as presented earlier in this paper in a concrete mix design the water-cement ratio, permeability, diffusion coefficient, resistivity and corrosion current intensity can be represented as functions of concrete compressive strength. In this section, as per Table (2) five different durability design specifications are considered to evaluate the sensitivity of the results to these design parameters.

The results of time-dependent reliability analysis are presented as the mean time to corrosion initiation, crack initiation and propagation in Table 2, for all durability design specifications. It can be concluded that cover depth has a paramount effect on service life of RC decks subject to chloride ingress. For example while the mean time to corrosion initiation for the reference case, with 50 mm cover depth and 35 MPa compressive strength, is 56 years, a 75 mm cover depth will result in a 132 years of predicted mean time of corrosion initiation while using a higher class concrete with 42 MPa compressive strength will yield to a 109 years of service life if corrosion initiation is selected as the criterion. In the other hand cover decrease to 25 mm in case 3 will have a very adverse effect in service life as it diminishes the predicted initiation time of corrosion by 76 percent to 13 years. This is confirming the importance of providing enough concrete cover in chloride laden environments, the requirement which most codes emphasize on.

Furthermore as per Table 2 for all design specifications in service life cycle of a RC deck subject to chloride ingress the time since construction to corrosion initiation is the greatest part of the whole service life and just few years after corrosion initiation one can expect crack initiation and afterwards increase to limit crack width of 0.3 mm. This is shown again for the reference case in Fig 4 where fitted lognormal distribution to simulated initiation and propagation times are almost coinciding.

The sensitivity analysis of the extent and likelihood of severe cracking to cover depth and concrete strength is conducted with time-dependent spatial Monte Carlo analysis. Again the results confirm the importance of providing enough cover depth. As per Fig. 10 the mean predicted extent of cracked area for reference case deviates much more from reference case for variation of cover depth in contrast to compressive strength. For example if the extent of 30 percent is determined as the limit for applying of repair action, a poor cover depth of 25 mm will result in just less than 10 years service life while a 75 mm cover depth with the same concrete strength extends the service life to 80 years in contrast to reference case with predicted service life equal to 32 years. In the same analysis for case 4 and 5 with a mean compressive strength equal to 42 MPa and 28 MPa respectively, the predicted service life is about 20 and 57 years.

This is an interesting result since it is obvious that 25 mm increase in cover depth is more economical than 7 MPa

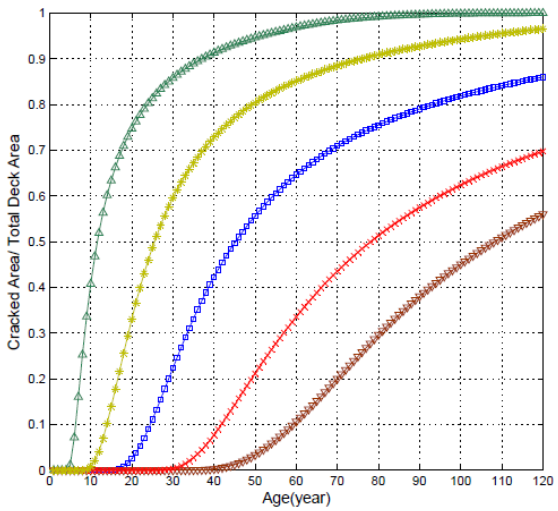

Fig. 10. Sensitivity of mean extent of severely cracked area to cover depth and compressive strength

Table 2. Statistical description of variables

Cases	Mean Value	T_i :Initiation Time	T_{1st} :Crack Initiation	T_{scr}: Crack Width limit w_{lim}=0.3 mm
Case 1 : (Ref. case)	f'_c= 35 MPa Cover = 50 mm	56	2.79	7.14
Case 2 : (Cover Increase)	f'_c = 35 MPa Cover = 75 mm	132 (+135%)	4.84 (+73%)	17.6 (+146%)
Case 3 : (Cover Decrease)	f'_c = 35 MPa Cover =25 mm	13.2 (-76%)	1.47 (-0.47%)	1.18 (-83%)
Case 4: (Strength Increase)	f'_c = 42 MPa Cover = 50 mm	109 (+95%)	3.0 (+7%)	9.56 (+34%)
Case 5: (Strength Decrease)	f'_c = 28 MPa Cover = 50 mm	31.3 (-44%)	2.75 (-1%)	4 (-43%)

Note1: The reference time for every phase is the occurrence of its preceding phase

Note 2: The reported values are the mean of Monte Carlo simulation results with N=1000

Note 3: The values inside the () are the percent of increase or decrease in respect to reference Case 1 corresponding values

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increase in compressive strength for a reference concrete with 50 mm cover depth and 35 MPa compressive strength. On the other hand not only designers shall be aware of adverse results of poor cover depth but also strict quality control and good workmanship and practice is required in construction stage for assurance of providing enough cover for RC structures prone to chloride ingress.

4. Conclusions

This paper presents the spatial time-dependent reliability analysis of RC bridge decks subject to chloride ingress. The spatial variability of concrete cover depth, compressive strength and surface chloride concentration was considered whilst other material and deterioration parameters were treated as dependent spatial or random variables. The results of the analysis are presented as the extent and likelihood of the corrosion damage during life cycle of a hypothetical bridge deck. The sensitivity of the results to cover depth and compressive strength of concrete as the main controlling parameters in design stage is presented. In summery the following conclusion can be made:

• Considering spatial variability of governing parameters of reinforcement corrosion can lead to substantial decrease in predicted service life of RC structures. In the other words ignoring spatial variability in reliability based maintenance management of bridges will lead to underestimating failure probability of cracking limit state.

• The main part of service life of RC structures subject to chloride ingress, e.g. bridge decks, is within initiation phase of corrosion. In the other words just a few years after corrosion initiation, occurrence of first symptoms of cracking in concrete cover is inevitable and then propagation of the severity and extent of cracks will result in spalling of concrete. For this reason it is suggested that simple diagnostic NDE tests, like half cell tests, be accomplished on RC bridge decks during routine inspections to prevent costly repair and maintenance of chloride contaminated RC structures.

• It is of paramount importance to provide enough cover depth as the major economical strategy in extending service life of RC bridge decks. Negligence in design or construction stage will result in costly repair and maintenance interventions and or shortened service life of the bridge deck.

• In selecting maintenance strategies if visual inspection and cracking is determined as the criterion of service life, the limit extent of cracking shall be determined carefully based on optimum life cycle costs. This can be formulated as a multi objective decision making based on reliability, extent of damage and life cycle costs to determine the optimum maintenance strategy for one bridge or a couple of bridges in a network.

• The analytical model used for crack width calculation requires iteratively solving simultaneous equations in every time step. In the Monte Carlo simulation method these equations shall be solved many times for every discretized element. Consequently spatial time-dependent reliability analysis is very time consuming. Utilizing artificial intelligence as a surrogate to Monte Carlo simulation will be promising in efficiency of the method.

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