An Efficient Selection Operator for Genetic Search of Pipe Networks Optimal design

M. H. Afshar[1] and M. R. Ghasemi[2]

1.Assistant. Prof., College of Civil Engineering, Iran Univ. of Science and Tech., Narmak, Tehran, Iran 16844.
E-mail: mhafshar@iust.ac.ir

2.Assistant. Prof., Dept. of Civil Eng., University of Sistan-Baluchestan, Zahedan, Iran

Abstract: *An efficient selection operator for use in genetic search of pipe networks optimal design is introduced in this paper. The proposed selection scheme is the superior member of a family of improved selection operators developed in an attempt to more closely simulate the main features of the natural mating process which is not reflected in existing selection schemes. The mating process occurring in the nature exhibits two distinct features. First, there is a competition between phenotypes looking for the fittest possible mate which usually ends up with choosing a mate with more or less the same fitness. Second, and more importantly, the search for a mate is often confined to a community of phenotypes rather than the whole population. Four different selection operators simulating these features in a random and pre-determined manner are developed and tested. All the selection schemes exhibit good convergence characteristics, in particular the one in which both the size of the sub-community and the pair of the mates in the sub-community are determined randomly. The efficiency of the proposed selection operator is shown by applying the method for the optimal design of three well-known benchmark networks, namely two-loop, Hanoi and New-York networks. The proposed scheme produces results comparable to the best results presented in the literature with much less computational effort.*

Keywords: *Pipe Networks; Optimal design, Genetic Algorithm, Selection operator*

Introduction

The problem of network optimization requires the determination of pipe sizes from a set of commercially available diameters ensuring feasible least cost solution. Amongst all the methods developed for pipe network optimization, GA is gaining more popularity in recent years. The early research was primarily concentrated on developing a methodology for applying GA to pipe network optimization problem using simple genetic algorithms (Dandy et al., 1993; Simpson et al., 1993; Murphy et al., 1993; Simpson el al., 1994). More recent researches on the application of GA to pipe network optimization have been focused on the development of new genetic algorithms to yield cheaper solution than already existing algorithms. These improvements are mostly achieved via modifications of the simple genetic algorithm or introducing new operators and features to the basic algorithms (Dandy et al., 1996; Walters et al., 1999; Halhal et al., 1997). This paper presents a new selection operator with improved convergence characteristics compared to simple and recently introduced genetic algorithms. The proposed selection operator is the superior member of a family of selection schemes developed to more closely simulate two distinguished natural mating features. First, the phenotypes in nature have a tendency to look for the fittest possible mate which usually ends up with choosing a mate with more or less the same fitness. Second, and more importantly, the search for a mate is often confined to a community of phenotypes rather than the whole population. Four different selection operators are developed based on this idea and the superior one is selected. The selected scheme is shown to out-perform the existing algorithm from convergence characteristics point of view while producing results comparable to the cheapest solutions available in the literature .

PROPOSED GA FORMULATION

In what follows, the steps taken in a standard GA formulation of the pipe network optimal design (Simpson et al., 1993) is outlined. The modifications made to the simple GA formulation, if applicable, is explained at the end of each step along with their justifications.

1. Encoding design variables. The genetic algorithm requires that any trial solution of the design problem be represented by a coded string of finite length similar to the structure of a chromosome of genetic code. This is usually achieved by defining a selected mapping between the possible values of the design variables and a set of coded sub-strings with required number of binary bits. For example, a 4-bit sub-string can be coded to represent any of the 16 commercially available pipe diameters. Here a standard binary coding is used to represent the possible values of the pipe diameters.

2. Generation of an initial population. GA randomly generates an initial population , of size N, of coded strings representing some trial solutions to the pipe network design problem.

3. Computation of network cost. Each of N members of the population is considered and decoded to the corresponding pipe networks. The cost of each trial solution of the current population is then calculated.

4. Hydraulic analysis of the network. A steady state analysis is carried out for each network of the current population to find the pressure and velocity constraint violations. In this work, an element-by-element simulation program is used to solve the set of hydraulic constraints for nodal heads (Afshar 2001).

5. Computation of the total cost. The total cost of the network for each member of the population is considered as the sum of the network cost calculated in step (3) and the penalty cost. The penalty cost of each network in the population is computed if the trial design does not satisfy the pressure and flow constraints. It is a common practice to use the maximum constraint violation for calculating the penalty cost (Dandy et al., 1996; Abebe and Solomatine, 1999). In this method, GA could not distinguish between two different designs with the same maximum constraint violation but different number of constraint violations. Here the penalty cost is calculated according to:

$$C_p = \sum_{l=1}^{m} \alpha_l (q_l - q_{\min})^2 + \sum_{k=1}^{n} \alpha_k (H_k - H_{\min})^2$$

in which all the constraint violations are used for penalty cost calculation. Here a_l ,a_k are the pipe flow and nodal head penalty parameters, respectively, with large values when corresponding constraints are violated, and zero values otherwise. This method ensures that infeasible networks would have higher penalty costs and, therefore, leads to a better distribution of the fitness function in the search space compared to conventional method and thus helping GA to locate more useful genes.

Penalty parameters used in the GA formulation of the pipe network optimizations are usually specified by the user (Dandy et al., 1996; Abebe and Solomatine, 1999). This method of specifying the penalty parameter requires a trial-and-error procedure to get the proper value of the parameters as a low value of penalty parameters could lead to a constraint-violating solution while a high value of parameter would result in rejecting some of constraint-violating solutions with useful genes from the evolution process (Gen and Cheng, 1997). Here the proper values of penalty parameters are determined before starting the genetic search by using a mathematical programming method to solve the problem for its continuous solution. For this purpose, an iterative procedure is chosen, whereby the minimization of the penalized cost function is substituted with a series of

minimization problems with different penalty parameter values. The procedure starts with a value of unity for penalty parameters and the design problem is solved. The solution is then checked for constraints violation, upon which the next design problem is defined with the increased value of the penalty parameters by an order of magnitude if the corresponding constraints are not satisfied. This procedure is continued until all the constraints are satisfied, i.e. all the penalized terms of Eq. (1) become equal to zero. The computational effort required for the setting of the penalty parameters is negligible compared to the one required by the genetic search.

6. Computation of the fitness. The fitness of a trial design is usually taken as a function of the total network cost. Different forms of the fitness functions are used by different researchers (Gen and Cheng, 1997; Dandy et al., 1996). Here the inverse of the total cost is used as the fitness of each network.

7. Selection of the mates to generate new population. The simple GA generates the members of the new generation by a roulette wheel selection scheme. In this scheme, the probability of a string i , , to be selected as a parent for the next generation is given by

$$p_i = \frac{f_i}{\sum_{i=1}^{N} f_i}$$

This scheme, however, is believed to be the source of the so-called premature convergence, especially, in small population searches (Goldberg,1989). Different remedies in the form of scaling or alternative selection operators are proposed to prevent the dominance of the extraordinary fit strings in the early stages of the search (Goldberg, 1989). Brindle (1981) used a ranking procedure originally suggested by Wetzel (1983) called stochastic tournament procedure on several test functions. In this method, selection probabilities are calculated normally and successive pairs of individuals are drawn using roulette wheel selection. After drawing a pair, the string with higher fitness is declared the winner and inserted in the new population. Here the tournament selection scheme is modified in such a way to more closely simulate the natural mating process. In the proposed method, a sub-community of the current population is randomly generated using a uniform probability density function and the two fittest members are drawn as the pair of parents going to the mating pool. This procedure is repeated to form *npop/2* pairs of parents needed to form the new generation. This method is intended to simulate the well-observed tendency of the phenotypes to look for fitter mates in their communities. Different selection operators can be defined of based on this idea depending on the size of the sub-communities and the way their members are selected in each generation. Two methods of deterministic and random determination of the sub-communities' size and members are considered in this paper as follows.

Deterministic length determination: In this method, the length of the sub-communities is determined deterministically starting with $N^{0.5}$ for the first generation and then decreasing linearly during the evolution process to the lowest possible value of 2 at the end of the search.

Random Length determination: In this method, the length of the sub-communities in each generation is randomly selected between 2 and $N^{0.5}$.

Deterministic member determination: In this method, the members of each sub-community are selected uniformly across the population starting from a randomly selected position in the population.

Random member determination: In this method, the members of each sub-community are selected randomly from the

whole population excluding the ones which are already selected in order to prevent selection of the same strings as both parents. Four different tournament selection operators, namely Deterministic Length - Deterministic Members (DLDM), Deterministic Length - Random Members (DLRM), Random Length - Deterministic Members (RLDM) and finally Random Length - Random Members (RLRM) are developed and tested in this study and the superior one is selected.

8. The crossover operation. Two off-springs are formed via partial exchange of bits between two selected parents by using a crossover operator. Crossover occurs with some specified probability of crossover for each pair of parents selected in the previous step. Here a one point crossover operator with p_c=1 is used.

9. Mutation. A bit-wise mutation with some specified probability of mutation p_m is carried out for each of the strings which have undergone crossover. The bit-wise mutation changes the value of the selected bit to the opposite value (i.e., 0 to 1 or 1 to 0). Here a one-bit mutation operator with p_m =1 is used throughout the evolution process.

10. Production of successive generations. The three operators described above produce a new generation of pipe network trial designs. This procedure is repeated to create successive generations. One of the recognized problems with GA is the uncertainty about the termination of the search since the best solution of the generation could oscillate during the evolution process. Too loose convergence criteria could therefore lead to an early termination while a too stringent termination criteria would increase the computational cost. Here an elitist strategy is also used in which the fittest string of the current generation is directly copied into a randomly chosen position of the next generation. This mechanism has two advantages. First, the useful information of the fittest string is directly passed to the next generation as a member in addition to the information supplied by its off-springs. This ensures that this information is never lost during the evolutionary process. Second, the oscillatory nature of the best solution of the generations is eliminated, paving the way for the definition of a logical convergence criteria, ie; the GA search can be considered to have converged if the best solution of the successive generations remains constant for a specified number of generations.

Test Problems

The first problem to be considered is a two-loop network with 8 pipes, 7 nodes and one reservoir shown in Fig. 1 (Abebe and Solomatine 1999). All the pipes are 1000m long and Hazen-Williams coefficient is assumed to be 130 for all the pipes. The minimum nodal head requirement for all demand nodes is 30m. There are 14 commercially available pipe diameters as listed in Table 1. Figs. 2.1 and 2.2 show the convergence curve, best of the generations cost versus number of network analysis required, of the four selection operators for 100 and 200 population genetic searches, respectively. Table 2 shows the results produced by four selection operators for 100

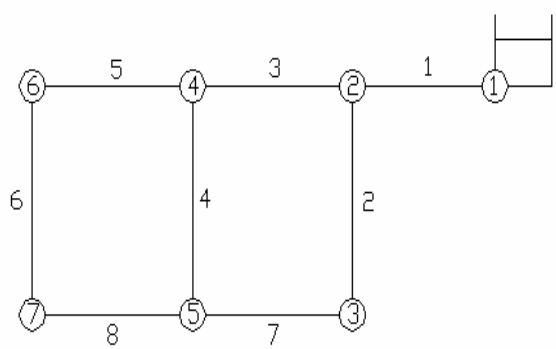

Fig.1. Two loop network

and 200 population size. The selection operators DLRM and DLDM clearly show a sign of premature convergence exhibited in the roulette wheel selection scheme. The methods RLDM and RLRM produced better results with increasing number of populations. RLDM exhibits superior convergence characteristics while RLRM was capable of producing best ever solution of 419000 units in only 38 generations and 7600 evaluations. This compares with more than 6500 evaluation required by the method of Savic and walters (1997) to get the solution of 419000 units. Table 3 compares the best results produced by the presented methods to some of the available random search results in the literature (Abebe and Solomatine, 1999; Savic and Walters, 1997).

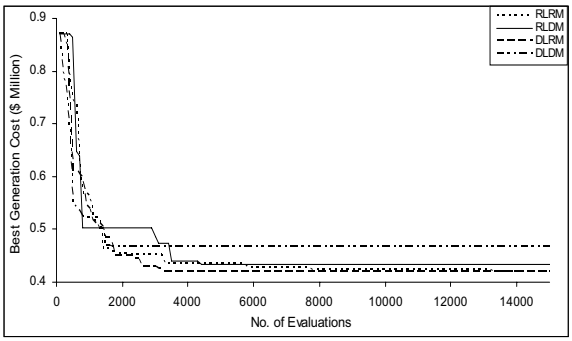

Fig. 2.1 Best of generations cost for two loop network (100 population size)

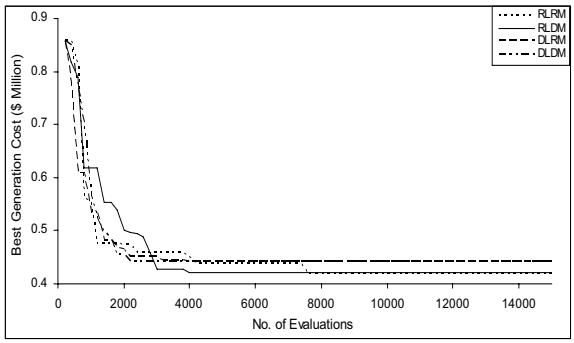

Fig. 2.2 Best of generations cost for two loop network (200 population size)

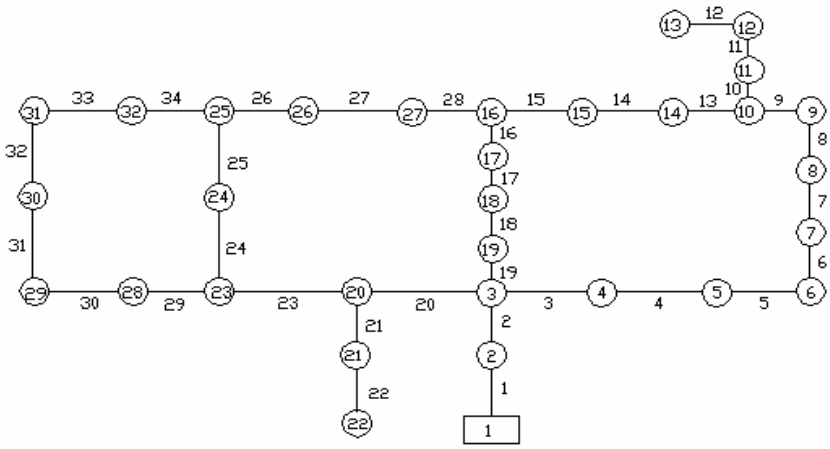

Fig. 3. Hanoi network

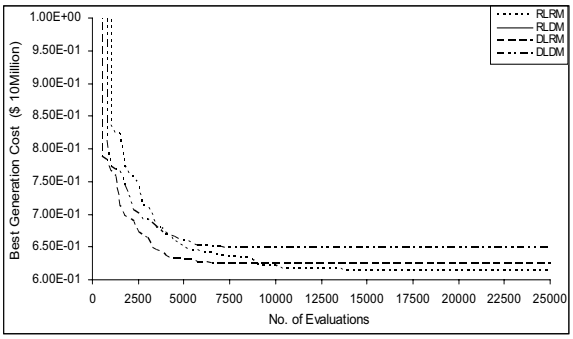

Fig. 4.1 Best of generations cost for Hanoi network (250 population size)

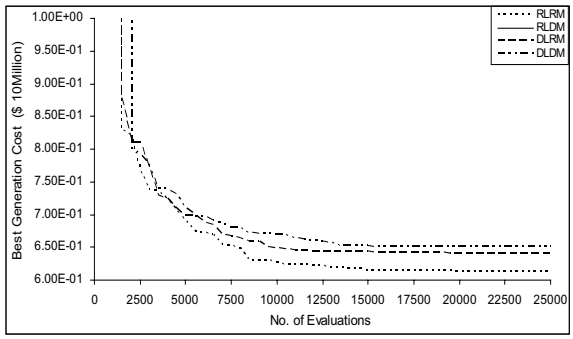

Fig. 4.2 Best of generations cost for Hanoi network (500 population size)

Table 1. Cost data for the two-loop network

Diameter (inch)	1	2	3	4	6	8	10	12	14	16	18	20	22
Cost (units/m)	2	5	8	11	16	23	32	50	60	90	130	170	300

Table 2. Optimal pipe diameters produced by the presented methods for two loop network

Pipe	RLRM		DLRM		RLDM		DLDM	
	P100	P200	P100	P200	P100	P200	P100	P200
1	18	18	20	18	18	18	18	18
2	14	10	10	8	14	14	8	8
3	14	16	16	18	14	14	18	18
4	1	4	1	10	1	1	14	10
5	14	16	14	16	14	14	14	16
6	6	10	10	10	1	6	1	10
7	14	10	10	1	14	14	1	1
8	10	1	1	1	14	10	12	1
Cost (units)	420000	419000	420000	447000	441000	420000	457000	441000
Evaluations	13300	7600	3300	4000	4400	4000	16100	3600
Success rate	7/10	8/10	6/10	7/10	5/10	6/10	7/10	8/10

Table 3. Best runs along with some of the available discrete results for two loop network

Pipe	Present work (Best run)		Abebe and Solomatine (1999)		Savic and Walters (1997)*	
	P100	P200	GA	ACCOL**	GA2	GA1
1	18	18	18	22	20	18
2	14	10	14	18	10	10
3	14	16	14	20	16	16
4	1	4	1	3	1	4
5	14	16	14	16	14	16
6	6	10	1	4	10	10
7	14	10	14	18	10	10
8	10	1	12	16	1	1
Cost (units)	420000	419000	424000	447000	420000	419000

Table 4. Cost data for the Hanoi network

Diameter (inch)	12	16	20	24	30	40
Cost (units/m)	45.73	70.40	98.39	129.33	180.75	278.28

Table 5. Optimal pipe diameters obtained by the proposed methods for Hanoi network.

Pipe	RLRM		DLRM		RLDM		DLDM	
	P250	P500	P250	P500	P250	P500	P250	P500
1	40	40	40	40	40	40	40	40
2	40	40	40	40	40	40	40	40
3	40	40	40	40	40	40	40	40
4	40	40	40	40	40	40	40	30
5	40	40	40	40	40	40	40	30
6	40	40	40	40	40	40	40	30
7	40	40	40	40	40	40	40	30
8	40	40	40	30	40	40	40	30
9	40	40	40	30	40	30	40	20
10	24	30	30	30	24	24	24	30
11	30	24	30	24	30	30	24	30
12	24	24	24	30	24	24	30	24
13	16	20	24	16	16	12	16	24
14	16	16	16	12	16	12	16	30
15	12	12	20	12	12	12	12	30
16	12	12	12	24	12	30	12	40
17	20	20	20	30	20	30	20	40
18	20	20	24	30	24	30	20	40
19	24	20	20	30	20	30	24	40
20	40	40	40	40	40	40	40	40
21	20	20	20	20	20	20	20	20
22	12	12	12	12	12	12	12	12
23	40	40	40	30	40	30	40	30
24	30	30	30	24	30	20	30	20
25	30	30	24	24	30	16	24	12
26	20	20	16	12	20	20	20	20
27	16	12	16	24	12	24	16	30
28	12	12	20	30	12	30	12	24
29	16	16	16	20	16	20	20	20
30	12	12	12	16	12	16	30	16
31	12	12	12	12	12	12	20	12
32	12	16	12	16	20	12	12	12
33	20	16	16	16	16	16	12	12
34	24	24	30	30	24	24	16	20
Cost ($M)	6.13	6.10	6.25	6.39	6.12	6.31	6.48	6.48
Evaluations	16250	36000	6500	27500	20250	24000	8250	27000
Success rate	6/10	7/10	5/10	7/10	5/10	6/10	6/10	8/10

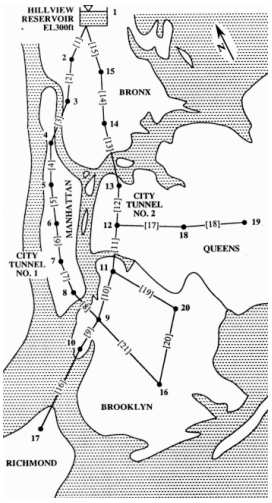

Fig. 5. New York tunnel network

Table 6. Optimal pipe diameters obtained by different methods for Hanoi network.

Pipe	Present work (Best run)		Abebe and Solomatine (1999)		Savic and walters (1997)*	
	P250	P500	GA	ACCOL	GA1	GA2
1	40	40	40	40	40	40
2	40	40	40	40	40	40
3	40	40	40	40	40	40
4	40	40	40	40	40	40
5	40	40	30	40	40	40
6	40	40	40	30	40	40
7	40	40	40	40	40	40
8	40	40	30	40	40	40
9	40	40	30	24	40	30
10	24	30	30	40	30	30
11	30	24	30	30	24	30
12	24	24	30	40	24	24
13	16	20	16	16	20	16
14	16	16	24	16	16	16
15	12	12	30	30	12	12
16	12	12	30	12	12	16
17	20	20	30	20	16	20
18	20	20	40	24	20	24
19	24	20	40	30	20	24
20	40	40	40	40	40	40
21	20	20	20	30	20	20
22	12	12	20	30	12	12
23	40	40	30	40	40	40
24	30	30	16	40	30	30
25	30	30	20	40	30	30
26	20	20	12	24	20	20
27	16	12	24	30	12	12
28	12	12	20	12	12	12
29	16	16	24	16	16	16
30	12	12	30	40	16	16
31	12	12	30	16	12	12
32	12	16	30	20	12	12
33	20	16	30	30	16	16
34	24	24	12	24	20	20
Cost ($M)	6.13	6.10	7.0	7.836	6.07	6.19

* These solutions are obtained using different numerical conversion constant for the head loss equations

Table 7. Pipe cost data for New-York network

Diameter (inch)	0	36	48	60	72	84	96	108
Cost ($/ft)	0	93.5	134.0	176.0	221.0	267.0	316.0	365.0
Diameter (inch)	120	132	144	156	168	180	192	204
Cost ($/ft)	417.0	469.0	522.0	577.0	632.0	689.0	746.0	804.0

Table 8. Pipe & nodal data for New York tunnel network.

Pipe data					Nodal data		
Pipe	Start Node	End node	Length (ft)	Existing diameter (inch)	Node	Demand (Cft/s)	Min total head (ft)
1	1	2	11600	180	1	reservoir	300
2	2	3	19800	180	2	92.4	255
3	3	4	7300	180	3	92.4	255
4	4	5	8300	180	4	88.2	255
5	5	6	8600	180	5	88.2	255
6	6	7	19100	180	6	88.2	255
7	7	8	9600	132	7	88.2	255
8	8	9	12500	132	8	88.2	255
9	9	10	9600	180	9	170	255
10	11	9	11200	204	10	1	255
11	12	11	14500	204	11	170	255
12	13	12	12200	204	12	117.1	255
13	14	13	24100	204	13	117.1	255
14	15	14	21100	204	14	92.4	255
15	1	15	15500	204	15	92.4	255
16	10	17	26400	72	16	170	260
17	12	18	31200	72	17	57.5	272.8
18	18	19	24000	60	18	117.1	255
19	11	20	14400	60	19	117.1	255
20	20	16	38400	60	20	170	255
21	9	16	26400	72			

Table 9. Optimal pipe diameters obtained by the proposed methods for New York network.

Pipe	RLRM		DLRM		RLDM		DLDM	
	P250	P500	P250	P500	P250	P500	P250	P500
1	120	144	120	0	144	0	96	0
7	96	0	120	0	0	0	84	0
15	0	0	0	120	0	108	0	120
16	84	96	84	84	96	96	96	84
17	96	96	96	96	96	96	96	96
18	84	84	84	84	84	84	84	84
19	72	72	108	72	72	72	72	72
21	72	72	60	72	72	72	72	72
Cost ($M)	40.20	39.68	42.05	38.80	39.68	39.28	39.85	38.80
Evaluations	16000	19500	8000	7500	14250	29000	15500	10000
Success rate	6/10	7/10	6/10	7/10	6/10	6/10	5/10	7/10

Table 10. Optimal pipe diameters obtained by different methods for New York network.

Pipe	Present work (Best run)		Dandy et al. (1996)	Morgan and Goulter (1985)	Gessler (1982)	Savic and Walters (1997)*	
	P250	P500				GA1	GA2
1	144	0	0	0	0	0	0
7	0	0	0	144	100	104	0
8	0	0	0	0	100	0	0
15	0	120	120	0	0	0	144
16	96	84	84	96	100	96	84
17	96	96	96	96	100	96	96
18	84	84	84	84	80	84	84
19	72	72	72	60	80	72	72
21	72	72	72	84	80	72	72
Cost ($M)	39.68	38.80	38.80	39.20	41.8	37.13	40.42

* These solutions are obtained using different numerical conversion constant for the head loss equations.

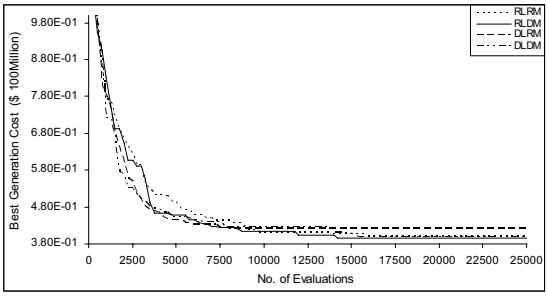

Fig. 6.1 Best of generations cost for New-York network (250 population size)

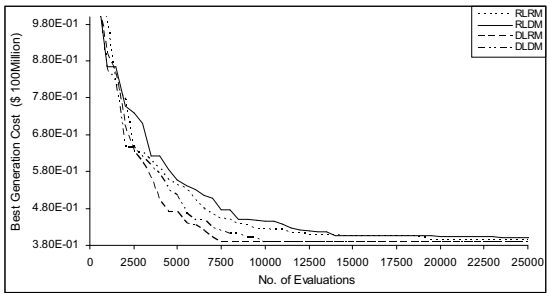

Fig. 6.2 Best of generations cost for New-York network (500 population size)

Table also shows the success rate of the methods for ten runs using different initial populations.

The second test problem that is of the Hanoi network shown in Fig. 3 with 34 pipes, 31 demand nodes and one reservoir (Abebe and Solomatine, 1999). The minimum nodal head requirement at all demand nodes is 30m. Diameters of commercially available pipes and their cost calculated based on the analytical cost function 1.1 D1.5 used in references is shown in Table 4 (Fujiwara and Khang 1990). Figs. 4.1 and 4.2 shows the best generation cost against the number of network analysis required by the proposed schemes for 250 and 500 population sizes in each generation. Table 5 shows the results produced by four selection operators for 250 and 500 population size. The selection operators DLRM and DLDM again show a sign of premature convergence exhibited in the previous example. The RLDM method produced better results than RLRM for smaller population size but failed to exhibit the same performance for larger populations. RLRM was capable of producing solutions of 6.13 and 6.1 million dollars in only 16250 and 36000 evaluations for the 250 and 500 populations, respectively. This is compared favorably with about 100,000 evaluations required by the method of Savic and Walters (1997) to get the solution of 6.07 million. Table 6 compares the best results produced by the proposed methods with some of the available random search solutions (Abebe and Solomatin, 1999; Savic and Walters, 1997). Table also shows the success rate of the methods for ten runs using different initial populations.

The third test problem concerns the rehabilitation of the New York City water supply network with 21 pipes, 20 demand nodes and one reservoir shown in Fig. 5 (Dandy et. al. 1996). Diameters of commercially available pipes and their costs are shown in Table 7 while the pipe and nodal data of the existing network is shown in Table 8. Figs. 6.1 and 6.2 shows the best generation costs against the number of network analysis required by the proposed selection schemes for 250 and 500 population sizes. Table 9 show the results produced by four selection operators for 250 and 500 population sizes. All the selection

operators performed consistently in the sense that they produced better results with larger population sizes. DLRM and DLDM operators performance was astonishingly well as they were capable of producing the cheapest ever solution wit only 7500 and 10000 evaluations, respectively. This is compared favorably with about 200,000 evaluations required by the method of Dandy et. Al. (1996) to get the solution of 38.8 million dollar. The best solution obtained by the presented methods for this problem is shown in Table 10 along with some of the available GA solutions. Table also shows the success rate of the methods for ten runs using different initial populations. A note has to be added here regarding the poor performance of the DLRM and DLDM methods in previous examples. This could be attributed to the fact that the third example has a different nature from the first two examples. This is a rehabilitation problem while the first two are not. Other experience of this kind has been reported by Dandy et al. (1996) where a gray coding has been successfully used for the rehabilitation problem.

CONCLUDING REMARKS

A family of highly convergent selection operators for genetic search of the pipe network optimal design is presented in this paper. The proposed selection operators are devised in an attempt to better simulate the process of natural mating, leading to an improvement in the accuracy and convergence characteristics of the GA search. The efficiency of the proposed methods is shown by applying the method for the optimal design of three well-known benchmark networks, namely two-loop , Hanoi and New-York networks. The method produces results comparable to the best results presented in the literature with much less computational effort.

References

[1]. Abebe, A.J. and D.P. Solomatine, 1999, 'Application of Global Optimization to the Design of Pipe Networks', 3rd Int. Conf. On Hydroinformatics, Copenhagen, balkema, Rotterdam, PP 989-996

[2]. Afshar, M. H. 2001 , 'An Element - by - Element Algorithm for the Analysis of Pipe Networks', Int. J. for Eng. Science, Vol. 12, No. 3, pp. 87-100.

[3]. Alperovits, E. and U. Shamir 1977 ,'Design of Optimal Water Distribution systems', Water Res. Research, 13(6), 885-900

[4]. Bhave, P.R., 1985, 'Optimal Expansion of Water Distribution Systems', J. Environ. Eng. N.Y., 111(2), 177-197

[5]. Brindle, A., 1981, 'Genetic Algorithms for Function Optimisation', Unpublished Doctoral Dissertation, University of Alberta, Edmonton

[6]. Dandy, G.C. , A.R. Simpson and L.J. Murphy, 1993, 'A review of pipe network optimisation techniques' Proceedings of Watercomp 93, pp. 373-383, Melboune, Australia.

[7]. Dandy, G.C. , A.R. Simpson and L.J. Murphy, 1996, 'An Improved Genetic Algorithm for Pipe Network Optimization', Water Resources Research, Vol. 32, No. 2, 449-458

[8]. Eiger, G., U. Shamir and A. Ben-Tal , 1994, 'Optimal Design of Water Distribution Networks', Water Res. Research, 30(9), 2637-2646

[9]. Fujiwara, O and D.B khang , 1990 , 'A two-phase Decomposition Method for Optimal Design of Looped Water Distribution Networks', Water Resources Research, 26 (4), 539-549

[10] Gen, M and R. Cheng, 1997 , 'Genetic Algorithms and Engineering Design', 412pp, JOHN-WILEY & SONS, New-York, USA.

[11]. Goldberg, DE. 1989 ,'Genetic Algorithms in Search, Optimization & Machine Learning', Addison- Wesley publishing co, Reading. U.K.
[12]. Gessler, J., 1982 , 'Optimization of Pipe Network', Paper presented at International Symposium on Urban Hydrology, Hydraulics and Sediment Control, Univ. of Kentucky, Lexington
[13]. Halhal, D., Walters, G.A., Quazar, D., and Savic, D.A., 1997, 'Water network rehabilitation with structured messy genetic algorithm. ' J. Wat. Resour. Plng. and Mgmt, ASCE 123(3), 137-146.
[14]. Morgan, D.R., and I.C. Goulter, 1985, 'Optimal Urban Water Distribution Design', Water Ressources Research, 21(5), 642-652
[15]. Murphy, L. J., Simpson, A. R., and Dandy, G. C., 1993, 'Design of a network using genetic algorithms. ' Water 20, 40-42.
[16]. Savic, D.A. and G. A.Waters, 1997 ,'Genetic Algorithms for Least-Cost Design of Water Distribution Networks', Water Res. Planning and Management, ASCE ,123 (2) 67-77
[17]. Simpson, A.R. and D.E. Goldberg , 1993 , 'Pipeline Optimization Using an Improved Genetic Algorithm , Dept. of Civil and Env. Engg , University of Adelaide, Research Report No. R109 , Adelaide, Australia
[18]. Simpson, A, R. , G.C. Dandy and L.J. Murphy, 1994 ,'Genetic Algorithms Compared to Other Techniques for Pipe Optimization', Water Res. Planning and Management, ASCE, 120 (4), 423-443
[19]. Walters, G. A., Halhal, D., Savic, D., and Quazar, D., 1999, 'Improved design of anytown distribution network using structured messy genetic algorithms. ' Urban Water 1(1), 23-38.
[20]. Wetzel, A., 1983, 'Evaluation of the Effectiveness of Genetic Algorihms in Combinatorial Optimisation' , Unpublished Manuscript, University of Pitsburg, Pitsburg