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An analytycal method for calculating the natural frequency of retaining walls

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Abstract

For calculating the natural frequency of structures such as buildings, chimneys, bridges and silos appropriate analytical formulas exist. However, in the case of retaining walls undergoing the soil pressure at one side, calculating the natural frequency is not a straightforward task and requires the effects of soil-structure interactions to be considered. By modeling the soil as series of linear springs, a new formulation is presented in this article, to calculate the natural frequency of retaining walls. This formula considers the vertical cross sectional width change, and hence, enables us to calculating the natural frequency of retaining walls with different types of backfill. The geometrical properties of the retaining walls and its bending rigidity together with the soil's modulus of elasticity and its Poisson's ratio are the most important parameters to calculate. A comparison of the results for retaining walls with constant cross section obtained from the suggested method with those of the software analyses was carried out and good agreement was detected. A second comparison of the results with those of other researchers revealed that the natural frequency of flexible retaining wall is an upper bound for natural frequency of rigid walls. The Selected shape function is also very close to the real shape mode.

Keywords: *Retaining wall, Analytical method, Soil-structure interaction, Shape function, Natural frequency.*

1. Introduction

For any structure, calculating the natural frequency or the natural period time is the essential part for studies on its dynamic behavior. Therefore, introducing an appropriate technique to determine the natural frequency of any structure is highly important. Since cantilever retaining walls are the structures that are widely used, studying their dynamic response against the earthquake loads appears to be crucial. Furthermore, the soil-structure interaction makes the natural frequency calculation for retaining walls very complicated. The natural frequency calculation of retaining walls with its backfill soil is usually carried out by applying the one-dimensional shear beam technique based on the height of the wall and the shear wave velocity in the soil.

Matsu and Ohara [1] ,Wood [2], Scott [3], and Wu [4] based on two parameters namely height of the backfill soil and the shear wave velocity, applied analytical method to parametrically analyze and predict the natural frequency variation range of retaining walls. In order to calculate the natural frequency of soil with linear elastic characteristics under horizontal vibrations of the ground, Matsu and Ohara [1] defined two limiting boundaries where they believed the real solution lied within. Using a numerical method, Wood [2] obtained a solution for the soil's frequency calculation embedded between two rigid walls, which, in fact was a boundary condition problem. He calculated the natural frequency of backfill for plane strain conditions assuming it was homogeneous and elastic.

Scott [3] modeled the soil as a one-dimensional shear beam attached to the wall by Winkler springs and obtained the natural frequency of a rigid retaining wall. Yeh [5] using the same model as Scott [3] included the rigid transitions and rotations for the wall in his calculations and solved the associated partial differential equations by applying the Galerkin method. Assuming the rigidity of the wall is always one of the main assumptions made in the shear beam method. In all the above mentioned equations which are based on the shear beam technique this assumption is essential. However, Jain and Scott [6] considered the deformability of the wall in his solution.

Elgamal et al. [7] instrumented a retaining wall with measuring devices and recorded the response of soil-wall system in a wide range of resonating frequencies. They then

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modeled the two categories of walls by finite element method. They noticed that walls with variable heights along the length, (i.e. wing walls), might experience a significant 3D resonance. They concluded that the three-dimensional analyses would provide better and more realistic results for the wing walls compared to the simple 2D analysis. By numerical modeling of the reinforced walls, Hatami and Bathurst [8] studied the effects of the following parameters: wall's height, the backfill width, stiffness and the length of reinforcements, soil's angle of friction, condition of the toe's abutment and magnitude of the earth movements.

Hatami and Bathurst's [8] study showed that the principle frequency of the modeled reinforced retaining walls can be estimated using the elastic wave theory and the shear wave velocity in backfill with sufficient width and the wall height. Their numerical analyses revealed that the influence of the reinforcements' stiffness, length of reinforcements, toe restraint condition and the strength of granular backfill, which depends on the angle of friction was very little. Several researches on dynamic behavior of soil-wall systems were carried out in the recent years. Researches like: Whitman [9], Hatami and Bathurst [10], Li and Aguilar [11], Gazetas et al. [12], Lanzoni et al. [13], Chen and Kianoush [14], Tang and Yeh [15], Bashaa and Babub [16] and Menona and Magenesa [17].

One other common technique of calculating the natural frequency of a structure is the Rayleigh method. Natural frequency of structures like chimneys, towers and concrete liquid reservoirs have been determined by this method. In this approach for flexible systems with distributed mass, the natural frequency of the system depends on the selected shape function. Since the cantilever retaining walls can be categorized under the flexible systems with distributed mass, the first step to calculate the frequency of these structures, would be to find an appropriate shape function.

In this paper, by using the supposition of the beam on elastic foundations theory and using spring model to model the soil along the height of the wall, the first shape mode of a uniform wall was estimated. Then by using calculated shape mode as a shape function of non-uniform wall, the natural frequency of the wall with variable cross section was devised. To verify the results, finite element analysis software was used to determine the natural frequency of the uniform and non-uniform walls with different heights. Then comparisons between the results were drown.

2. The principles and the assumptions

1. The retaining wall is assumed to be flexible, cantilevered and with variable cross section along the height.
2. The granular backfill is dry with a constant modulus of elasticity at any point.
3. The backfill is modeled as a series of springs with a linear elasticity behind the wall. The springs' stiffness is assumed constant along the wall (Fig. 1).
4. The Rayleigh principle was applied to calculate the natural circular frequency of the wall.

In this research, behavior of soil is assumed linear elastic; therefore, all obtained results are valid only for this assumption. Other advanced conditions like non-constant and nonlinear spring stiffness are also possible.

3. Dynamic equilibrium equations

For any arbitrary differential element as shown in Fig. 2, dynamic equilibrium equations for its vertical forces and moments are shown. Equation (1) can be driven using these equilibrium equations.

$$-\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 y}{\partial x^2}\right)-k(x)y=m(x)\frac{\partial^2 y}{\partial t^2} \tag{1}$$

where, $EI(x)$ is flexural stiffness of wall, y is wall transverse displacement , $k(x)$ is variable Winkler spring stiffness and $m(x)$ is mass per unit length of wall with variable section.

4. The differential equation's solution

Equation (1) shows a partial differential equation modeling the free vibrations of a cantilever beam with variable linear springs underneath. The separation of variables technique was applied to solve the differential equation. In fact, this method assumes that the solution for this differential equation represents a coefficient for the shape function at different times. This assumption sounds reasonable for our model here. Therefore, a solution to the equation (1) can take the following form:

$$y(x,t)=Y(x)F(t) \tag{2}$$

where, $Y(x)$ represents the wall shape or configuration, a

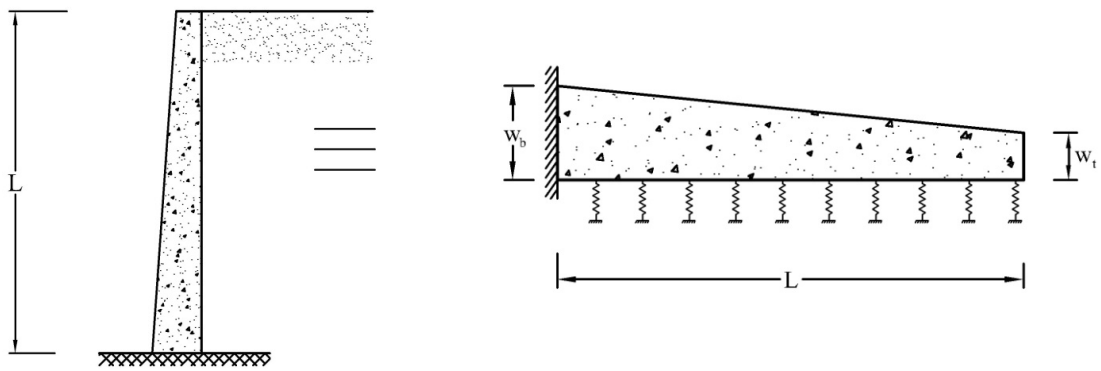

Fig. 1 Retaining wall with variable cross section and its equivalent beam-like model on an elastic foundation: (a)Modeling the retaining wall, and(b) Reinforced concrete cantilever retaining wall

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function of x alone, and $F(t)$ indicates how the amplitude of the profile varies with time t. Substituting the equation (2) into equation (1) and simplifying, the following expression can be obtained :

$$-\frac{1}{m(x)Y(x)}\frac{d^2}{dx^2}\left(EI(x)Y''(x)\right)-\frac{k(x)}{m(x)}=\frac{\ddot{F}(t)}{F(t)} \quad (3)$$

where prime and dot denote derivatives with respect to x and t. Taking the right hand side of the relationship (3) and equating it with a constant a solution for $F(t)$ can be driven as follows:

$$F(t)=C\cos(\omega t-\Phi) \quad (4)$$

where C and Φ are constant coefficients obtained from the initial conditions and ω is the frequency of vibration. Similarly, the solution for $Y(x)$ can be found as:

$$\frac{d^2}{dx^2}\left(EI(x)Y''(x)\right)+k(x)Y(x)=\omega^2 m(x)Y(x) \quad (5)$$

Since the analytical solution for the above differential equation is not an easy task, the Rayleigh approximation method was used to solve the equation and hence obtained the natural frequency of retaining walls with varying cross sections and taking the effect of backfill soil into the consideration.

Rayleigh [18] based on the theory of sounds and the principle of conservation of energy, showed that the produced natural frequency for a mechanical system, using a particular shape function is equal to/or greater than the actual and real natural frequency of the system (i.e. the retaining wall in this case). Rayleigh theory represents a unique concept in vibrations, which its importance is unparalleled over a broad range of problems. Indeed, it can be used to obtain a quick estimation of the lowest natural frequency and it serves as a key component in an algorithm for computing eigensolutions for discrete systems. Moreover, it plays a central role in a theory concerned with the derivation of approximate eigensolutions for distributed systems. Equally important is the fact that the concept can be used to gain physical insights into the behavior of vibrating systems [19].

Although the principle of virtual displacements provides an approximate result for natural vibration frequency of any structures, it is instructive to get the same result by another approach, developed by Rayleigh. In this method, by considering systems with distributed mass and elasticity, natural frequency of system can be found by equating the maximum potential energy of system to the maximum kinetic energy of the system over a vibration cycle. The result of this equation is known as Rayleigh's quotient for a system with distributed mass and elasticity. Rayleigh's quotient is valid for any natural vibration frequency of multi-degree of freedom system, although its greatest utility is in determining the lowest of fundamental frequency.[18]

For this purpose and by considering the soil-structure interactions, in order to choose an appropriate shape function to model the vibrations of the retaining walls with variable cross sections, the first mode of vibration in retaining walls with constant cross sections has been used (Fig. 3).

Accordingly, considering the cross section and material type being constant along the beam, equation (5) can be written as:[19]

$$EI\frac{d^4Y(x)}{dx^4}+kY(x)=\omega^2 mY(x) \quad (6)$$

Deflection and slope at the beginning of the wall (fixed end) and moment and shear values at the crown of the wall (free end) are all equal to zero. Hence, applying these four boundary conditions, natural frequency of the retaining wall with constant cross section can be obtained as follows:

$$\omega_c=\sqrt{\frac{12.362EI}{mL^4}+\frac{k}{m}} \quad (7)$$

Consequently, the first mode of vibration can be written as following normalized form:

$$Y_1^n(x)=0.367\sin(\frac{1.88}{L}x)-0.5\cos(\frac{1.88}{L}x)-0.367\sinh(\frac{1.88}{L}x)+0.5\cosh(\frac{1.88}{L}x) \quad (8)$$

where $Y_1^n(x)$ is the first mode of vibration and L is the height of wall.

5. Calculating the natural frequency of the walls with variable cross section

The normalized form of first vibration mode in a retaining wall with constant cross section has been used as the shape function for determining the natural frequency of retaining wall with variable cross section. Therefore, the kinetic and potential energies are calculated as below. The maximum kinematic energy is equal to:

$$T_{\max}=\frac{1}{2}\int_0^L m(x)\left(\frac{\partial y}{\partial t}\right)^2 dx=\frac{1}{2}\omega^2\int_0^L m(x)Y^2(x)dx \quad (9)$$

where $m(x)$ is mass per unit length of wall with variable section. The maximum potential energy is equal to:

$$V_{\max}=\frac{1}{2}\int_0^L EI(x)\left(\frac{d^2Y(x)}{dx^2}\right)^2 dx+\frac{1}{2}\int_0^L kY^2(x)dx \quad (10)$$

where $I(x)$ moment of inertia of wall with variable section.

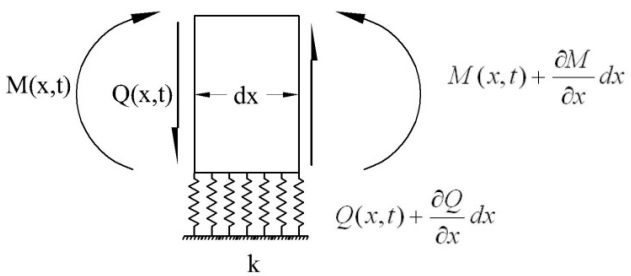

Fig. 2 Differential element of the modeled beam

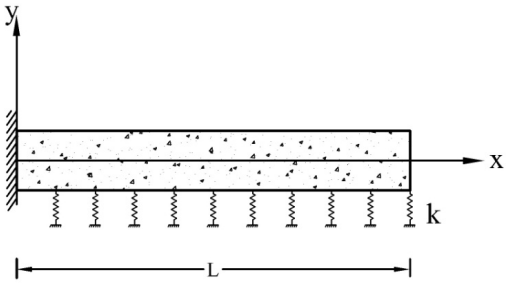

Fig. 3 A schematic retaining wall with constant cross section

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By equating the maximum potential energy and the maximum kinetic energy, the value of ω^2 was obtained as equation (11):

$$\omega^2 = \frac{\int_0^L EI(x)\left(\frac{d^2Y(x)}{dx^2}\right)^2 dx + \int_0^L kY^2(x)dx}{\int_0^L m(x)Y^2(x)dx} \quad (11)$$

Assuming the perpendicular dimension to the wall's plane to be a unit in scale and regarding the Fig. 1, $m(x)$ and $I(x)$ functions for the retaining wall with variable cross section are driven as follows:

$$m(x) = \rho\left(w_t + \frac{(L-x)(w_b - w_t)}{L}\right) \quad (12)$$

$$I(x) = \frac{1}{12}\left(w_t + \frac{(L-x)(w_b - w_t)}{L}\right)^3 \quad (13)$$

where w_b is wall base width and w_t is wall top width. Thus the value of ω^2 can be written as equation (14):

$$\omega^2 = \frac{\frac{5E}{300L^3}\left(1.701w_b w_t^2 + 4.457w_b^2 w_t + 9.032w_b^3 + 3.866w_t^3\right) + 0.252kL}{\rho\left(0.049Lw_b + 0.203w_t L\right)} \quad (14)$$

Equation (14) gives the natural frequency of a retaining wall with variable cross section by taking the interactions between backfill material and the wall, into account.

In equations (7) and (14), the parameter k is the stiffness per unit length of Winkler's spring. The stiffness of Winkler's spring can be calculated using the subgrade's reaction modulus, K.

All researchers have proposed different relations for calculating the subgrade reaction modulus. Table 1 shows a number of these relations along with their associated parameters. [20-27]

Table 1 Subgrade Reaction Modulus

Equation	Researcher(s)	Parameters
$K = \frac{\pi E_s}{2b(1-\nu_s^2)\log(L/b)}$	Galin [19]	E_s= Soil Modulus of Elasticity ν_s= Soil possion Ratio H = Thikness of a Layer L= Beam Length b = Beam Width
$K = \frac{0.65E_s}{b(1-\nu_s^2)}\left(\frac{E_s b^4}{E_b I}\right)^{\frac{1}{12}}$	Vesic and Johnson [20]	E_s= Soil Modulus of Elasticity ν_s= Soil possion Ratio b = Beam Width I = Beam Moment of Inertia E_b = Beam Modulus of Elasticity
$K = \frac{0.65E_s}{b(1-\nu_s^2)}$	Barden [21]	E_s= Soil Modulus of Elasticity ν_s= Soil possion Ratio b = Beam Width
$K = \frac{E_s}{H(1+\nu_s)(1-2\nu_s)}$	Vlazov and Leontiev's [23]	E_s= Soil Modulus of Elasticity ν_s= Soil possion Ratio H = Thikness of a Layer
$K = \frac{4E_s(1-\nu_s)}{H(1+\nu_s)(1-2\nu_s)}$	Scott [3]	E_s= Soil Modulus of Elasticity ν_s= Soil possion Ratio H = Thikness of a Layer
$K = 1.2E_s$	Makris and Gazetas [24]	E_s= Soil Modulus of Elasticity
$K = \frac{C_2 G}{H}$	Richards et al. [25]	C_2 = lumps all the geometric variables H = scale factor G= shear modulus of soil

6. Obtained results from the suggested method

A loose sand backfill with elastic modulus of E=15 MPa and with the Poisson's ratio of ν= 0.2 is considered in calculations. A retaining wall with 1m width (w_b=1m) in the foundation and the width of the crest equal to 0.4 m (w_t=0.4m) with the elastic modulus of $2.6\times10^{10} N/m^2$ and the mass per unit length equal to ρ=2320kg/m^2 is assumed to support the mentioned backfill. In order to study the effect of the ratio of the backfill width to the wall length (H/L), natural frequency of the wall versus H/L has been plotted.

Based on the suggested method, Fig. 4 illustrates the variations in natural angular frequency of the retaining wall against the H/L ratio. According to Fig. 4, increasing the flexibility of the wall causes a non-linear decrease in angular frequency.

In order to investigate the effect of soil type, the ratios of wall top to the bottom width w_t/w_b and the height of the wall L in calculating the natural frequency of retaining walls, three types of backfills have been considered. These backfills include loose, medium and dense sand with elastic modulus (E_s) equal to $15MN/m^2$, $30MN/m^2$ and $60MN/m^2$ and the Poisson's ratios (v) of 0.2, 0.27 and 0.3, respectively. The elastic modulus of the wall (E_w) was taken to be equal to $2.6\times10^{10} N/m^2$ and the frequencies of the walls with heights ranging from 3 to 10 m under three ratios of width to length (H/L) have been investigated.

At first, with a loose sand backfill and flexible wall with the height ranging from 3 to 10 m, under three ratios of H/L equal to 10, 5 and 1; the variations of natural angular frequency of the wall for different ratios w_t/w_b have been studied. As can be observed in Fig. 5, by increase in the wt/wb ratio, ω decreases non-linearly. In addition, it can be observed that by 10 percent decrease in the H/L ratio in a wall with certain height (e.g. 3m) and w_t/w_b= 0.25, the magnitude of ω only increases by about 3%. On the other hand by doubling the height of the wall, 20% increase in ω be observed. The results show that the natural frequency of free vibrations in shorter walls is more sensitive to the changes in w_t/w_b ratio. Changing the backfill type to the medium sand and maintaining all the other conditions as before, changes in natural angular frequency of the wall for different w_t/w_b ratios have been studied. As it can be observed in Fig. 6, by increasing the wt/wb ratio, the natural frequency of the wall decreases non-linearly, too. However, for the ratios greater than 0.7 ($w_t/w_b \geqq 0.7$), these variations approach to a linear characteristics.

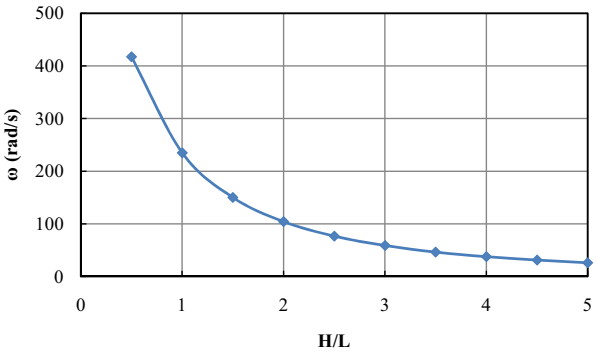

Fig. 4 Natural angular frequency of the wall versus H/L for loose sands

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Finally, considering a dense sand backfill and a flexible wall and keeping all the previous conditions as the same, variations in natural angular frequency have been studied for different *wt/wb* ratios. In Fig. 7, similar to those previous Figs, an increase in the wt/wb ratio causes a non-linear decrease in the natural frequency of the wall. However, at *H/L*=1 by increase in the height of wall, this trend terminates in such a way that for a 4m long wall the increase in wt/wb ratio, yields to irregular variations in the natural frequency of the wall. This pattern can be observed in other heights such as 5m to 10 m, too. Nevertheless, the natural frequency of the wall in this condition is greater than in the case with loose sand backfill.

Furthermore, assuming a 5 m high, flexible and variable cross section wall (*L*=5 m) with *H/L*=1, 5, 10, upper and lower bounds have been obtained for the natural frequency of retaining walls for three types of loose, medium and dense sand backfills. Variations of the wall natural frequency versus top-bottom width ratios are demonstrated in Figures 8 to 10.

Based on the obtained results, increase in the elastic modulus of the soil yields to increase in the natural frequency of vibrations in retaining walls.

Moreover, the ratio w_t/w_b plays a determining role in calculation of angular frequencies so that in some cases, by doubling the said ratio, a 200% reduction in the natural frequency was observed. It was also observed that when the *H/L* ratio increased, the natural frequency range of the walls decreased.

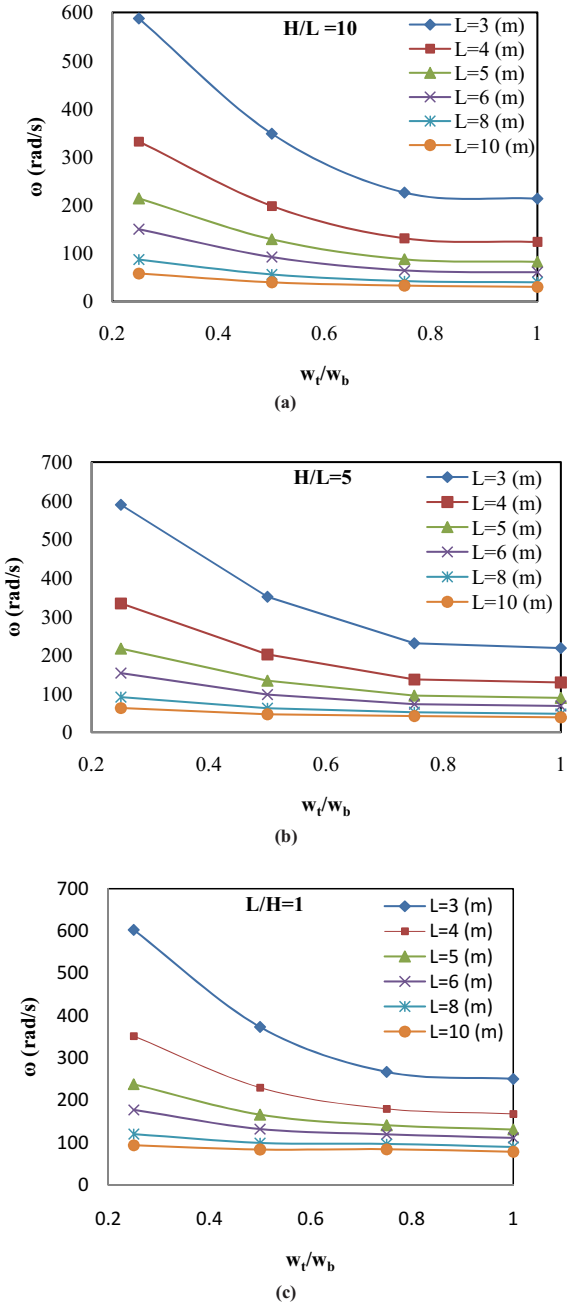

Fig. 5 Variations of natural angular frequency versus the top-bottom width ratio of a wall with E_s=15 *MN/m²*: (a) *H/L*=10, (b) *H/L*=5, and (c) *H/L*=1

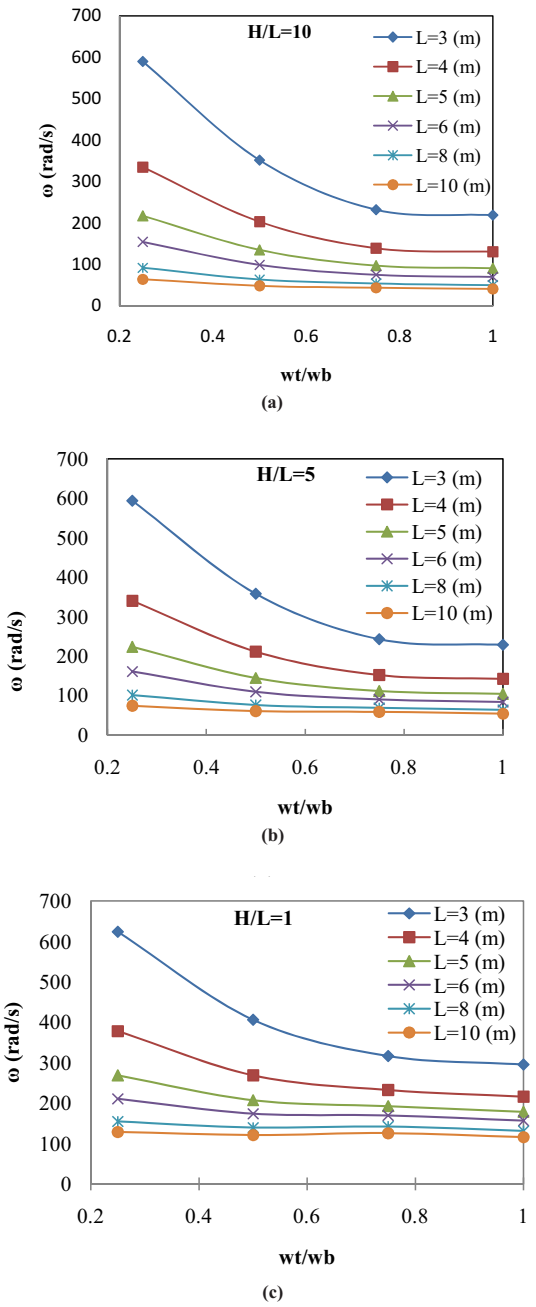

Fig. 6 Variations of natural angular frequency versus the top-bottom width ratio of a wall with E_s=30 *MN/m²*: (a) *H/L*=10, (b) *H/L*=5, and (c) *H/L*=1

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7. Comparison of the results with those of other researchers and FEM

In order to verify the formula for the calculation of the natural frequency of retaining walls with variable cross sections proposed in this article, it is essential to compare the obtained results from this formula with those obtained from numerical techniques and with those obtained from the other researchers proposed formulas. For this purpose, a non-uniform retaining wall was considered. The geometrical dimensions and mechanical properties of this wall are listed in Table 2.

In order to carry out the finite element analyses, a 2-dimensional model was used to model the wall and its backfill with both constant and variable cross sections. For this purpose,

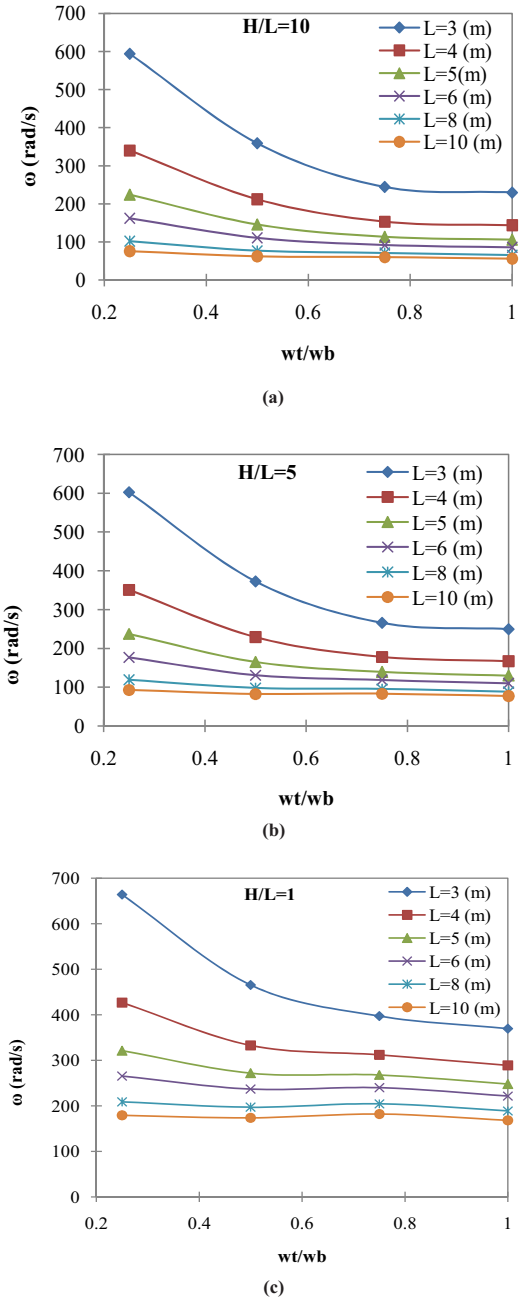

Fig. 7 Variations of natural angular frequency versus the top-bottom width ratio of a wall with E_s=60 MN/m^2: (a) *H/L*=10, (b) *H/L*=5, and (c) *H/L*=1

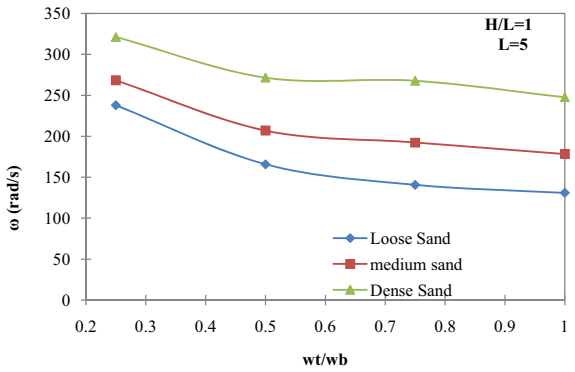

Fig. 8 Upper and lower bounds for main frequencies of a flexible wall (*H/L= 1*) with variable cross section

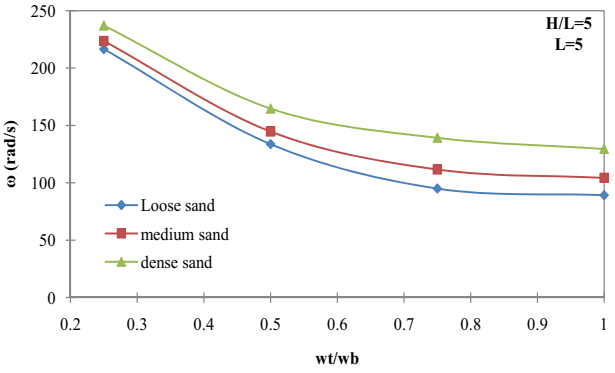

Fig. 9 Upper and lower bounds for main frequencies of a flexible wall (*H/L= 5*) with variable cross section

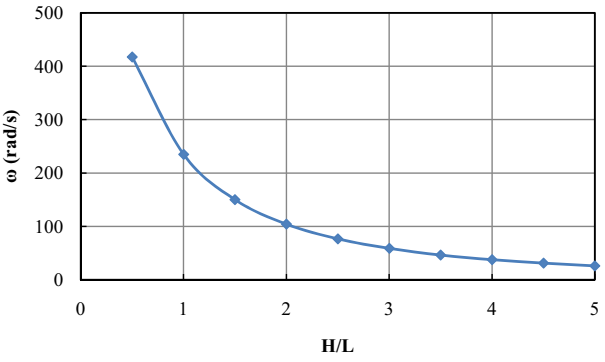

Fig. 10 Upper and lower bounds for main frequencies of a flexible wall (*H/L= 10*) with variable cross section

Table 2 Material properties and geometrical dimensions of the concrete retaining wall

Part	Parameter	Value
Retaining Wall	E (GPa)	26
	υ	0.2
	ρ(kg/m^3)	2320
	w_b (m)	1
	w_t (m)	0.5
Backfill	C (KPa)	0
	ϕ (deg)	30
	ρ (kg/m^3)	1900
	υ	0.3

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the CPE4R element in ABAQUS software [26], which is a 4-node bilinear plane strain quadrilateral element, was used. The interface between wall and backfill was modeled using a surface-to-surface contact with small-sliding formula. For the contact interaction properties of the tangential behavior, a penalty friction formula with friction coefficient of 0.32 was used. Also for the normal behavior, a hard contact with penalty constraint enforcement method was used. The finite element model for the wall with variable cross section is shown in Fig. 11. In this figure, the red part represents the wall, which all three DOF's of the underside is constrained. The gray part represents the backfill, which its underside is constrained in vertical direction. In order to model sufficient width of the backfill soil, the infinite CINPE4 element that is a 4-node linear one-way infinite one. is used. Mohr-Colomb constitution law was employed to modeling backfill. Moreover, the finite element model for the wall with constant cross section was used; all of its parameters were the same of variable one except its cross section, which is constant.

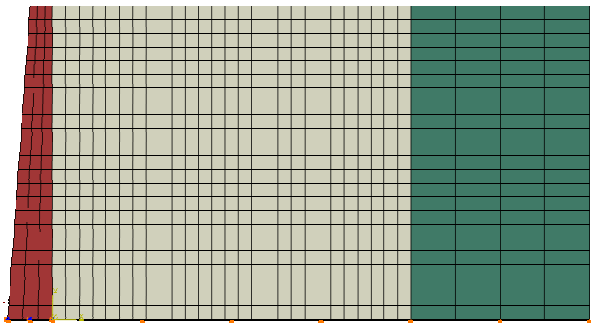

Fig. 11 Finite element model of the retaining wall and backfill

Table 3 and 4 show a comparison between the obtained results from finite element analysis and the proposed method. As it can be observed, the maximum relative difference between the results of the two methods in a wall with variable cross section was 17.91% for the wall with 5m length. In addition, it can be observed that the relative difference for the wall with constant cross section was smaller than the case with variable cross section. Fig. 12 shows the variations of natural angular frequency for the walls of different lengths with variable and constant cross sections.

Assuming the modulus of elasticity, specific mass and the Poisson's ratio of the soil are E_s =17.3 MN/m^2, ρ_s=1500 Kg/m^2 and v= 0.3 , respectively, variations of the natural frequency for a flexible wall with respect to H/L ratio was compared with

Table 3 Circular Natural Frequency for L=3m to 5m (rad/s)

	L = 3 m (k = 3.1 MN/m/m)		L = 4 m (k = 2.32 MN/m/m)		L = 5 m (k =1.86 MN/m/m)	
	Constant Section	Variable Section	Constant Section	Variable Section	Constant Section	Variable Section
Proposed Method	379.16	434.22	214.63	246.23	138.78	159.65
FEM	350.75	374.63	185.49	205.32	125.42	131.05
Relative Error	7.49%	13.72%	13.58%	16.61%	9.63%	17.91%

Table 4 Circular Natural Frequency for L=6m to 10m (rad/s)

	L = 6 m (k = 1.55 MN/m/m)		L = 8 m (k =1.16MN/m/m)		L = 10 m (k =0.93 MN/m/m)	
	Constant Section	Variable Section	Constant Section	Variable Section	Constant Section	Variable Section
Proposed Method	97.82	112.97	57.59	67.25	39.42	46.69
FEM	88.74	100.53	47.87	60.24	32.77	40.45
Relative Error	9.28%	11.01%	16.88%	10.42%	16.87%	13.36%

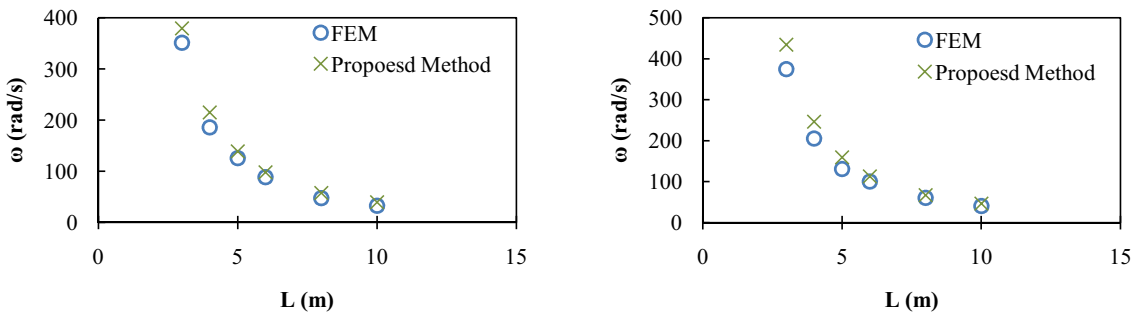

Fig. 12 Variations of natural angular frequency versus length of the wall: (a) Constant cross section wall, and (b) Variable cross section wall

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the results reported by Scott [3] and Wu [4] in Fig. 13. The results of the suggested method are an upper bound for the results of the mentioned researchers due to consideration of the flexibility of the wall in current study.

Given the Poisson's ratio, shear modulus and specific mass of soil to be v_s=0.4, G_s=3.6 ×10^7*N/m*^{2} and ρ_s=1600 *Kg/m*^{3} , respectively, and also considering a 6 meters long wall, the suggested method was compared with that of Jain and Scott [6] as shown in the Table 5. Nevertheless, the weight of backfill soil has not been taken into account in the present method where as it was considered in the Jain and Scott [6] study; hence the weight per unit length of the wall in the suggested method and their technique has been equal to *m*=2900*Kg/m*^{2} and *m*=4500*Kg/m*^{2} ,respectively.

8. Conclusion

Several methods are used by researchers to analysis of earth structures [27-29]. In this research based on the theory of beams on elastic foundations a new analytical formula has been derived for calculating the natural frequency of retaining walls with constant or variable cross sections. The suggested formula is capable of taking the geometrical properties and the stiffness of the wall as well as the stiffness of the backfill soil into account to calculate the natural frequency.

The obtained results from the suggested formulation reveal that given the normal conditions for the soil in regular walls with heights ranging from 3 to 10 m, vibrating frequency of the concrete retaining wall varies between 56 to 663 rad/s and thus the common period of the retaining walls varies from 0.01 to 0.11s depending on the ratio of backfill's width to the wall's length (*H/L*). On the other hand based on the proposed formulation by increase in the *H/L* ratio the natural frequency of vibrations for the wall drastically decreases. In addition, the results show that by widening the wall its natural frequency of vibrations decreases too. As an example for medium sand, decreasing the ratio of width of the crest to the width of the foundation by a half factor can decrease the frequency of vibrations.

On the other hand, an increase in the modulus of elasticity of soil leads to an increase in the natural frequency of vibrations of the wall. Therefore, the natural frequency of vibrations in the dense soils is a little bit greater than the natural frequency of vibrations of the wall in the loose soils. Comparison of the results obtained from the suggested method with those from the software analyses reveals that the presented method in this study is less erroneous. However, the obtained frequencies from the formulations proposed by the other researchers are remarkably smaller than the values obtained from the suggested method.

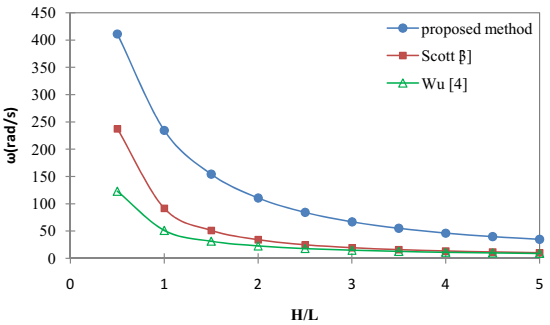

Fig. 13 Comparison between the variations of angular frequency versus H/L ratio

Table 5 Comparison of natural angular frequencies

H/ L	EI (N.m^2/m)	k (N/m^3)	ω (rad/s)	
			Jain and Scott (1989)	Proposed Method
10	9×10^8	14.4×10^6	71.5	89.03

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List of notations:

E	concrete Modulus of Elasticity
E_s	soil Modulus of Elasticity
H	backfill width
I	moment of inertia of uniform wall
$I(x)$	moment of inertia of non-uniform wall
k	constant Winkler spring stiffness
$k(x)$	variable Winkler spring stiffness
K	subgrade reaction modulus
L	height of wall
m	mass per unit length of uniform wall
$m(x)$	mass per unit length of non-uniform wall
T_{max}	maximum kinetic energy
V_{max}	maximum potential energy
w_b	wall base width
w_t	wall top width
y	wall transverse displacement
$Y_1^n(x)$	normalized first shape mode
ρ	wall unit mass
ρ_s	soil unit mass
υ	wall Poisson's ratio
υ_s	soil Poisson's ratio
ω	natural circular frequency of non-uniform wall
ω_c	natural circular frequency of uniform wall

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