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Characterization of the correlation structure of residual CPT profiles in sand deposits

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Abstract

Among the different ways of in-situ soil investigation, cone penetration test data are selected to evaluate the spatial variability of geomaterials and the scale of fluctuations is chosen to evaluate the correlation structure of CPT data. In this regard six case studies in sandy materials from Australia, U.S.A. and Iraq are selected. Various techniques for the calculation of the scale of fluctuation of geotechnical parameters are suggested in literature e.g. VXP, SAI, AMF, BLM and VRF without any preference or privilege for any specific procedure. In order to isolate the stochastic portion of cone tip resistance, deterministic trend was first removed by regression analysis. This study suggests that quadratic trend removal is more suitable for selected CPT data soundings. The closeness of the estimated scale of fluctuation using different approaches is assessed too. Mean value of the scale of fluctuation by five established methods ranges between 0.44 to 1.52 meter for six different cases and the coefficient of variation for the scale of fluctuation calculated by these methods varies between 12 to 27 % showing that available established methods produce almost compatible and comparable results.

Keywords: Spatial variability, Cone tip resistance, Correlation structure, Scale of fluctuation.

1. Introduction

The geomechanical characterization of a soil deposit involves both deterministic and statistical approaches. The correct approach for this characterization consists of defining a trend, expressed by a regression analysis, and the residual variability of the geotechnical properties which is termed stochastic heterogeneity. However, Baecher [1] emphasizes that the distinction between trend and stochastic variation is not inherent to the soil but to the modeler. The evaluation of stochastic variations which can be seen as a convenient way to describe the variability of a soil property has attracted extreme attention of researchers for last three decades. This variation of soil properties can be precisely described by three parameters: mean, coefficient of variation and the scale of fluctuation.

Since the first advent of the correlation concept and the definition of the scale of fluctuation by Vanmarcke [2], various techniques have been developed by subsequent researchers for the identification of the correlation structure of geotechnical data. Vanmarcke's expeditive method [2], direct integration of sample autocorrelation function [3], autocorrelation model fitting [4], variance reduction function [5, 6, 7] and Bartlett's limit methods [8] are suggested methods in literature; However there is no bias to any specific method. Current study focuses on the calculation of the scale of fluctuation in order to identify the correlation structure of CPT data. Cone penetration profiles performed in sandy materials adopted to evaluate the correlation structure of cone tip resistance of sandy materials and to compare different methods. In this way some inaccuracies raised by trend removal techniques are pointed out and appropriate trend model for sandy materials are recommended.

2. CPT data sets

Among the various ways of subsurface investigations, Cone Penetration Tests are an especially useful and inexpensive ways of evaluating soil profiles. Retrieving data continuously with depth (with electronic ones) or at very close intervals (with mechanical ones), the CPT is able to detect fine changes in the stratigraphy and spatial variability of soil properties. Six sets of CPT soundings are selected from different sites in U.S.A., Iraq and Australia indicated by case number 1 to 6 in

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Table 1; also summarized the original name, site location, number of data points for each sounding, N_d and sampling interval in each case. It is evident that all the experiments are carried out in sandy materials with 20 cm data intervals. Figure Figure 1 shows the soil description for investigated sites. The soil classification is conducted based on Eslami-Fellenius classification chart [9] which is shown in Figure 2.

There are several technical problems embedded in the evaluation of soil vertical variability using CPT data, but cone bearing, q_c profiles are usually preferred for processing. The sleeve friction, f_s is also measured during the tests, but this is generally considered to be unreliable due to technical drawbacks, e.g., sleeve wear [16]. At what follows, the cone tip resistance from CPT data was chosen to evaluate stochastic variation of soil properties and the correlation structure identification in other words.

Table 1. Summary of CPT soundings

Case number	Original case name	Site location	Sampling interval (cm)	N_d*	Reference
Cn1	BGHD1	Iraq	20	101	Altaee et al. 1992a, 1992b [10,11]
Cn2	BGHD2	Iraq	20	105	Altaee et al. 1992a, 1992b [10,11]
Cn3	L&D314	IL., U.S.A.	20	96	Briaud et al. 1989 [12]
Cn4	A&N2	Australia	20	86	Haustorfer and Plesiotis, 1989 [13]
Cn5	N&SB144	FL., U.S.A.	20	124	Nottingham, 1975 [14]
Cn6	SEATW	WA., U.S.A.	20	91	Horviz et al., 1981 [15]

* Number of data in depth

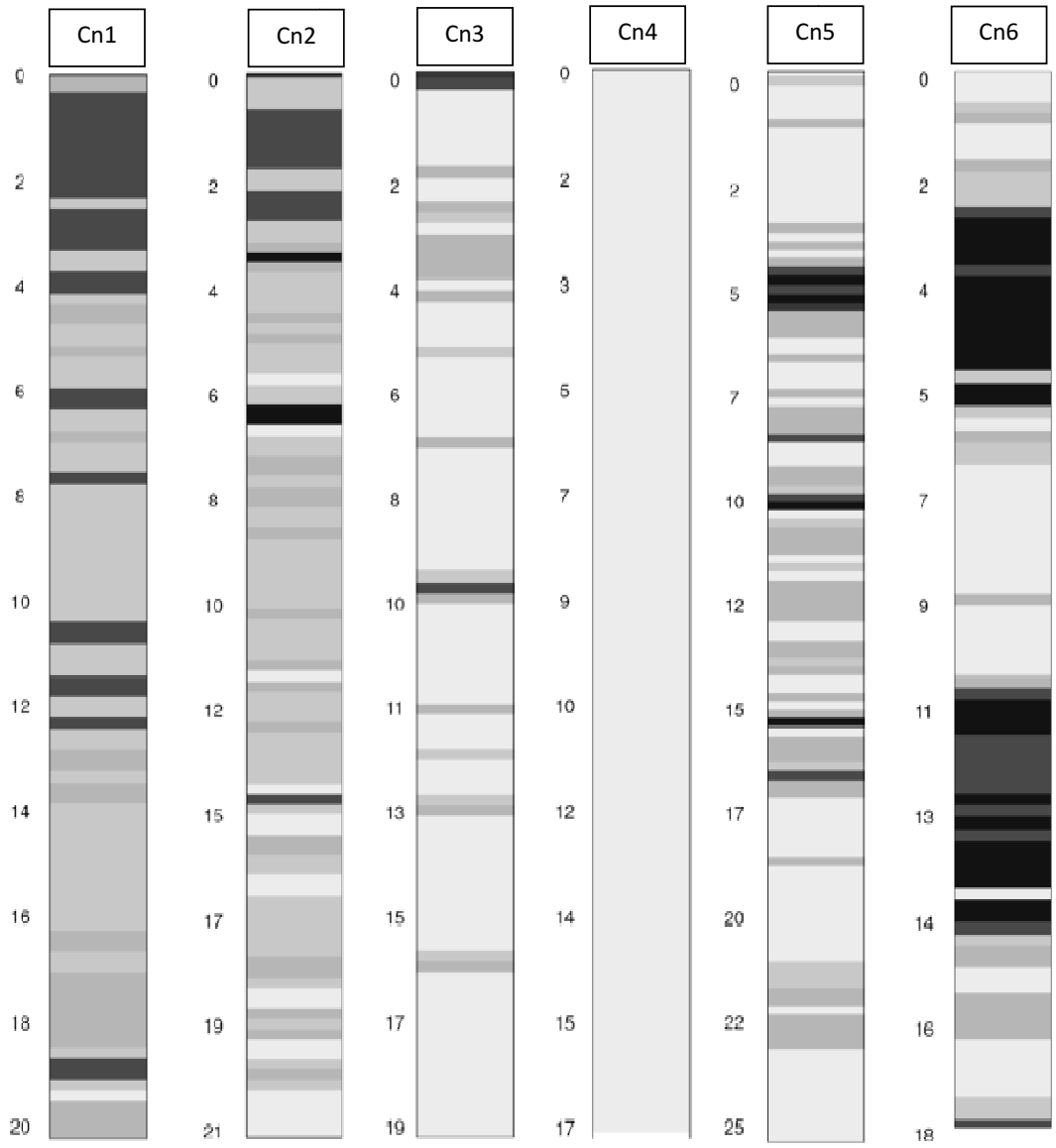

Fig. 1. Soil profile based on CPT data; Cn1 to Cn6

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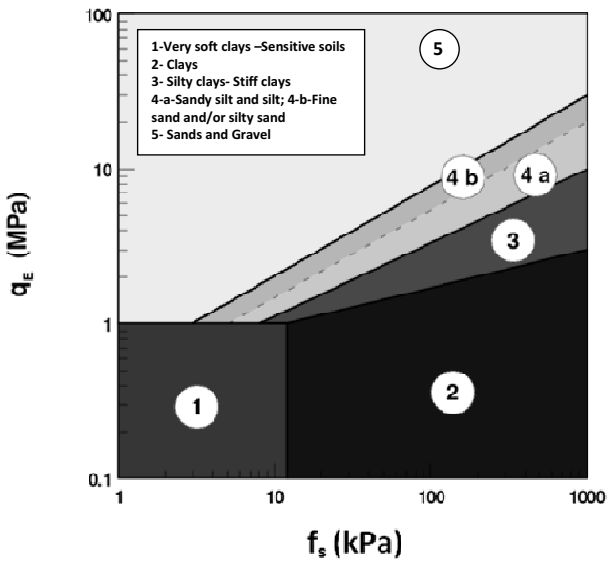

Fig. 2. Eslami-Fellenius soil profiling chart [9]

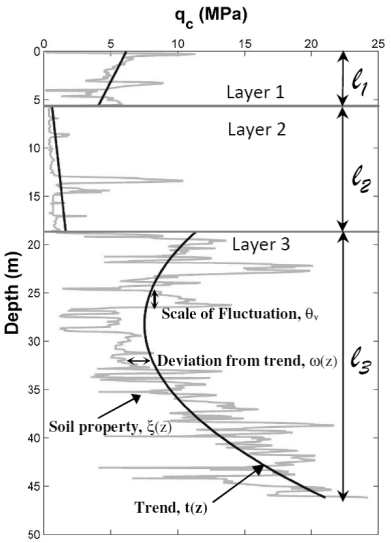

Fig. 3. Inherent and lithological soil variability (data from Yen et al., 1989 [21])

3. Spatial Soil Variability

The unique nature of soil and rock materials which are often highly variable, even within a short distance, makes geotechnical engineering much more an art than the other disciplines within civil engineering. The variation of properties from one location to another within a soil or rock mass is termed spatial variability. During the last decades, several models have been proposed to make explicit the affecting factors on overall variability and magnitude of each source of uncertainty. These models identify that the primary sources of geotechnical variability are inherent soil variability, measurement error and transformation uncertainty [17, 18, 19, 20]. Inherent soil variability is due to complex process of geo-materials formation such as sedimentation, weathering, stress history and time; Measurement error resulted from equipment, procedural-operator and field effects while transformation errors occur when fields or laboratory measurements are transformed to design soil properties with empirical or other correlation models. Although the inherent variability is common in soil layer which is homogeneous, in terms of composition, in majority of cases in geotechnical engineering, one will encounter with soil strata with different lithological origins. This type of variability called lithological heterogeneity results from the formation of soil layers from decomposition of different parental materials. So, along with the inherent variability in natural alluvial deposits, there is generally another source of variability manifested in the form of soft/stiff layers embedded in a stiffer/softer media or the inclusion of pockets of different lithology within a more uniform soil mass, but this is excluded in this study.

Inherent variability in geotechnical properties can be modeled by eq. (1) in which a depth dependent geotechnical property, ξ is decomposed into the deterministic component, t and the fluctuating component, ω that totally represent the inherent soil variability. Figure 3 shows schematically the inherent and lithological variabilities where different layers are resulted from lithological heterogeneity.

$$\xi(z) = t(z) + \omega(z) \quad (1)$$

A rational means of quantifying inherent variability is to model $\omega(z)$ as a homogeneous (stationary) random function or field [3]. This function is considered to be statistically homogeneous if (i) the mean and variance of ω do not change with depth; and (ii) the correlation between the deviations at two different depths is a function only of their separation distance, rather than their absolute positions [19]. Fenton [5] asserted that data detrending is performed essentially to obtain a spatially independent fluctuating component, $\omega(z)$. This condition is desirable because the statistical procedures employed are based on the assumption that data samples consist of statistically independent and identically distributed observations.

4. Correlation Structure

The first step in evaluation of the correlation structure of CPT data lines in the estimation of the sample autocovariance function and the sample autocorrelation functions of the detrended data. Such functions may be estimated for stochastic processes which are not homogenous as defined earlier but at least weakly stationary. In CPT testing, values are read at discrete, possibly constant, spatial interval, Δz. Hence, the autocovariance and autocorrelation functions are also estimated at a discrete number of points over the spatial interval L_d.

The sample autocovariance function may be obtained as an unbiased estimate of the autocovariance function $\hat{C}^*(\tau_j)$, through eq. (2), where N_d is the total number of data points and bias of an estimator is the difference between the estimator's expected value and the true value of the parameter being estimated while an estimator or decision rule with zero bias is called unbiased.

$$\hat{C}^*(\tau_j) = \frac{1}{N_d - |j|} \sum_{i=1}^{N_d - |j|} (\omega_i - \mu_\omega)(\omega_{i+|j|} - \mu_\omega) \quad for \quad |j| = 0,1,...,N_d - 1 \quad (2)$$

As $\hat{C}^*(\tau_j)$ is an even function, the unbiased estimate should

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also be an even function; thus, the absolute value of the lags is considered in the formula. $\hat{C}^*(\tau_j)$, however, it is not exactly unbiased, but, as shown by Priestley [22], is only asymptotically unbiased, as the effect of estimating the mean by the sample mean must be taken into account.

$$\hat{\mu}_\omega = \frac{1}{N_d}\sum_{i=1}^{N_d}\omega_i \tag{3}$$

Parzen [23] suggested that a biased estimate of the autocovariance function may be used instead:

$$\hat{C}(\tau_j) = \frac{1}{N_d}\sum_{i=1}^{N_d-|j|}(\omega_i - \mu_\omega)(\omega_{i+|j|} - \mu_\omega) \quad for \quad |j| = 0,1,...,N_d - 1 \tag{4}$$

While several authors have used unbiased autocorrelation functions [24, 25, 26], the use of a biased estimator, $\hat{C}^*(\tau_j)$ in random field modeling of stochastic processes in the context of geotechnical engineering is also warranted [5, 22] for the following reasons: a) the expected error variance is slightly smaller than that for the unbiased case; and b) the biased estimator, when estimating covariances, leads to a tractable nonnegative covariance matrix.

The sample covariance $\hat{C}^*(\tau_j)$ is estimated here for separation distances $\tau_j = j\Delta z$ corresponding to j=1, 2, …, N_d/4, as suggested by Box & Jenkins [27], where Δz is the sampling interval.

The biased estimate of the autocorrelation function, or sample autocorrelation function, is given by:

$$\hat{\rho}(\tau_j) = \frac{\hat{C}(\tau_j)}{\hat{C}(0)} \tag{5}$$

It may easily be inferred from eq. (4) that $\hat{C}(\tau_j)$ (covariance at lag zero) is equal to the sample variance:

$$\hat{\sigma}_\omega^2 = \frac{1}{N_d}\sum_{i=1}^{N_d}(\omega_i - \hat{\mu}_\omega)^2 \tag{6}$$

The autocorrelation function represents the correlation coefficient between pairs of transformations of a stochastic process $\Omega(\tau)$, ω_i and ω_{i+j} with source values z_i and z_{i+j} separated by an interval of length $\tau_i = z_{i+j} - z_i$ in the index set. The autocorrelation function may thus be interpreted as a measure of the similarity between a realization of the stochastic process, $\Omega(z)$, and the same realization shifted by τ_j units. As the residuals of q_c are zero-mean data vectors, eq. (5) can be rewritten as:

$$\hat{\rho}(\tau_j) = \frac{\sum_{i=1}^{N_d-j}\omega_i\omega_{i+j}}{\sum_{i=1}^{N_d-j}\omega_i^2} \tag{7}$$

In the geotechnical literature various kinds of autocorrelation models have been employed describing an autocorrelation function [4, 8, 17, 26, 28, 29]. Some of these models are shown in Table 2. According to Spry et al. [4], none of these models are preferable on the basis of physical motivation. For geotechnical data, it is not always easy to identify the ''correct'' form of the autocorrelation function. The reason is that the empirical autocorrelation function might be: (i) inaccurate because the available sample size is typically quite small and (ii) incomplete as the estimated value from eq. (3) is only accurate up to a maximum lag of less than 1/4 of the total sample length [30]. However, it should be taken into account that for most applications, the exact form of the ACF may not be important [31]. The other three theoretical models shown in Table 2 are less commonly used compared to the single exponential model [20].

Table 2. Autocorrelation model and related definitions for the scale of fluctuation

Autocorrelation model	Equation	Scale of fluctuation
Single exponential	$R(\tau)=\exp(-\lambda\|\tau\|)$	$\theta_v=\frac{2}{\lambda}$
Cosine exponential	$R(\tau)=\exp(-\lambda\|\tau\|)\cos(\lambda\tau)$	$\theta_v=\frac{1}{\lambda}$
Second order Markov	$R(\tau)=(1+\lambda\|\tau\|)\exp(-\lambda\|\tau\|)$	$\theta_v=\frac{4}{\lambda}$
Squared exponential	$R(\tau)=\exp[-(\lambda\tau)^2]$	$\theta_v=\frac{\sqrt{\pi}}{\lambda}$

5. Scale of fluctuation

The scale of fluctuation (θ) is an indicator of the extension of the correlation structure. Within separation distances smaller than the scale of fluctuation, the deviations from trend or the residual components show relatively strong correlation, but as the separation distance exceeds this value, little correlation between the fluctuations in measurements is expected. Different methods of calculation for the Scale of fluctuation of geotechnical properties have been emerged since its first introduction by Vanamrcke in 1977.

At what follows, different methods and approaches are first introduced and then applied to some experimental real CPT data so as to make a comparison between them and draw conclusions about both the range of the scale of fluctuation of the residuals of q_c data and also the accuracy of different methods.

5.1. Vanmarcke's expeditive method (VXP)

Vanmarcke in 1977 [2] introduced a simple definition of the scale of fluctuation of stochastic processes for the first time and proposed a simple method for approximating the scale of fluctuation. In his opinion, the average distance between the consecutive intersection of the overall profile and the trend line of a given profile can be used to estimate this parameter. Eq. (8) can be used to calculate the scale of fluctuation based on the VXP method and Figure 4 schematically shows how to calculate the average distance for use in this method. Based on this

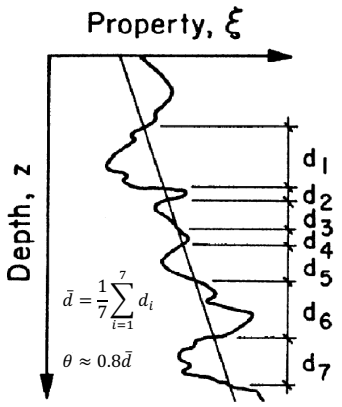

Fig. 4. Mean-crossing approximation for the estimation of the scale of fluctuation [10]

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method scale of fluctuation of cone tip resistance data set in Cn1 is equal with 0.54 meter for quadratic trend removed data.

$$\theta \approx \sqrt{\frac{2}{\pi}}\,\bar{d} = 0.8\bar{d} \tag{8}$$

5.2. Direct integration of sample autocorrelation function (SAI)

Following his preceding researches, in 1983, Vanmarcke [3] proposed a new technique for calculation of the scale of fluctuation of random processes. In this new procedure, the sample autocorrelation function is first plotted versus lag distance, and the area under this graph has been suggested as the scale of fluctuation. Eq. (9) shows how to calculate the scale of fluctuation from SAI method. In this relation ρ is the estimated sample autocorrelation function and τ is the separation distance between two measurements.

$$\theta = \int_{-\infty}^{+\infty} \rho d\tau = 2\int_{0}^{+\infty} \rho d\tau \tag{9}$$

This method is a non-parametric approach, although it assumes that the scale of fluctuation is finite and that the correlation function is monotonic. As the scale of fluctuation is not well defined for oscillatory correlation functions, the area is usually taken to be the area up to which the sample autocorrelation first becomes negative [32]. After substituting the detrended data, ω into eq. (7), the sample ACF is obtained and the area under this function is an indicator of the scale of fluctuation of CPT residuals. The calculated scale of fluctuation for the CPT data set in Cn1 is 0.49 m for quadratic trend removed data. Figure 5 provides a schematic representation of the SAI method.

5.3. Autocorrelation model fitting (AMF)

Curve fitting or regression technique attempt to find functions that best describe the relationship among variables, namely ACF and the lag distance, τ in this case. In effect, they attempt to build mathematical models of the correlation data set. After fitting the autocorrelation data with models introduced in Table 2, goodness of fit can be evaluated for different models so as to find the best one. However, as is common in statistical literature, the term goodness of fit is used in several senses, least square fitting, minimum uncertainty and the ability to explain a high proportion of the variability in the data are among all. A simple model that is common and easy to interpret might outweigh to other available models.

Single exponential model as a widely used model was selected in this study and the least squares error regression analysis was employed by applying linear and quadratic trend removal schemes as shown in Figure 6. For Cn1 CPT data set the scale of fluctuation was calculated 0.5 and 0.46 m for linear and quadratic trend removal respectively, while in both cases sample autocorrelation function inherits 0.8 m zero cut off values.

5.4. Variance reduction function (VRF)

The variance function introduced in eq. (10) measures the reduction in the variance of the moving average of a random process with the increase of the number of sequential random numbers included in the moving average, was introduced by Vanmarcke [3]. The variance reduction function (VRF) may be used to estimate the scale of fluctuation of the random process, as the rate at which the function decreases with increasing averaging size may be conceptually related to the spatial correlation structure as proved by Fenton [5] for 1D continuous case.

$$\Gamma_n{}^2(\tau) = \frac{\sigma_n{}^2}{\sigma_r{}^2} \tag{10}$$

Several studies [6, 7, 33, 34] have employed the VRF procedure to estimate the scale of fluctuation of cone bearing values. Such procedure is based on the hypothesis that, at large separation distances, the product of the variance reduction function and the separation distance approaches the scale of fluctuation:

$$\Gamma_n(\tau).\tau = \theta \tag{11}$$

A practical variant of the Vanmarcke's procedure, suggested by Wickremesinghe and Campanella [6], is to take the scale of fluctuation at the peak of a function, named by Cafaro and

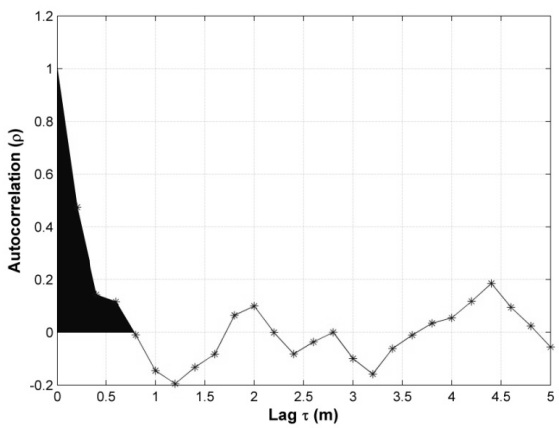

Fig. 5. Application of the SAI technique for the estimation of the scale of fluctuation for Cn1 CPT data set

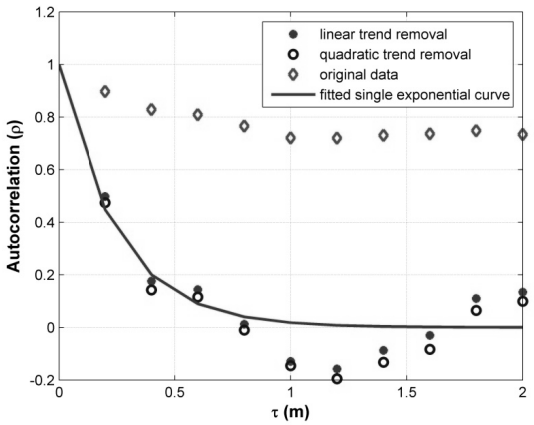

Fig. 6. Scale of fluctuation of Cn1 cone tip resistance by AMF method

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Cherubini [7] as ''fluctuation function'' which reaches a maximum with increasing the lag distance, which is the spacing size in other words. The fluctuation function is defined as follows:

Fluctuation function=(variance function)×(lag distance) (12)

As defined earlier the variance function is the ratio of the variance of moving average series of degree n to the variance of the original data; if the sample spacing is d, the degree n will be equal to (τ/d)+1, where τ=the lag distance. The variance function basically describes the decay of the variance due to a process of spatial averaging. For CPT data the calculation concerns residuals due to the presence of trend.

Jones et al [35] recommended calculating the scale of fluctuation of a random process based on Wickremesinghe and Chamapnella's proposed stepwise method as follow:

1. Calculate the variance for the series of data; this is the reference variance, σ_r;
2. Smooth the series of data by applying a moving average window of length n and substitute the original data value with the new smoothed value, x_k^* (e.g., for a windowsize, n = 3, $x_k^* = (x_{k-1} + x_k + x_{k+1})/n$);
3. Calculate the variance for the smoothed data: this is the windowed variance, σ_n and will be lower than σ_r due to cancellation of fluctuations due to spatial averaging [6];
4. Normalize the windowed variance by the reference variance and multiply by the window length to obtain, $SOF=(\sigma_k /\sigma_r)\times n$;
5. Repeat Steps 2 – 4 incrementing the window until the smoothing window is greater than about half the length of the data series;
6. Plot out the SOF as a function of window length;
7. Observe the behavior of the curve and take the first peak value as an estimate of the correlation length or scale of fluctuation, θ.

Seven steps, recommended by Jones et al. are applied to the CPT residuals off Cn1 data set and as seen in Figure 6, peak value which is assumed to represent the scale of fluctuation of residuals is 0.36 m.

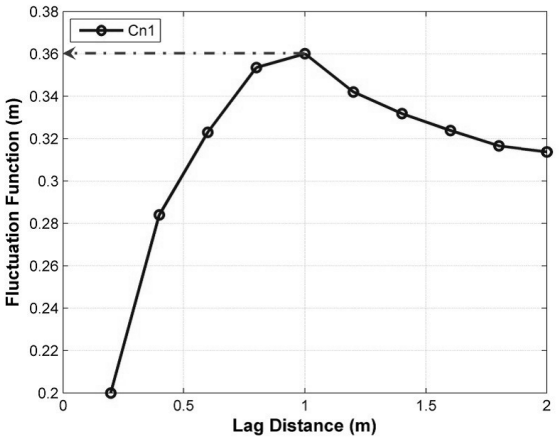

Fig. 7. Scale of fluctuation of Cn1 cone tip resistance by VRF procedure

5.5. Bartlett's limit method (BLM)

Jaksa [8] in a research on stiff clays observed that the scale of fluctuation of cone tip resistance can be estimated relatively simply by evaluating Bartlett's distance, which is the correlation distance defined by time series analysis. It is calculated by determining the lag at which the sample ACF first intersects Bartlett's approximation or limits, as given in eq. (12). Such method was proved computationally very efficient; however, it remained to be demonstrated whether such strong correlation would be observed in other soil types. Strength of this method of estimation of the scale of fluctuation, as observed by Jaksa [8], is the insensitivity to measurement interval [32]. For instance, variation of ACF with lag distance for data of Cn1 is drawn in Figure 8. This curve intersects Bartlett's limit in distance of 0.37 m which is assumed to be the scale of fluctuation of this data.

$$r_B = \pm \frac{1}{\sqrt{N_d}} \quad (13)$$

6. Trend removal models

Prior to evaluation of the correlation structure of cone tip resistance of CPT data, deterministic trend should be removed to access to the stationary data. Trends are low-order polynomial function no higher than a quadratic, obtained by regressing the cone tip resistance using ordinary least square method [25, 36, 37] which is used for estimating the unknown parameters in a linear regression model. This method minimizes the sum of squared vertical distances between the observed responses in the dataset, and the responses predicted by the linear approximation. The OLS method is generally used to solve a set of linear equations having more equations than unknown variables. Since there are more equations than variables, the solution will not be exactly correct for each equation; rather, the process minimizes the sum of the squares of the residual errors.

According to Li [38] an alternative technique based on generalized least squares (GLS) is more consistent with spatial variability analyses. In the GLS method, the least-squares method is often used to solve a set of non-linear

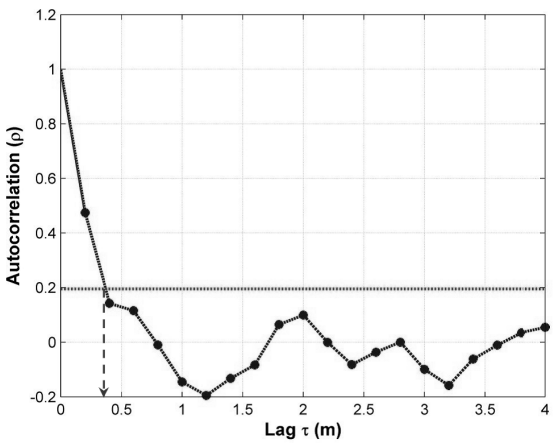

Fig. 8. Sample autocorrelation function and Bartlett's limit

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equations that have been linearized using a first-order Taylor-series expansion. Solving non-linear equations is an iterative process using Newton's method. The speed of convergence is dependent on the quality of an initial guess for the solution. The non-linear least-squares method is often referred to as a bundle adjustment since all of the values of an initial guess of the solution are modified together (adjusted in a bundle). This technique is also occasionally referred to as the Gauss-Newton method. However, Ripley [39] asserted that trends coming from OLS and GLS should only differ slightly.

As stated by Baecher [40] the selection of a particular trend function is a decision on how much of the spatial variability in the measurements is treated as a deterministic function of space and how much is treated statistically and modeled as random processes. Figure 9 shows cone tip resistance data for Cn1 and cn3 with their linear and quadratic regressed trends using OLS procedure. As seen in this figure, for Cn1, linear and quadratic trend are almost identical while for Cn3, two types of regressed trends are very different. The quadratic deterministic trend may be due to suction and capillary effect for fine sand upper crust layer in which strength and stiffness of this crust is higher than that of material immediately below it. Notwithstanding the fact that there is no unique trend in different soils, the deterministic variation of q_c profiles with depth in natural alluviums is assumed by authors to bear a quadratic trend which will render family fairly similar results in case linear trend prevails.

7. Comparison of methods

Tables 3 and 4 show the estimated values for the scale of fluctuation of CPT residual components using the VXP, SAI, AMF, BLM and VRF methods while both linear and quadratic trend models have been considered to acquire residuals off CPT data. First observation from these tables is that trend removal model has a significant impact on the estimation of the scale of fluctuation of qc profiles. As plotted in Figure 10, the relative difference in estimated scale of fluctuation between linear and quadratic trend removal models varies between 4 and 101%. The least error belongs to Cn1 which is easily confirmed by looking into Figure 9(a), too. It is evident

Table 3. Comparison of the scales of fluctuation of cone tip resistance adopting linear trend removal

Case number	Scale of fluctuation, θ (m)					μ_θ (m)	COV_θ (%)
	VXP	SAI	AMF	VRF	BLM		
Cn1	0.54	0.53	0.5	0.39	0.36	0.46	18.27
Cn2	0.52	0.93	1.05	0.9	0.47	0.77	33.78
Cn3	1.58	1.49	1.47	1.17	1.14	1.37	14.65
Cn4	1.28	2.44	2.23	2.34	1.71	2	24.49
Cn5	1.11	1.7	1.71	1.46	0.94	1.38	25.25
Cn6	1.22	2.12	1.8	2.02	1.57	1.74	20.66

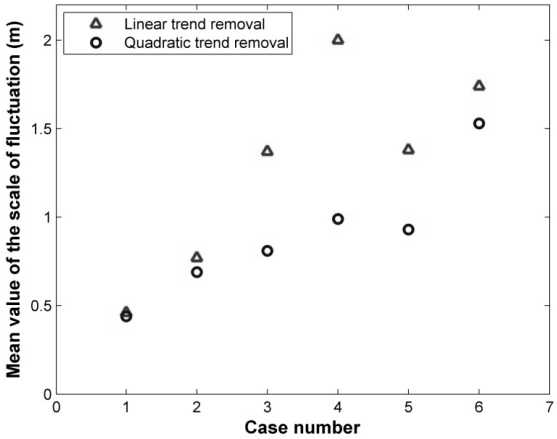

Fig. 10. Effect of trend removal model on mean values of the scale of fluctuation

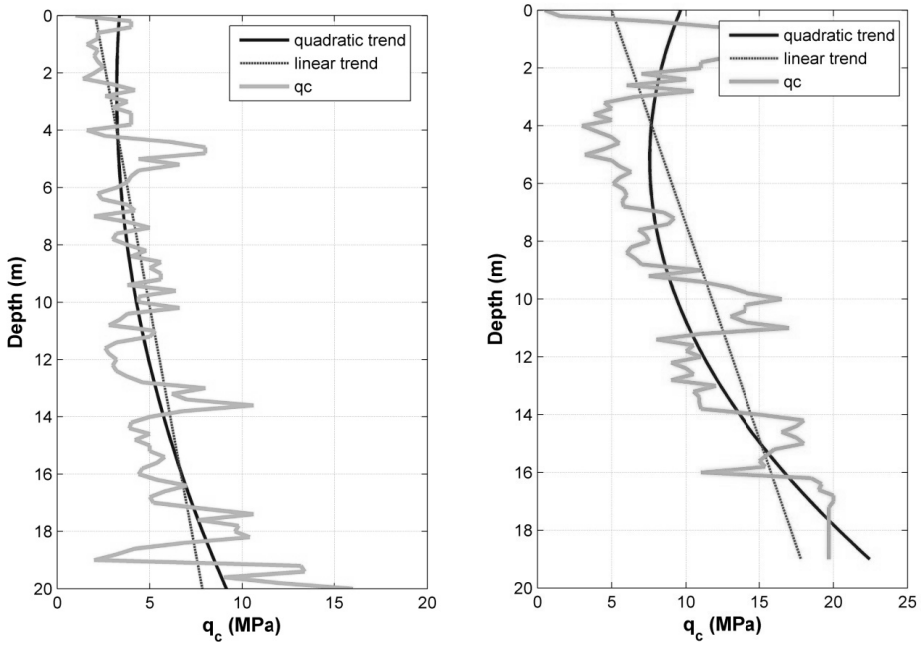

Fig. 9. Different trend models; a) Cn1; b) Cn3

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that quadratic trend removed in all cases renders lower values for the scale of fluctuation. This can be explained by the fact that higher order polynomials extract the deterministic behavior more efficiently and the residuals will show weaker correlation structure. Phoon et al. (2003) [26] examined the resulted scale of fluctuation trending off the CPT data after removing linear and quadratic trend and asserted that linear trend should be used if the difference is acceptably small; however in evaluated cases, quadratic trend removal model was chosen due to the spread range of error.

Another observation from these tables is that the mean estimated scales of fluctuation vary from 0.44 to 2m. This lies exactly in the previously published range for the scale of fluctuation of qc within sandy or clayey soils indicated to be 0.1 - 2.2 m by Phoon et al. [19]. Coefficient of variation of calculated scales of fluctuation from different procedures provides a qualitative indication of the closeness of various estimates. The results provided in Table 4 with quadratic trend removal show that the COV of the scales of fluctuation estimated from different procedures ranges between 12 and 27% for different cases. This indicates that employing different approaches will not lead to the same estimation for the scale of fluctuation of random process. However, if one excludes VXP and VRF procedures which are actually less common in geostatistics, the range for COV will dramatically decrease to 3-14%. This finding is visually confirmed by referring to Figure 11.

Table 4. Comparison of the scales of fluctuation of cone tip resistance adopting quadratic trend removal

Case number	Scale of fluctuation, θ (m)					μ_θ (m)	COV_θ (%)
	VXP	SAI	AMF	VRF	BLM		
Cn1	0.54	0.49	0.46	0.37	0.36	0.44	17.69
Cn2	0.55	0.8	0.8	0.84	0.47	0.69	24.13
Cn3	0.92	0.87	0.84	0.71	0.69	0.81	12.24
Cn4	0.72	1.1	1.08	1.22	0.85	0.99	20.31
Cn5	0.56	1.19	1.02	1.09	0.8	0.93	27.11
Cn6	1.07	1.85	1.56	1.81	1.36	1.53	21.27

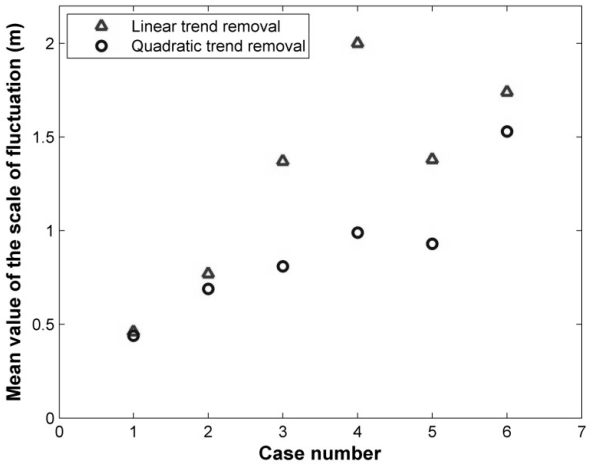

Fig. 11. Scale of fluctuation estimated by different procedures adopting quadratic trend removal

8. Conclusion

Cone penetration test is one of the most useful and versatile in-situ tests employed to determine the spatial variability of sandy soils. CPT data from six different sites, all representing sandy deposits were selected in order to study the stochastic properties of them. In this regard, five different established methods i.e. VXP, SAI, AMF, BLM and VRF were adopted to investigate the correlation structure of the CPT profiles. Scale of the fluctuation was calculated as the key parameter to evaluate the correlation behavior of CPT data using the above mentioned procedures.

Trend removal technique was shown to have a critical effect on the scale of fluctuation of the CPT residuals. It was emphasized that quadratic trend models will render more realistic correlation properties of CPT residuals due to the better regression of the deterministic component of the CPT profile.

The mean estimated scale of fluctuation acquired from different procedures proved to vary from 0.44-2 meter which lies within the available range in the literature.

Another observation of this study is regarding the comparison of different procedures to calculate the scale of fluctuation of CPT residuals. It was shown that the coefficient of variation of the estimated values ranges from 12 to 27% and the COV_θ decreases if VXP and VRF which are simple approximation of the scale of fluctuation are excluded. This means that other three methods namely SAI, AMF and BLM give more consistent results.

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